DEVIL PHYSICS
BADDEST CLASS ON CAMPUS

IB PHYSICS

OPTION B-1B:
ROTATIONAL DYNAMICS

Essential Idea:

- The basic laws of mechanics have an extension when equivalent principles are applied to rotation. Actual objects have dimensions and they require the expansion of the point particle model to consider the possibility of different points on an object having different states of motion and/or different velocities.

Nature Of Science:

- Modelling: The use of models has different purposes and has allowed scientists to identify, simplify and analyse a problem within a given context to tackle it successfully. The extension of the point particle model to actually consider the dimensions of an object led to many groundbreaking developments in engineering.

Theory Of Knowledge:

- Models are always valid within a context and they are modified, expanded or replaced when that context is altered or considered differently.
- Are there examples of unchanging models in the natural sciences or in any other areas of knowledge?

Understandings:

- Torque
- Moment of inertia
- Rotational and translational equilibrium
- Angular acceleration
- Equations of rotational motion for uniform angular acceleration
- Newton’s second law applied to angular motion
- Conservation of angular momentum

Applications And Skills:

- Calculating torque for single forces and couples
- Solving problems involving moment of inertia, torque and angular acceleration
- Solving problems in which objects are in both rotational and translational equilibrium

Applications And Skills:

- Solving problems using rotational quantities analogous to linear quantities
- Sketching and interpreting graphs of rotational motion
- Solving problems involving rolling without slipping

Guidance:

- Analysis will be limited to basic geometric shapes
- The equation for the moment of inertia of a specific shape will be provided when necessary
- Graphs will be limited to angular displacement–time, angular velocity–time and torque–time

Data Booklet Reference:

- $\Gamma = Fr\sin\theta$
- $I = mr^2$
- $\Gamma = I\alpha$
- $\omega = 2\pi f$
- $\omega_f = \omega_i + \alpha t$
- ${\omega_f}^2 = {\omega_i}^2 + 2\alpha\theta$
- $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
- $L = I\omega$
- $E_{K_{rot}} = \frac{1}{2}I\omega^2$

Utilization:

- Structural design and civil engineering rely on the knowledge of how objects can move in all situations

Aims:

- Aim 7: technology has allowed for computer simulations that accurately model the complicated outcomes of actions on bodies

REVIEW

Overview of Rotational Equations of Motion

Kinematics

Linear

Position: x (meters)

Velocity: $v = \frac{\Delta x}{\Delta t}$ (m/s)

Linear Acceleration: $a = \frac{\Delta v}{\Delta t}$

(m/s²)

Radial Acceleration: $a_r = \frac{v^2}{r}$

Rotational

Position: θ (radians)

Angular Velocity: $\omega = \frac{\Delta \theta}{\Delta t}$ (rad/s)

$v_{tan} = \omega r$

Angular Accl: $\alpha = \frac{\Delta \omega}{\Delta t}$ (rad/s²)

$a_{tan} = \alpha r$

$a_r = \omega^2 r$

Kinematics

Linear

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{u+v}{2}t$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Rotational

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{\omega_i + \omega_f}{2}t$$

$$\omega_f = \omega_i + \alpha t$$

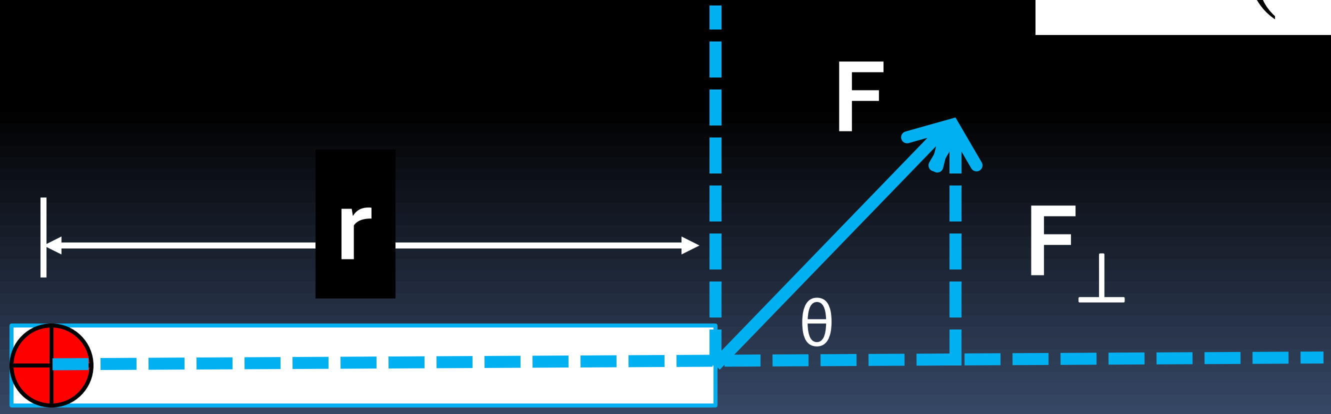
$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha s$$

Torque

- However, only the component of the force perpendicular to the moment arm produces torque

$$\Gamma = F_{\perp} r$$

$$\Gamma = (F \sin\theta) r$$



Equilibrium

- Translational
  - No movement in any direction or movement in a constant direction at constant velocity
  - The net ***force*** on the body treated as a point mass is equal to zero.
  - The net force acting on the center of mass is zero.

$$\Sigma F = F_{net} = 0$$

Equilibrium

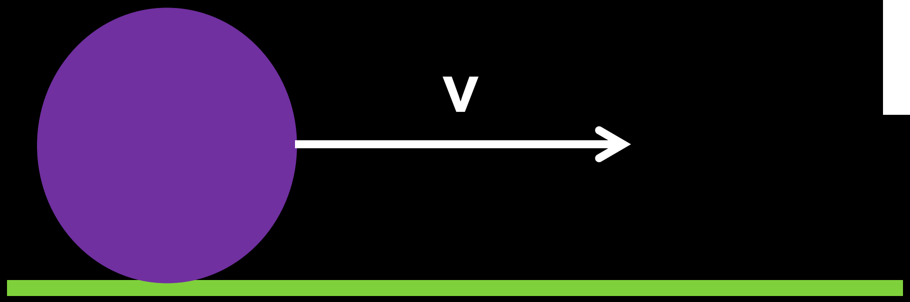
- Rotational
  - No rotation of the body
  - The net ***torque*** on the body is equal to zero.

$$\Sigma\Gamma = \Gamma_{net} = 0$$

Kinetic Energy of a Body

- Translational

$$KE = \frac{1}{2}mv^2$$



Kinetic Energy of a Body

- Rotational

$$KE = \frac{1}{2} m_1 \omega^2 r_1{}^2 + \frac{1}{2} m_1 \omega^2 r_2{}^2 + \ldots$$

$$KE = \frac{1}{2}\left(\Sigma m_i r_i{}^2\right)\omega^2$$

- ***There has to be a better way***

Moment of Inertia

$$I = \Sigma m_i r_i^2$$

- This works for a single point mass, but not much else

Moment of Inertia

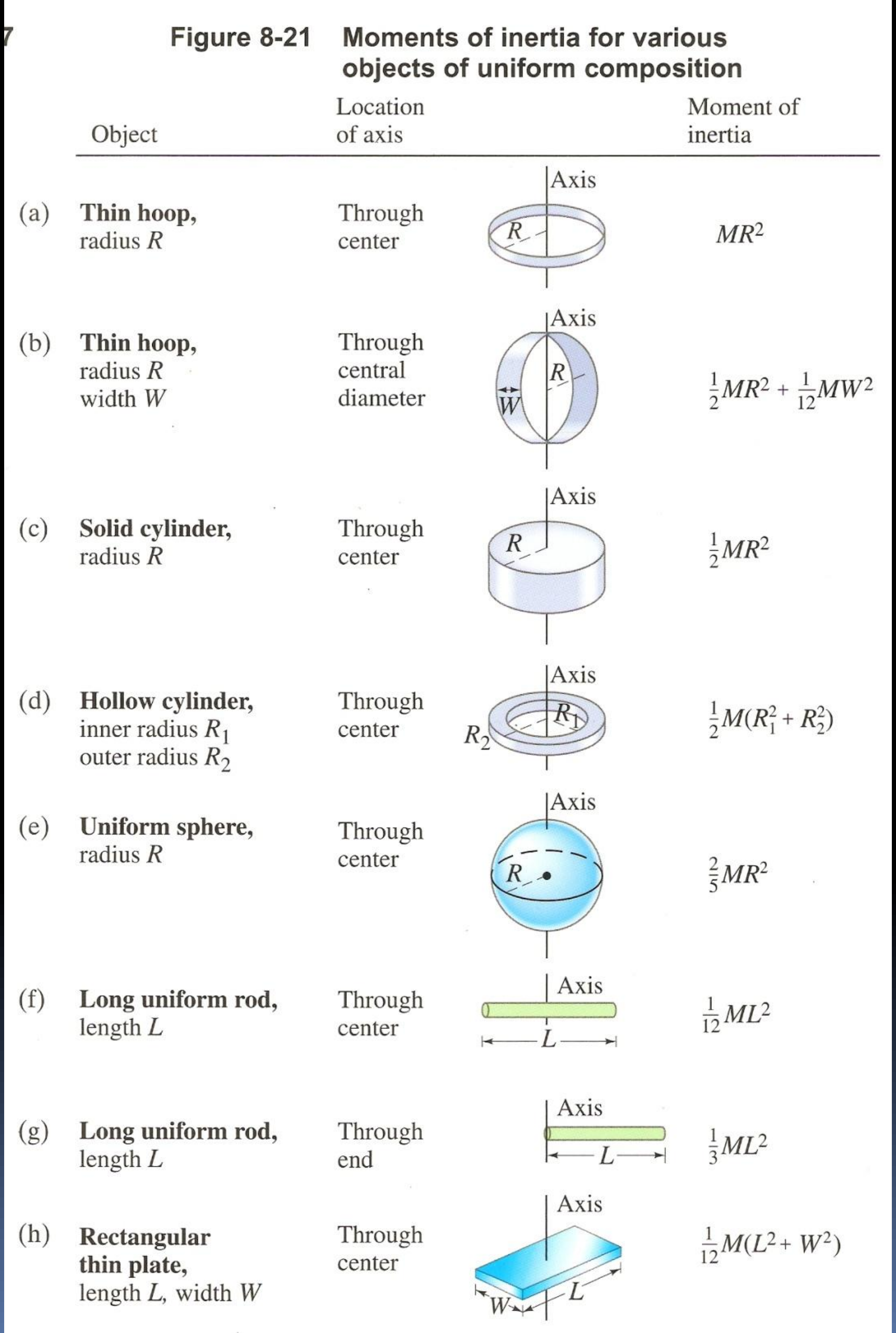


Figure 8-21 Moments of inertia for various objects of uniform composition

	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R	Through center	MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter	$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center	$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center	$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center	$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end	$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center	$\frac{1}{12}M(L^2 + W^2)$

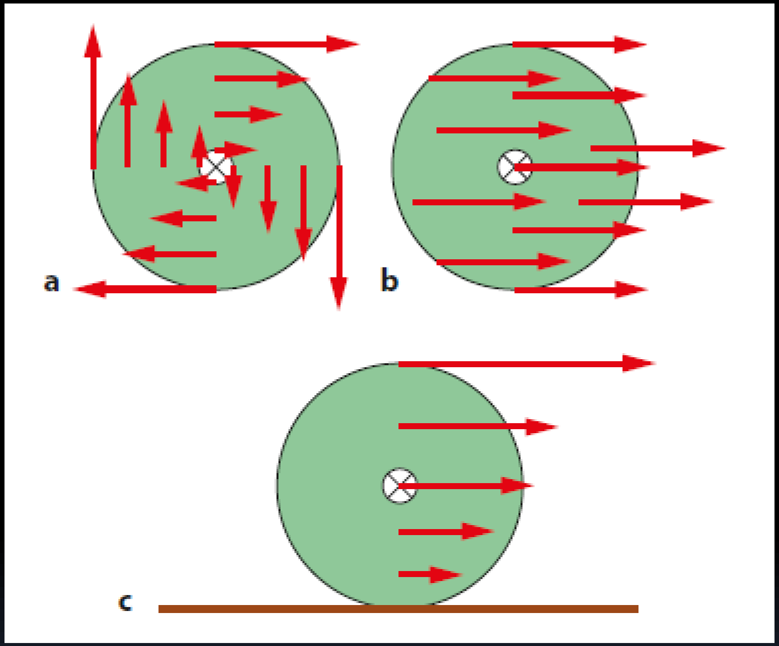
Kinetic Energy of a Body

- Rotational

$$KE = \frac{1}{2} I \omega^2$$

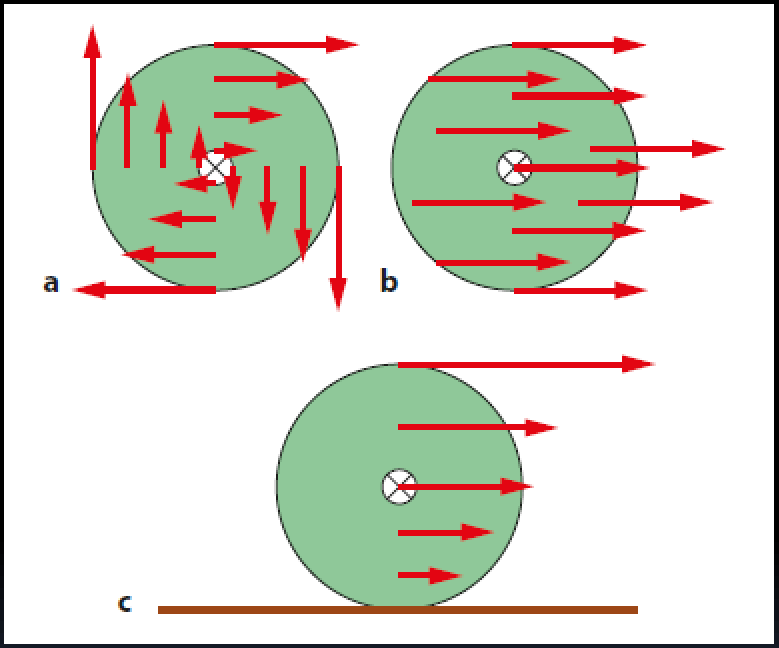
Rolling Motion (Without Slipping)

- Three Cases:
  a. Rotation
  b. Sliding
  c. Rolling without slipping



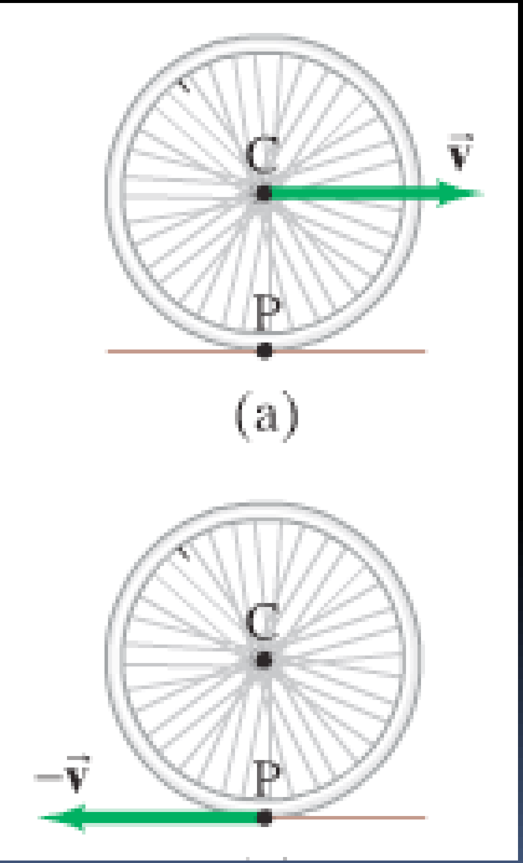
Rolling Motion (Without Slipping)

- Three Cases:
  a. Rotation
  b. Sliding
  c. Rolling without slipping
- ***Depends on where the force is applied and the relationship between torque and friction***



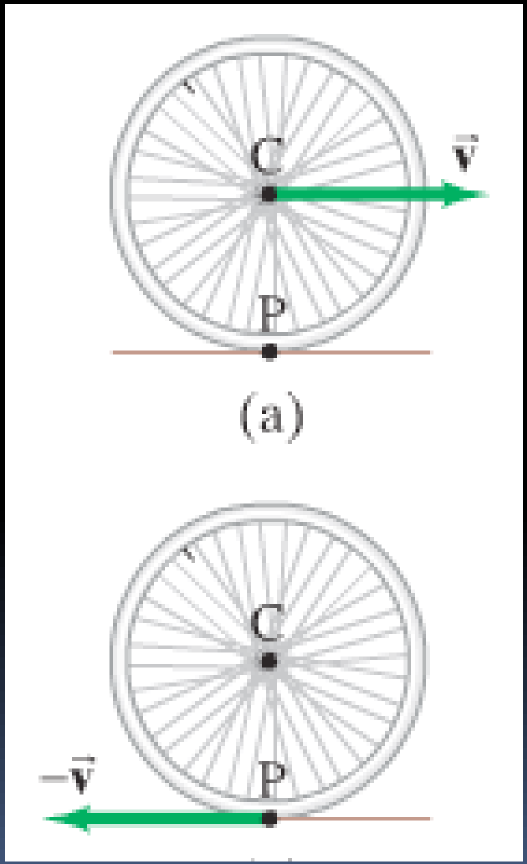
Rolling Motion (Without Slipping)

- Rolling motion involves translational and rotational motion
- In the diagram, the axis of rotation (C) is exhibiting translational motion
- Meanwhile, all other points on the body of the wheel are exhibiting clockwise rotational motion



Rolling Motion (Without Slipping)

- ***What will be the tangential velocity at point P?***

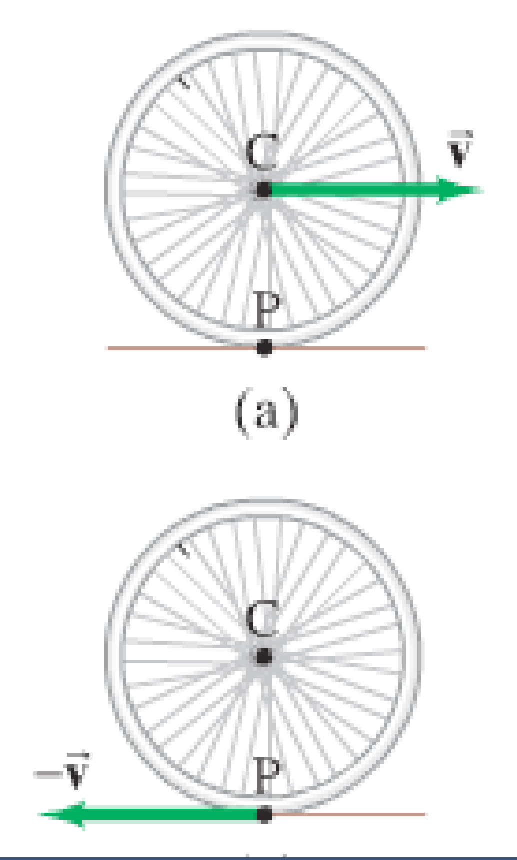


Rolling Motion (Without Slipping)

- ***What will be the tangential velocity at point P?***

$$v = 0$$

- ***What will be the tangential velocity at the top of the wheel?***



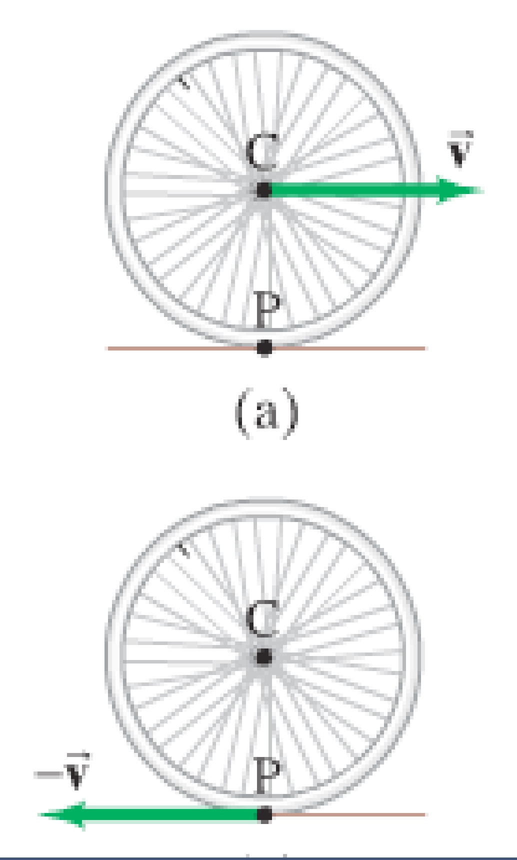
Rolling Motion (Without Slipping)

- ***What will be the tangential velocity at point P?***

$$v = 0$$

- ***What will be the tangential velocity at the top of the wheel?***

$$v = 2r\omega$$

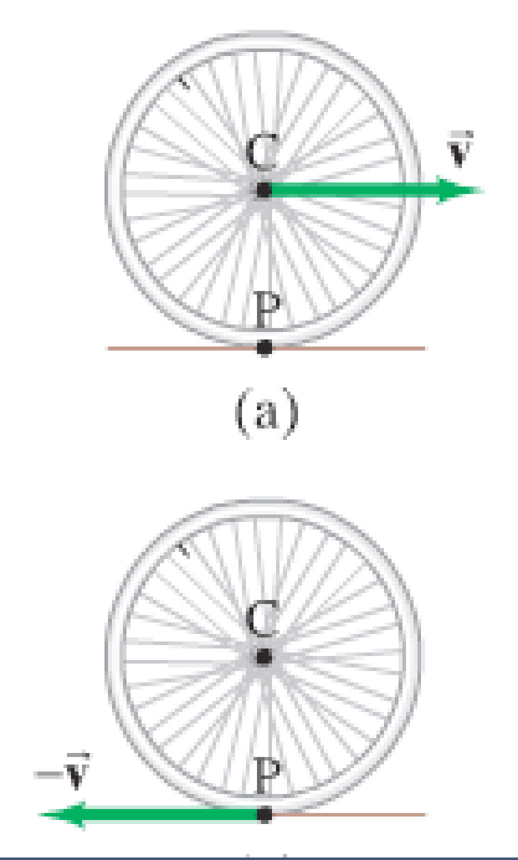


Rolling Motion (Without Slipping)

- The linear speed of the wheel will be

$$v = r\omega$$

- This equation is only valid if there is no slipping of the wheel on the surface
- No slipping depends on a high enough friction force between the wheel and the surface



Rotational Kinetic Energy

- Linear Kinetic Energy is $$KE = 1/2mv^2$$
- To convert tangential (linear) velocity to angular velocity $$v = r\omega$$
- Rotational kinetic energy is then $$KE = 1/2mr^2\omega^2$$
- For the entire mass it is $$KE = 1/2\left(\Sigma mr^2\right)\omega^2$$

Rotational Kinetic Energy

- Kinetic energy for the entire mass it is

$$KE = 1/2\left(\Sigma mr^2\right)\omega^2$$

- But, the moment of inertia is

$$I = \Sigma mr^2$$

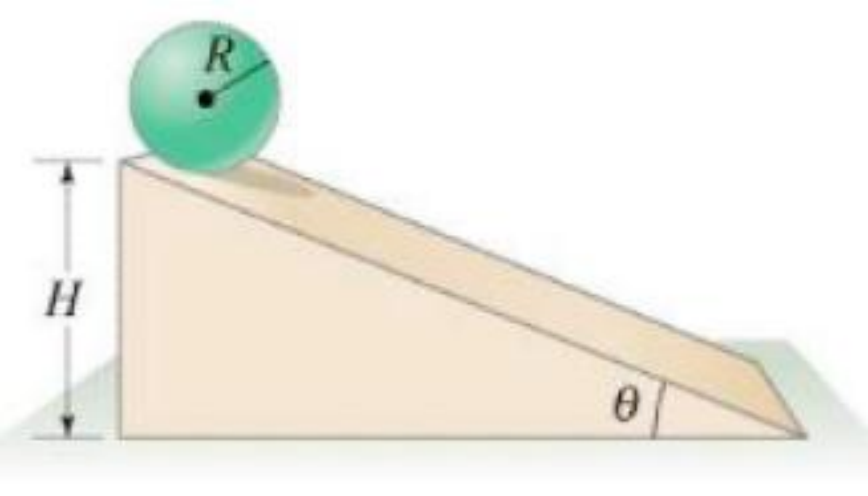
- So, kinetic energy in terms of the moment of inertia is

$$KE = 1/2 I\omega^2$$

Total Kinetic Energy

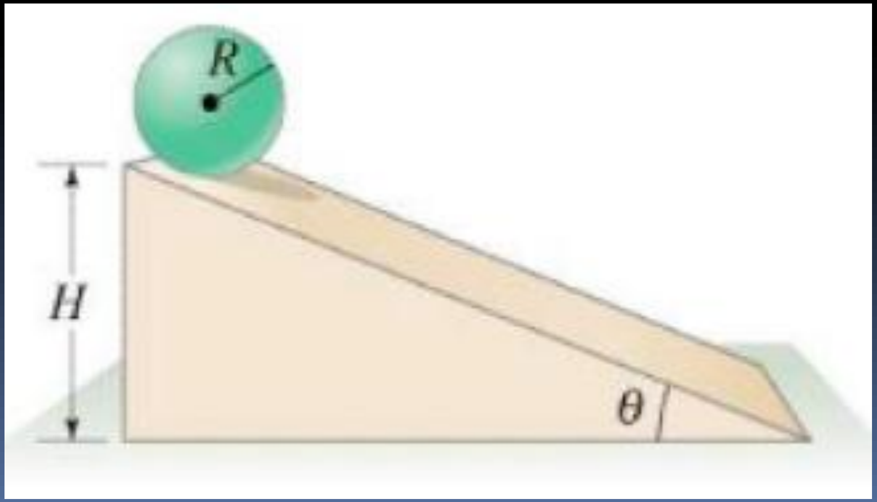
- But what happens if some-thing has both rotational and translational motion
- This is when depression sets in
- The total kinetic energy is the sum of the translational kinetic energy and the rotational kinetic energy

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$



Sample Problem

The height of the ramp is 3m, the mass of the ball is 3kg, and the radius of the ball is 0.3m. What is the velocity of the ball at the bottom of the ramp?

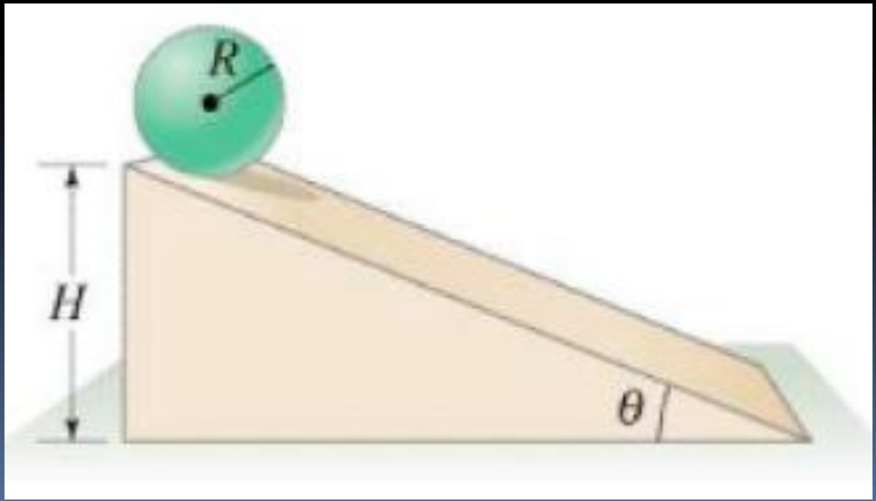


Sample Problem

The height of the ramp is 3m, the mass of the ball is 3kg, and the radius of the ball is 0.3m. What is the velocity of the ball at the bottom of the ramp?

It depends

Which velocity, angular or linear?



Sample Problem

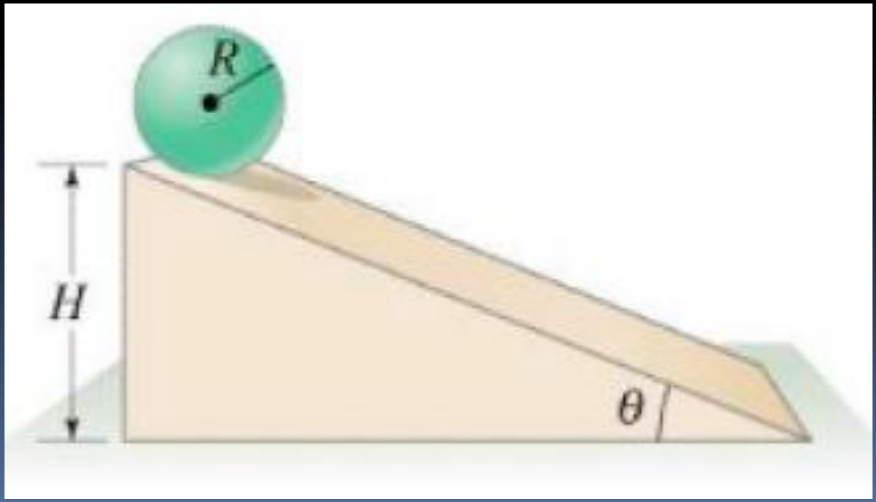
The height of the ramp is 3m, the mass of the ball is 3kg, and the radius of the ball is 0.3m. What is the velocity of the ball at the bottom of the ramp?

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$v = r\omega$$

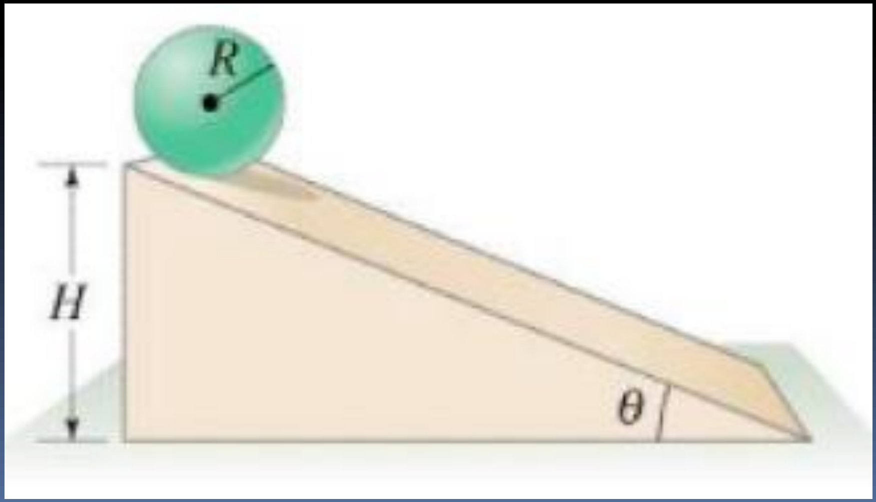
$$KE = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}I_{cm}\omega^2$$

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v}{r}\right)^2$$



Question 1?

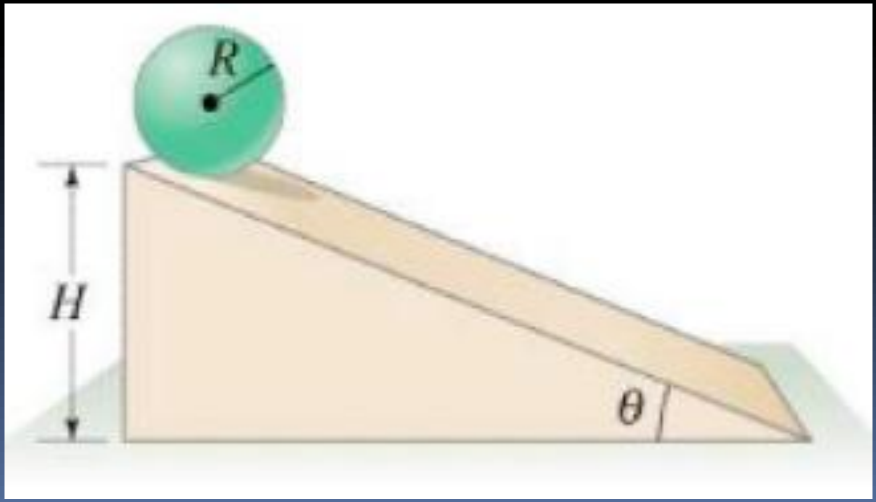
How would the situation change if I told you that there was a high coefficient of friction between the ball and the ramp?



Question 1?

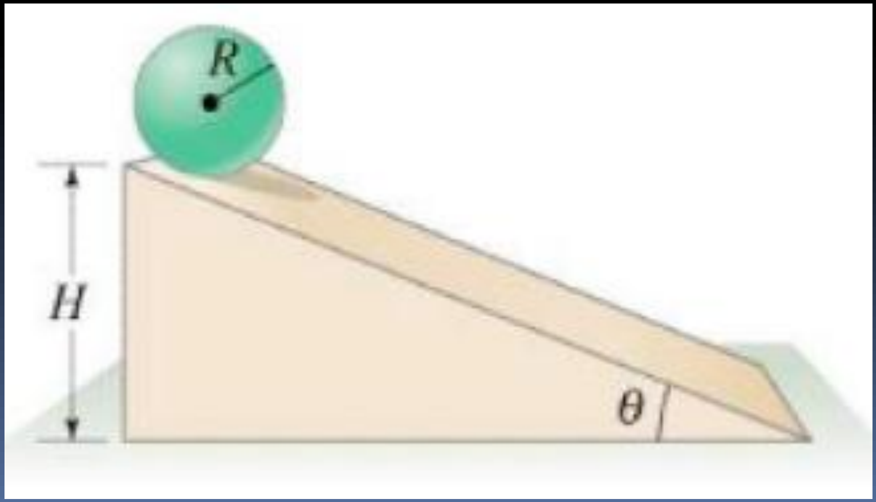
How would the situation change if I told you that there was a high coefficient of friction between the ball and the ramp?

Not at all. In fact the situation assumes a high coefficient of friction to make the ball roll instead of slide.



Question 2?

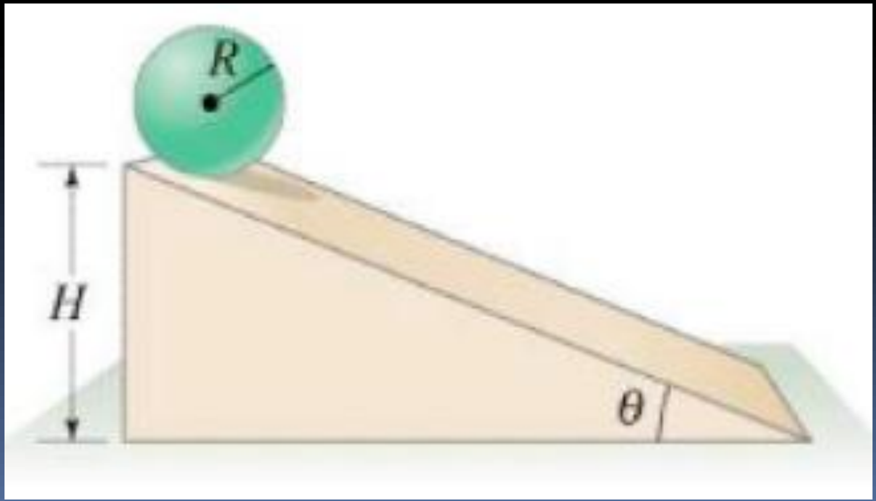
How would the situation change if I told you that the ramp was frictionless?



Question 2?

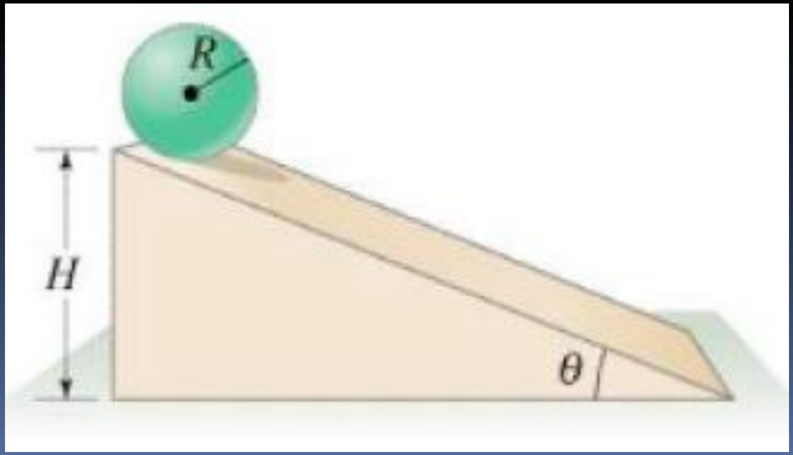
How would the situation change if I told you that the ramp was frictionless?

The ball would slide instead of rolling. There would be no rotational kinetic energy, just translational – the same as a box sliding down a ramp.



Question 3?

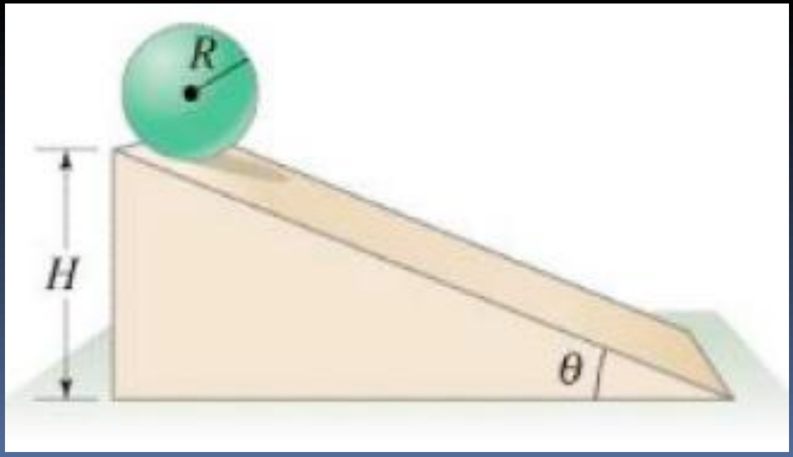
Which would be going faster at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?



Question 3?

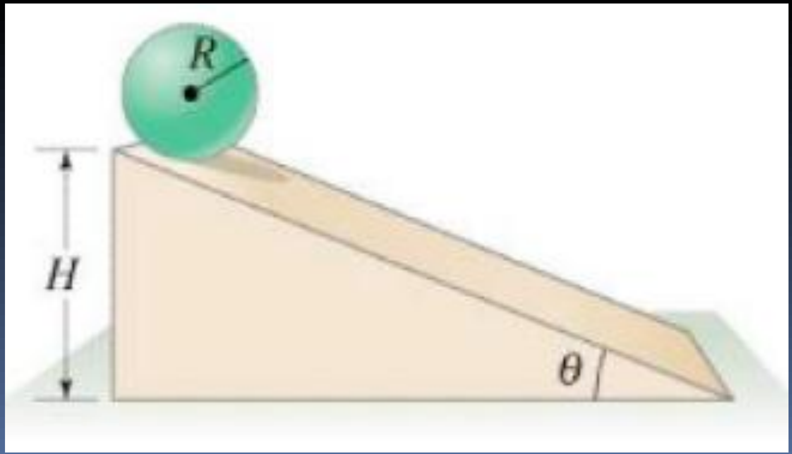
Which would be going faster at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?

The ball on the frictionless ramp. In this case, all of the potential energy is converted into translational kinetic energy. On a ramp with friction, some of the PE must be translated into rotational KE.



Work Done By Friction

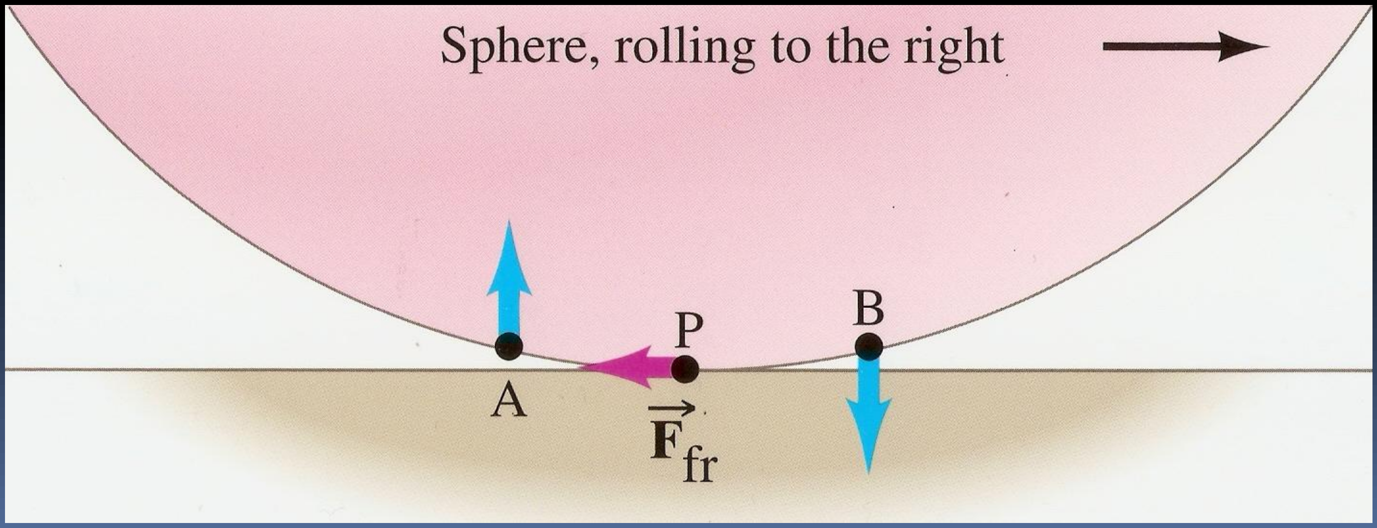
Speaking of friction, how do you calculate the work done by friction on a ball sliding down a ramp?



Work Done By Friction

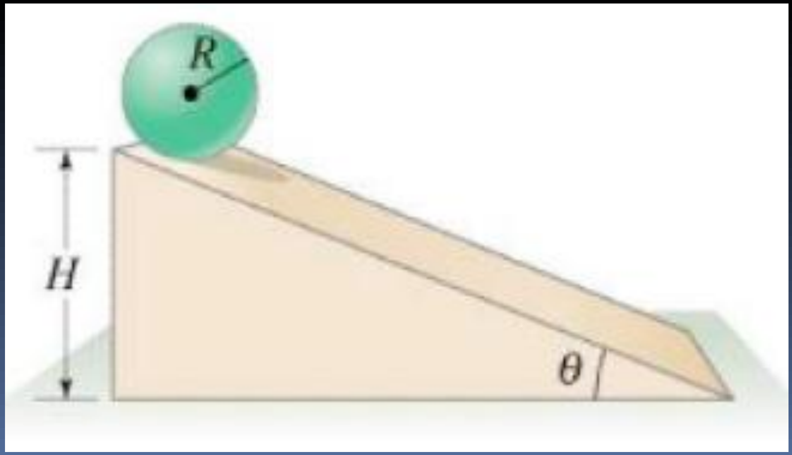
Speaking of friction, how do you calculate the work done by friction on a ball sliding down a ramp?

Trick question – you don't – there is no work done by friction. At the point of contact, the surface of the ball moves up, perpendicular to the force of friction. Force must be in direction of motion to do work.



Conservation of Energy

If a ramp has friction, is energy conserved when a ball slides down a ramp?

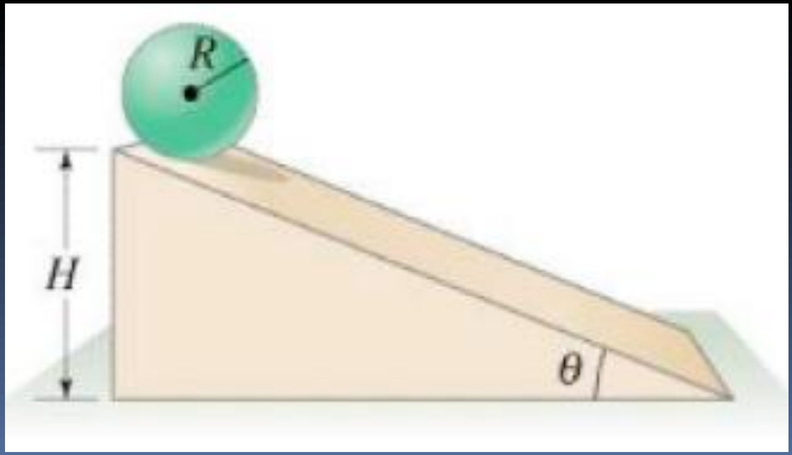


Conservation of Energy

If a ramp has friction, is energy conserved when a ball slides down a ramp?

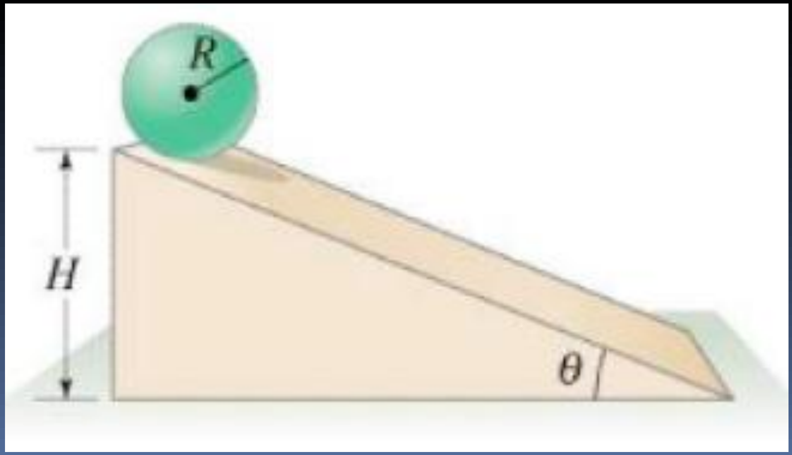
It depends

Total energy is always conserved. Mechanical energy is not conserved because friction converts some of the potential energy to heat.



Conservation of Energy

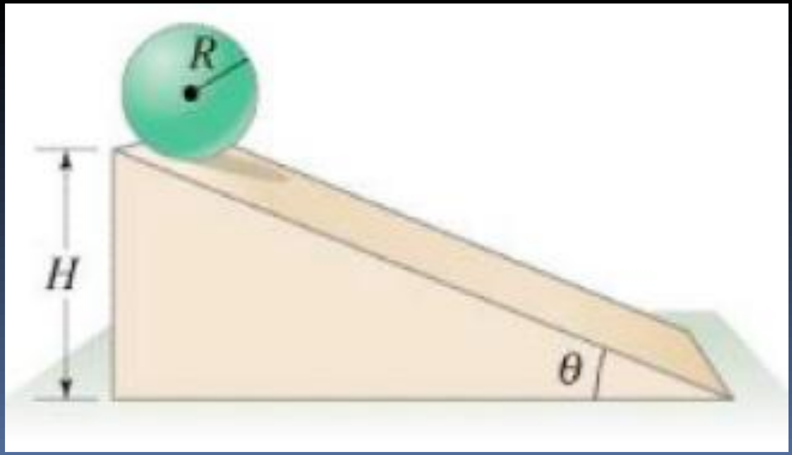
If a ramp has friction, is energy conserved when a ball rolls down a ramp without slipping?



Conservation of Energy

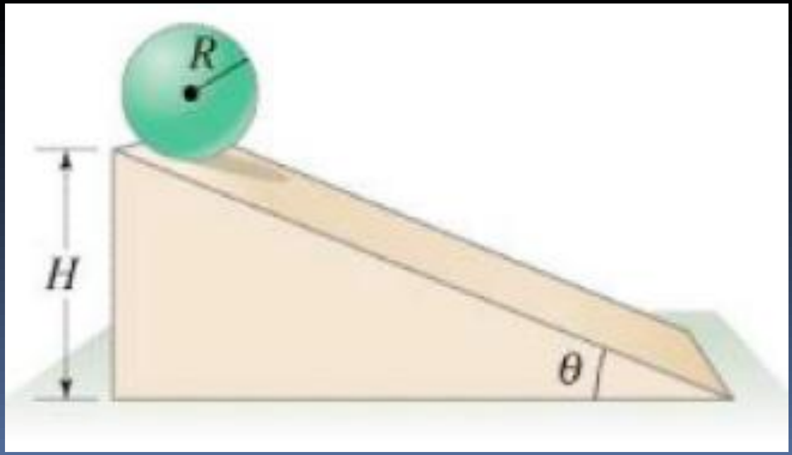
If a ramp has friction, is energy conserved when a ball rolls down a ramp without slipping?

Definitely. Since friction does no work when rolling without slipping, mechanical energy is conserved in the sum of rotational and translational kinetic energy.



Conservation of Energy

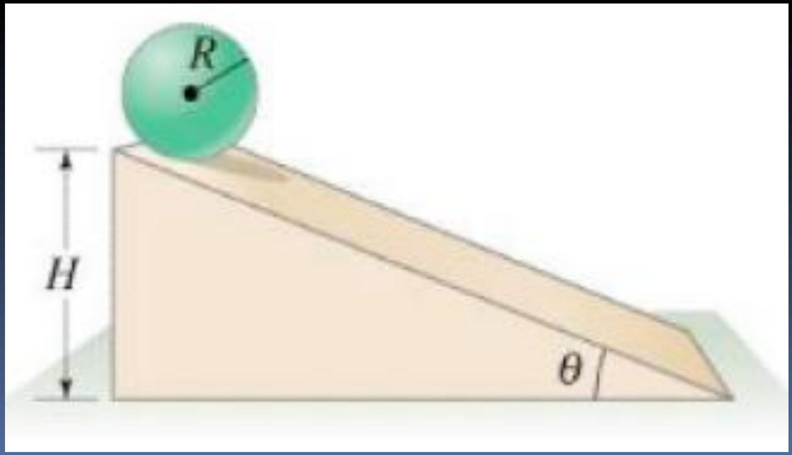
If a ball rolls down a ramp without slipping, does it gather any moss?



Conservation of Energy

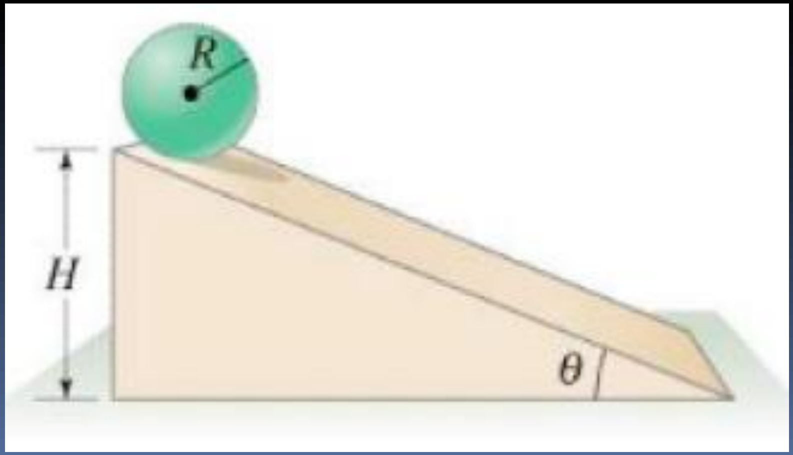
If a ball rolls down a ramp without slipping, does it gather any moss?

Definitely not. Everyone knows that a rolling stone (or bowling ball) gathers no moss.



Work Done By Torque

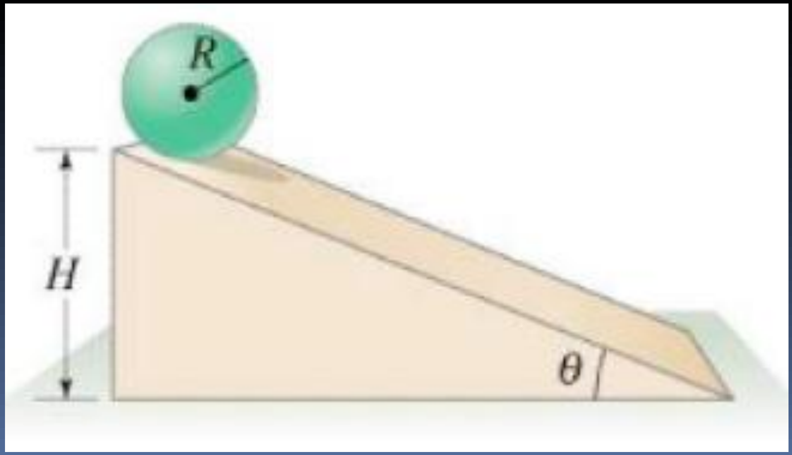
Does torque do work?



Work Done By Torque

Does torque do work?

Yes

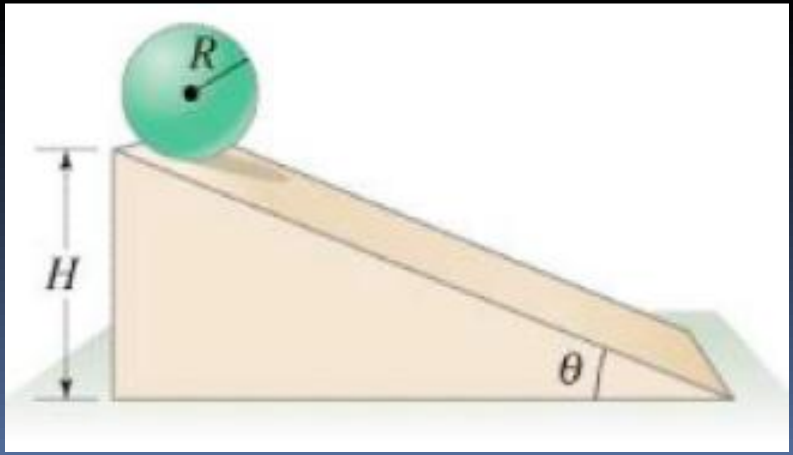


Work Done By Torque

Does torque do work?

Yes

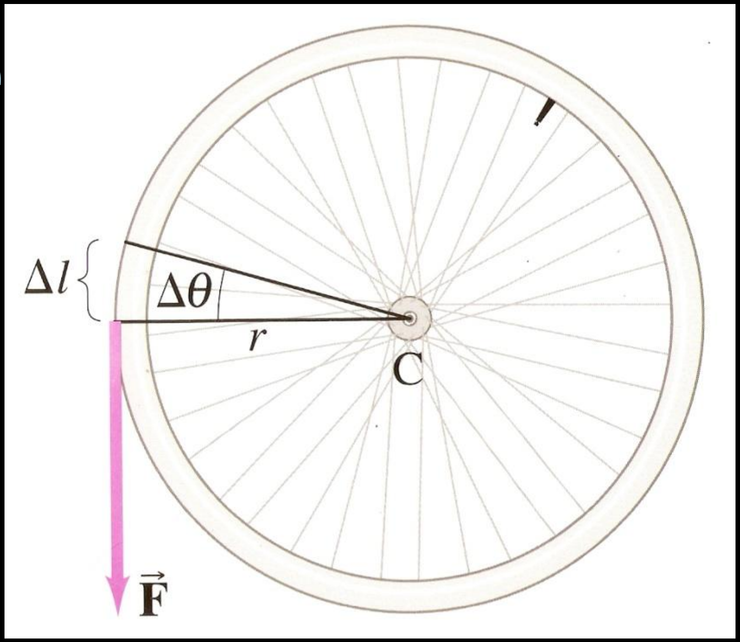
Oh, you want an explanation?



Work Done By Torque

- Torque is force times a moment arm

$$\tau = Fr$$



- Work is force times distance

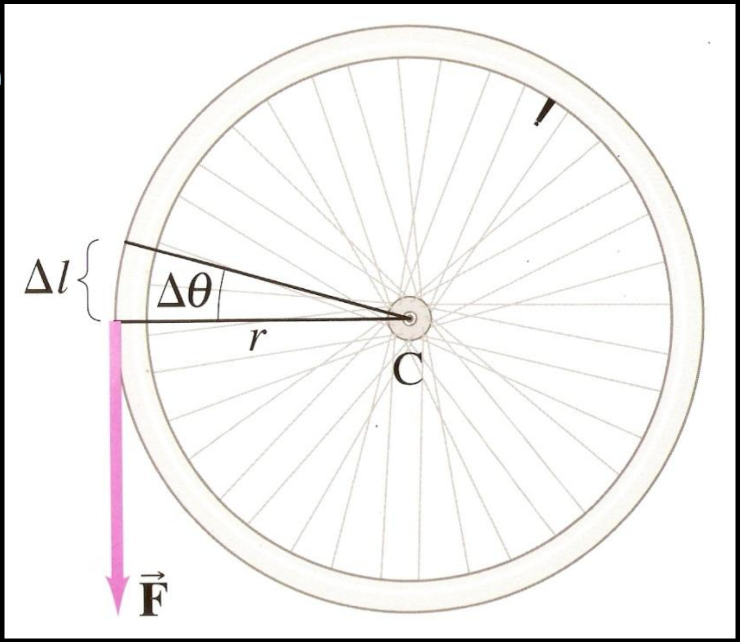
$$W = Fd$$

- A small amount of rotation (small Δθ) will produce a Δl and we know

$$\Delta\theta = \frac{\Delta l}{r}$$

$$r\Delta\theta = \Delta l$$

Work Done By Torque



- Torque is force times a moment arm

$$\tau = Fr$$

- Work is force times distance

$$W = Fd$$

- For small changes in θ, the force can be considered in the direction of motion so torque does work

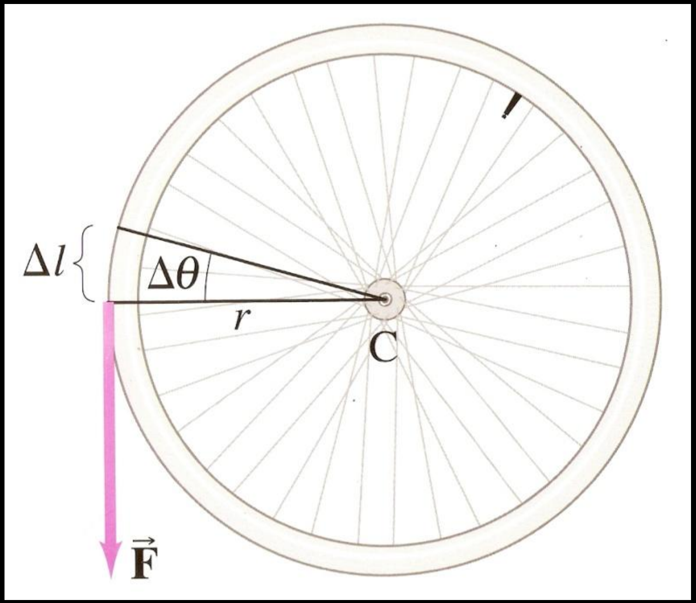
$$\Delta\theta = \frac{\Delta l}{r}$$

$$r\Delta\theta = \Delta l$$

Work Done By Torque

- Torque is force times a moment arm

$$\tau = Fr$$



- Work is force times distance

$$W = Fd$$

- Using the marvels of Algebra,

$$r\Delta\theta = \Delta l$$

$$W = Fr\Delta\theta$$

$$\tau = Fr$$

$$W = \tau\Delta\theta$$

Power Generated By Torque

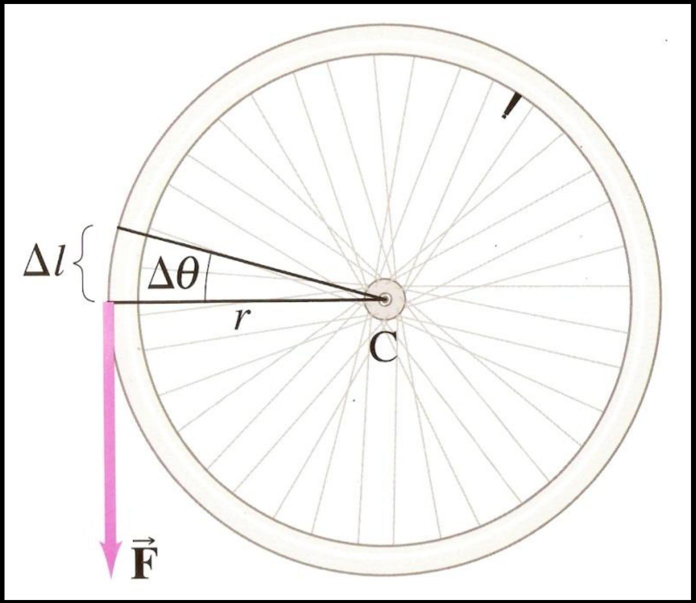
- Power is work divided by time so,

$$W = \tau \Delta \theta$$

$$P = \frac{W}{t} = \frac{\tau \Delta \theta}{t}$$

$$\omega = \frac{\Delta \theta}{t}$$

$$P = \tau \omega$$

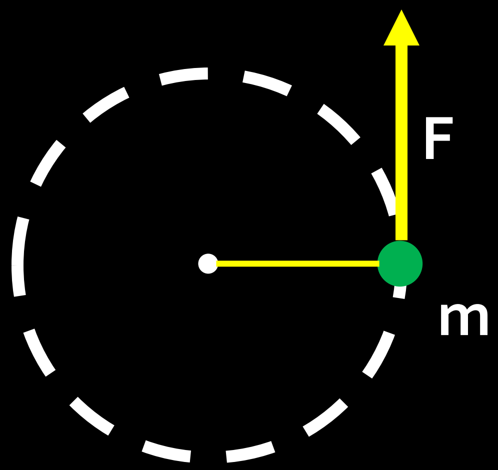


Work, Energy and Power

Linear motion	Rotational motion
$F = ma$	$\Gamma = I\alpha$
$W = F\,\Delta s$	$W = \Gamma\,\Delta\theta$
$P = Fv$	$P = \Gamma\omega$

Moment of Inertia

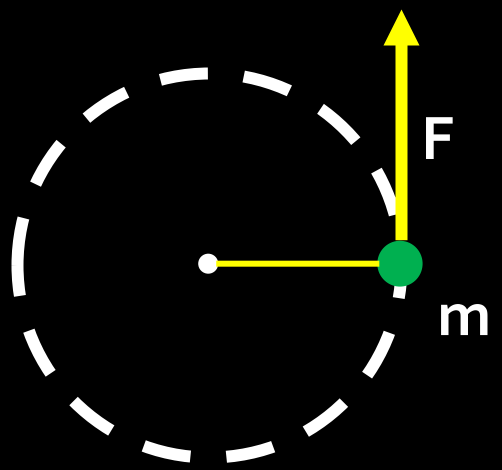
- Angular acceleration is proportional to the net torque applied to it
  - Force along a moment arm is torque
  - More force means more acceleration



$$\alpha \propto \sum \tau$$

Moment of Inertia

- Everything in life relates to Newton's Second Law



$$\Sigma F = ma$$

$$a_{\tan} = r\alpha$$

$$F = mr\alpha$$

$$\tau = rF$$

$$\frac{\tau}{r} = F$$

$$\frac{\tau}{r} = mr\alpha$$

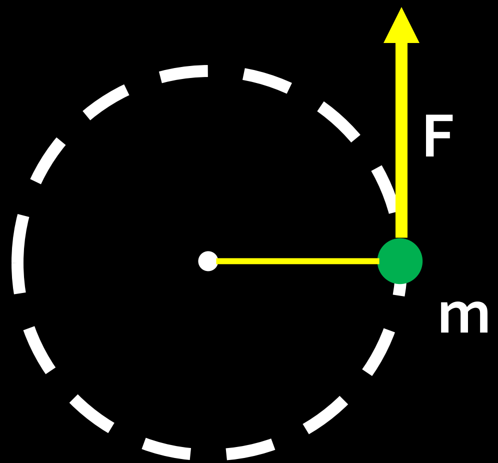
$$\tau = mr^2\alpha$$

$$\tau = \left(\Sigma mr^2\right)\alpha$$

Moment of Inertia

- Moment of Inertia defined:

$$I = \Sigma m r^2$$



Note that the moment of inertia is much more affected by distance than by mass.

Determination of the axis of rotation (reference point) is critical!

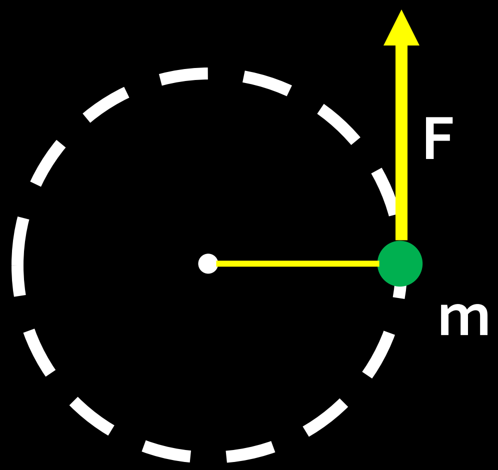
Moment of Inertia

- Moment of Inertia defined:

$$\tau = \left(\Sigma m r^2\right)\alpha$$

$$I = \Sigma m r^2$$

$$\tau = I\alpha$$



- *This is Newton's Second Law for rotation!*

ANGULAR MOMENTUM

Angular Momentum

Angular Momentum

- Linear momentum

$$p = mv$$

- Angular momentum (L)
  - I = moment of inertia
  - ω = angular velocity

$$L = I\omega$$

Angular Momentum

- Newton's Second Law
  - In terms of linear momentum

$$\Sigma F = ma$$

$$\Sigma F = \frac{\Delta p}{\Delta t}$$

Angular Momentum

- Newton's Second Law
  - In terms of angular momentum and re-written in terms of torque
  - $\Sigma\tau$ = net torque acting to rotate the object
  - ΔL = change in angular momentum
  - Δt = time interval

$$L = I\omega$$

$$\Sigma\tau = I\alpha$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Sigma\tau = \frac{I\omega}{t}$$

$$\Sigma\tau = \frac{\Delta L}{\Delta t}$$

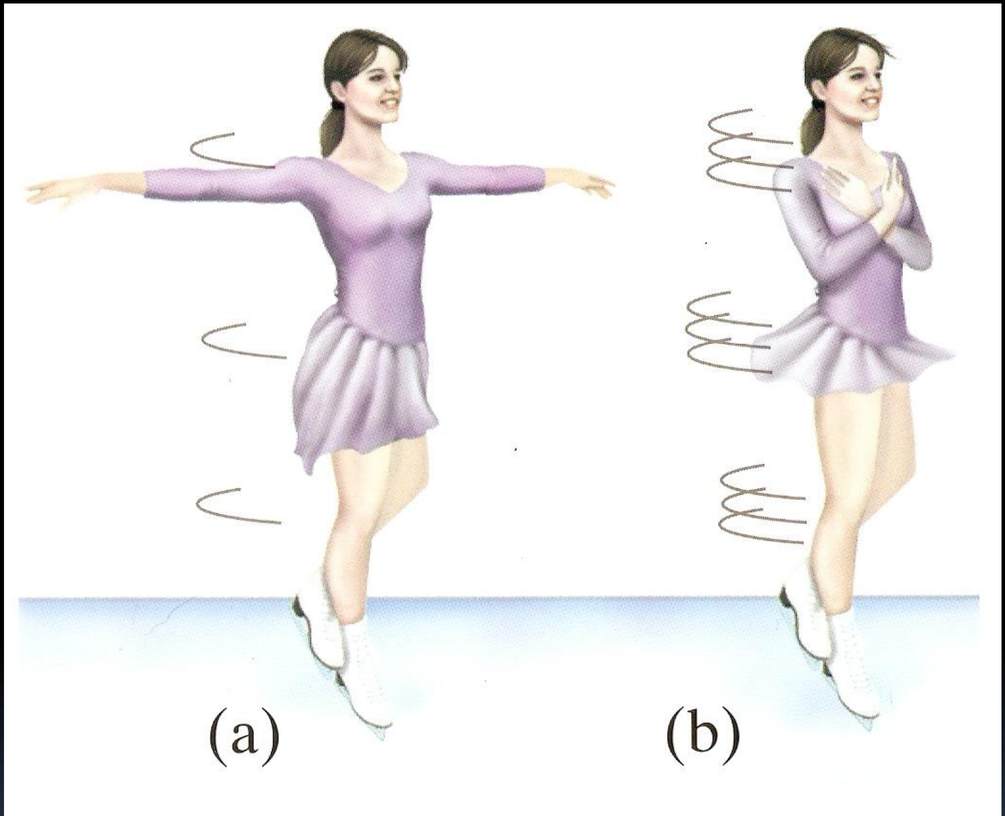
Conservation of Angular Momentum

- The total angular momentum of a rotating object remains constant if the net torque acting on it is zero

$$I\omega = I_0\omega_0 = constant$$

Conservation of Angular Momentum

- *What happens when a spinning figure skater extends and retracts her arms?*

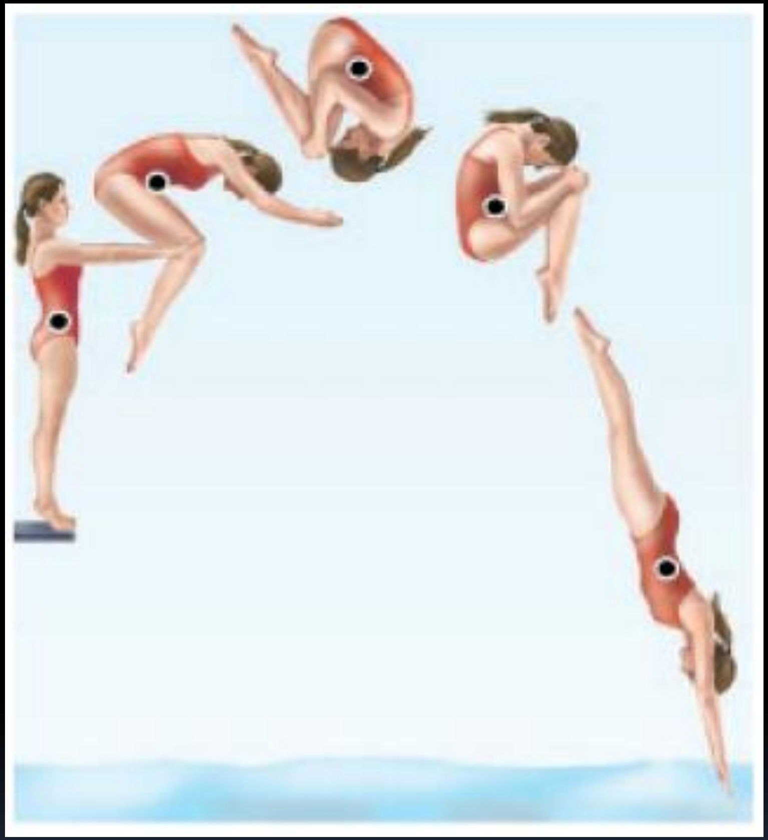


$$I\omega = I_0\omega_0$$

$$I = mr^2$$

Conservation of Angular Momentum

- *Why does a 2 ½ in the pike position have a higher degree of difficulty than the same dive in the tuck position?*



$$I\omega = I_0\omega_0$$

$$I = mr^2$$

Conservation of Angular Momentum

- *For angular momentum to be conserved,*
  a) *Net torque must be 0*
  b) *Net force must be 0*
  c) *Both a and b*
  d) *Neither a nor b*
  e) *It depends*

$$I\omega = I_0\omega_0$$

$$I = mr^2$$

Conservation of Angular Momentum

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$$I\omega = I_0\omega_0$$

$$I = mr^2$$

Summary Video: The Mechanical Universe-Angular Momentum

Guidance:

- Analysis will be limited to basic geometric shapes
- ***The equation for the moment of inertia of a specific shape will be provided when necessary***
- Graphs will be limited to angular displacement–time, angular velocity–time and torque–time

Understandings:

- Torque
- Moment of inertia
- Rotational and translational equilibrium
- Angular acceleration
- Equations of rotational motion for uniform angular acceleration
- Newton’s second law applied to angular motion
- Conservation of angular momentum

Data Booklet Reference:

- $\Gamma = Fr\sin\theta$
- $I = mr^2$
- $\Gamma = I\alpha$
- $\omega = 2\pi f$
- $\omega_f = \omega_i + \alpha t$
- ${\omega_f}^2 = {\omega_i}^2 + 2\alpha\theta$
- $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
- $L = I\omega$
- $E_{K_{rot}} = \frac{1}{2}I\omega^2$

Guidance:

- Analysis will be limited to basic geometric shapes
- The equation for the moment of inertia of a specific shape will be provided when necessary
- Graphs will be limited to angular displacement–time, angular velocity–time and torque–time

Applications And Skills:

- Calculating torque for single forces and couples
- Solving problems involving moment of inertia, torque and angular acceleration
- Solving problems in which objects are in both rotational and translational equilibrium

Applications And Skills:

- Solving problems using rotational quantities analogous to linear quantities
- Sketching and interpreting graphs of rotational motion
- Solving problems involving rolling without slipping

Essential Idea:

- The basic laws of mechanics have an extension when equivalent principles are applied to rotation. Actual objects have dimensions and they require the expansion of the point particle model to consider the possibility of different points on an object having different states of motion and/or different velocities.

Devil Physics
IB Physics

Questions?

HOMEWORK #8-15