Devil Physics
Baddest Class on Campus

IB Physics

OPTION B-1A: ROTATIONAL DYNAMICS

Essential Idea:

- The basic laws of mechanics have an extension when equivalent principles are applied to rotation. Actual objects have dimensions and they require the expansion of the point particle model to consider the possibility of different points on an object having different states of motion and/or different velocities.

Nature Of Science:

- Modelling: The use of models has different purposes and has allowed scientists to identify, simplify and analyse a problem within a given context to tackle it successfully. The extension of the point particle model to actually consider the dimensions of an object led to many groundbreaking developments in engineering.

Theory Of Knowledge:

- Models are always valid within a context and they are modified, expanded or replaced when that context is altered or considered differently.
- Are there examples of unchanging models in the natural sciences or in any other areas of knowledge?

Understandings:

- Torque
- Moment of inertia
- Rotational and translational equilibrium
- Angular acceleration
- Equations of rotational motion for uniform angular acceleration
- Newton’s second law applied to angular motion
- Conservation of angular momentum

Applications And Skills:

- Calculating torque for single forces and couples
- Solving problems involving moment of inertia, torque and angular acceleration
- Solving problems in which objects are in both rotational and translational equilibrium

Applications And Skills:

- Solving problems using rotational quantities analogous to linear quantities
- Sketching and interpreting graphs of rotational motion
- Solving problems involving rolling without slipping

Guidance:

- Analysis will be limited to basic geometric shapes
- The equation for the moment of inertia of a specific shape will be provided when necessary
- Graphs will be limited to angular displacement–time, angular velocity–time and torque–time

Data Booklet Reference:

- $\Gamma = Fr\sin\theta$
- $I = mr^2$
- $\Gamma = I\alpha$
- $\omega = 2\pi f$
- $\omega_f = \omega_i + \alpha t$
- ${\omega_f}^2 = {\omega_i}^2 + 2\alpha\theta$
- $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
- $L = I\omega$
- $E_{K_{rot}} = \frac{1}{2}I\omega^2$

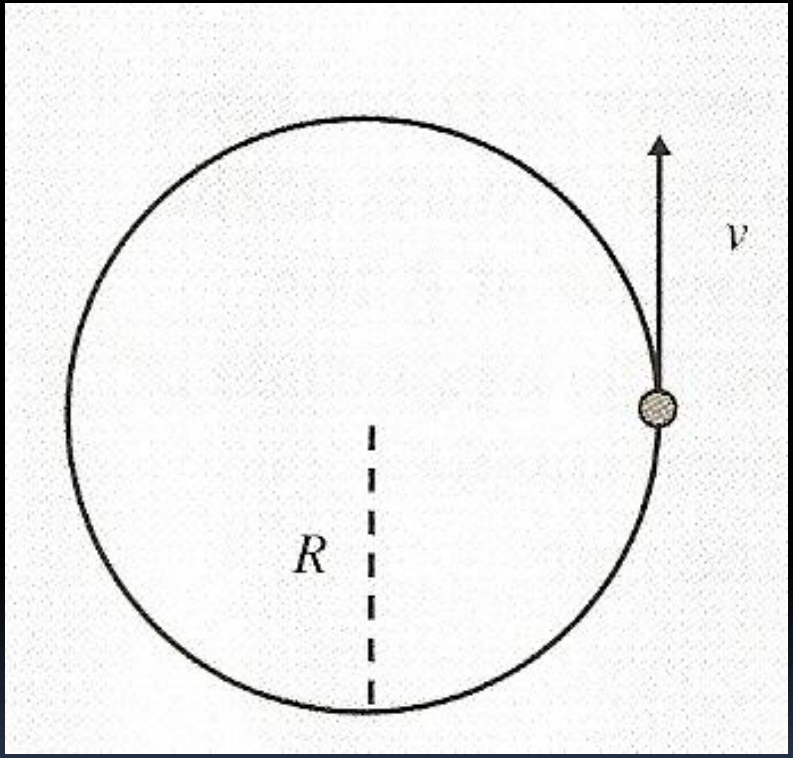
Utilization:

- Structural design and civil engineering rely on the knowledge of how objects can move in all situations

Aims:

- Aim 7: technology has allowed for computer simulations that accurately model the complicated outcomes of actions on bodies

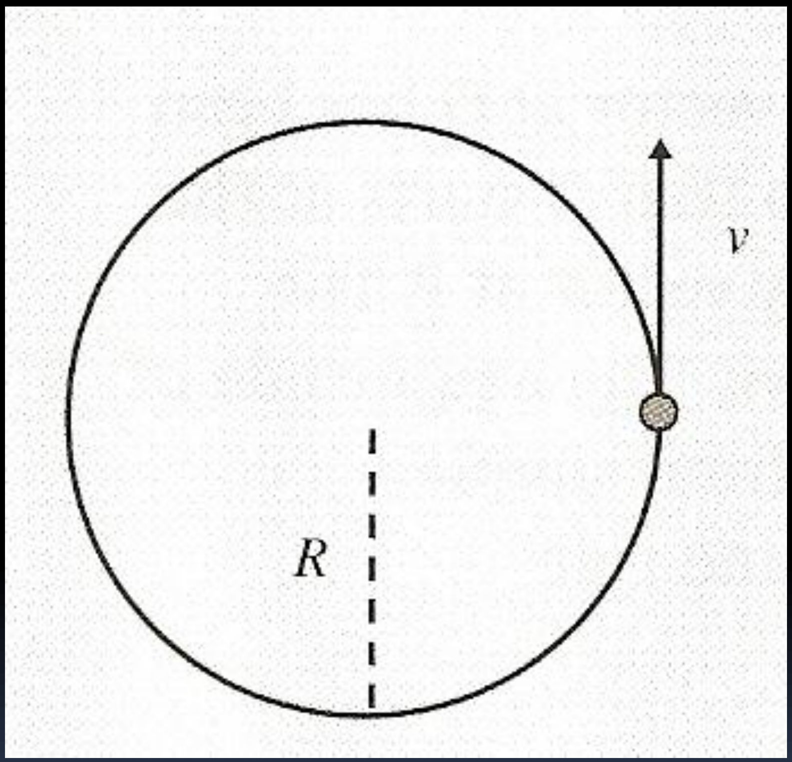
Circular Motion and Centripetal Acceleration



$$v = \frac{2\pi R}{T}$$

- T is the period in seconds – the time it takes to make one revolution
- 2πR is the circumference or distance around the circle

Circular Motion and Centripetal Acceleration



$$\omega = \frac{2\pi}{T}$$

- Angular Speed
- In one period, the object will sweep out an angle of 2π radians
- Units are radians per second or just s^{-1}

Circular Motion and Centripetal Acceleration

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta v = v \sin \theta$$

$$\theta_{rad} = \frac{\Delta s}{R}$$

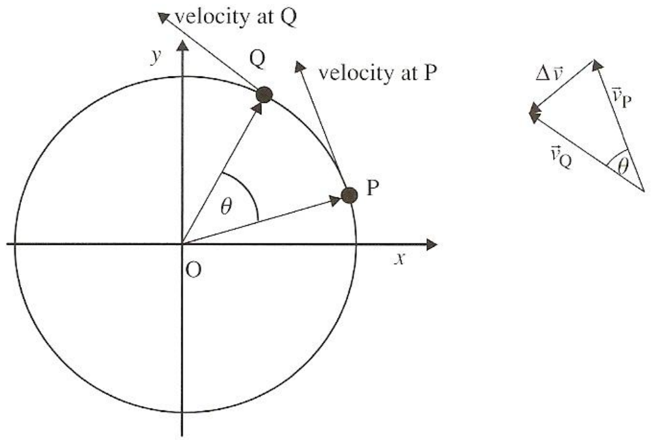


Figure 8.4 The velocity vector changes direction in circular motion, hence we have acceleration.

$$\Delta s = v \Delta t$$

$$\Delta v = \frac{v v \Delta t}{R}$$

$$a_c = \frac{v^2}{R}$$

Circular Motion and Centripetal Acceleration

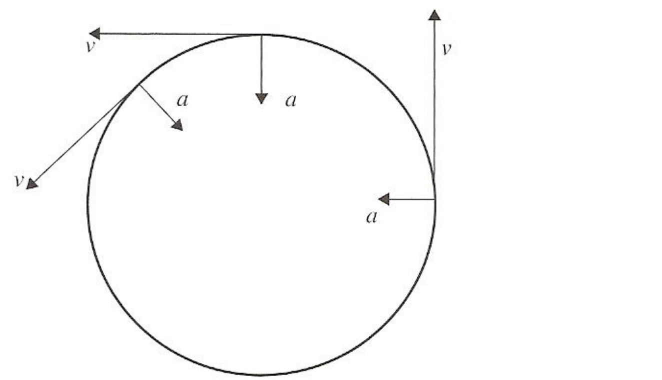


Figure 8.5 The centripetal acceleration vector is normal to the velocity vector.

- A body moving along a circle of radius R with a speed v experiences centripetal acceleration that has magnitude given by

$$a_c = \frac{v^2}{R}$$

and is directed toward the centre of the circle

Tangential Acceleration

- If the magnitude of the velocity changes, we have tangential acceleration.
- This is a vector in the same direction as the velocity vector if speed is increasing, in the opposite direction if decreasing

$$a_t = \frac{\Delta v}{\Delta t}$$

Tangential and Centripetal Acceleration

- When velocity direction and magnitude are changing, we have both centripetal acceleration and tangential acceleration
- The total acceleration is the vector sum of the vectors representing these accelerations

Kinematics

Linear

Position: x (meters)

Velocity: $v = \frac{\Delta x}{\Delta t}$ (m/s)

Linear Acceleration: $a = \frac{\Delta v}{\Delta t}$

(m/s²)

Radial Acceleration: $a_r = \frac{v^2}{r}$

Rotational

Position: θ (radians)

Angular Velocity: $\omega = \frac{\Delta \theta}{\Delta t}$ (rad/s)

$$v_{tan} = \omega r$$

Angular Accl: $\alpha = \frac{\Delta \omega}{\Delta t}$ (rad/s²)

$$a_{tan} = \alpha r$$

$$a_r = \omega^2 r$$

Kinematics

Linear

Position: x (meters)

Velocity: $v = \frac{\Delta x}{\Delta t}$ (m/s)

Linear Acceleration: $a = \frac{\Delta v}{\Delta t}$

(m/s²)

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$a_{tan} = \alpha r$

$a_r = \omega^2 r$

Kinematics

Linear

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{u+v}{2}t$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Rotational

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{\omega_i + \omega_f}{2}t$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha s$$

Kinematics

- What is the slope of an angle θ versus time t graph?

Kinematics

- What is the slope of an angle θ versus time t graph?
  - Angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Kinematics

- What is the slope of an angular velocity ω versus time t graph?

Kinematics

- What is the slope of an angular velocity ω versus time t graph?
  - Angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Kinematics

- What is the area under the curve of an angular velocity ω versus time t graph?

Kinematics

- What is the area under the curve of an angular velocity ω versus time t graph?
  - Angle turned

$$\theta = \omega t$$

Kinematics

- What is the area under the curve of an angular acceleration α versus time t graph?
  - Change in angular velocity

$$\Delta\omega = \alpha t$$

Torque

- The ability of a force to produce a rotation.
- It is equal to the force times the distance from the point of application of the force to the axis of rotation (***moment arm***)

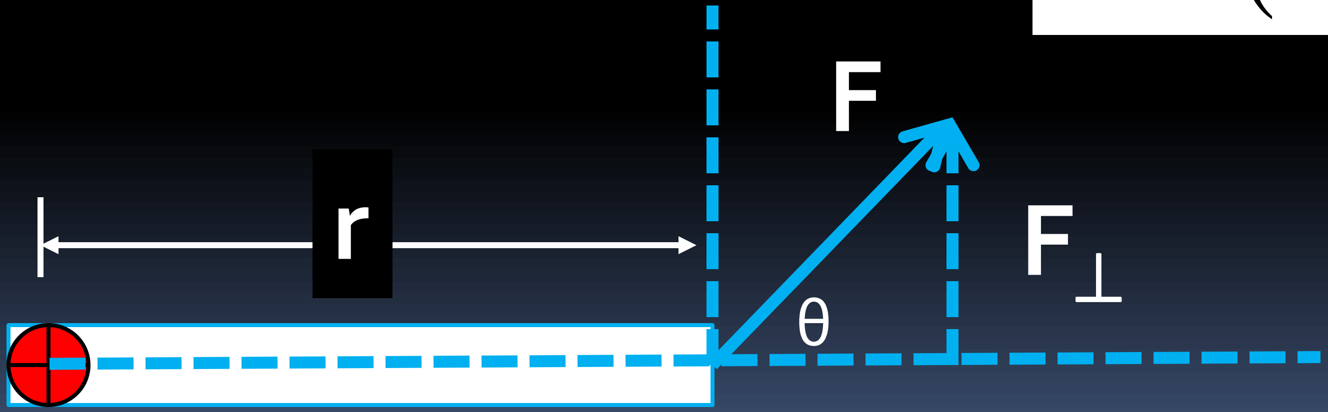
$$\Gamma = Fr$$

Torque

- However, only the component of the force perpendicular to the moment arm produces torque

$$\Gamma = F_{\perp} r$$

$$\Gamma = (F \sin \theta) r$$

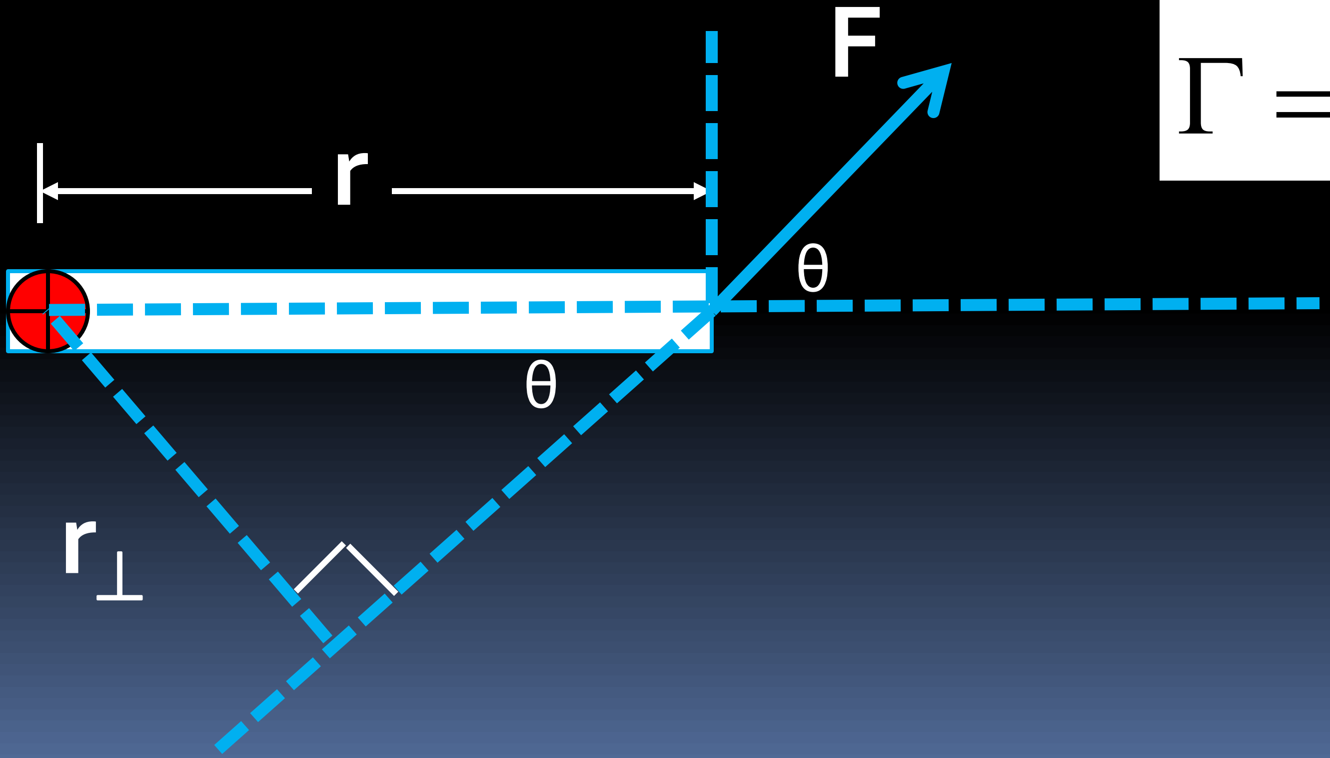


Torque

- You can also think of it as the force times the perpendicular distance

$$\Gamma = F r_{\perp}$$

$$\Gamma = F\left(r \sin\theta\right)$$



Torque

- A force applied at the axis of rotation produces zero torque

$$\Gamma = Fr = 0$$

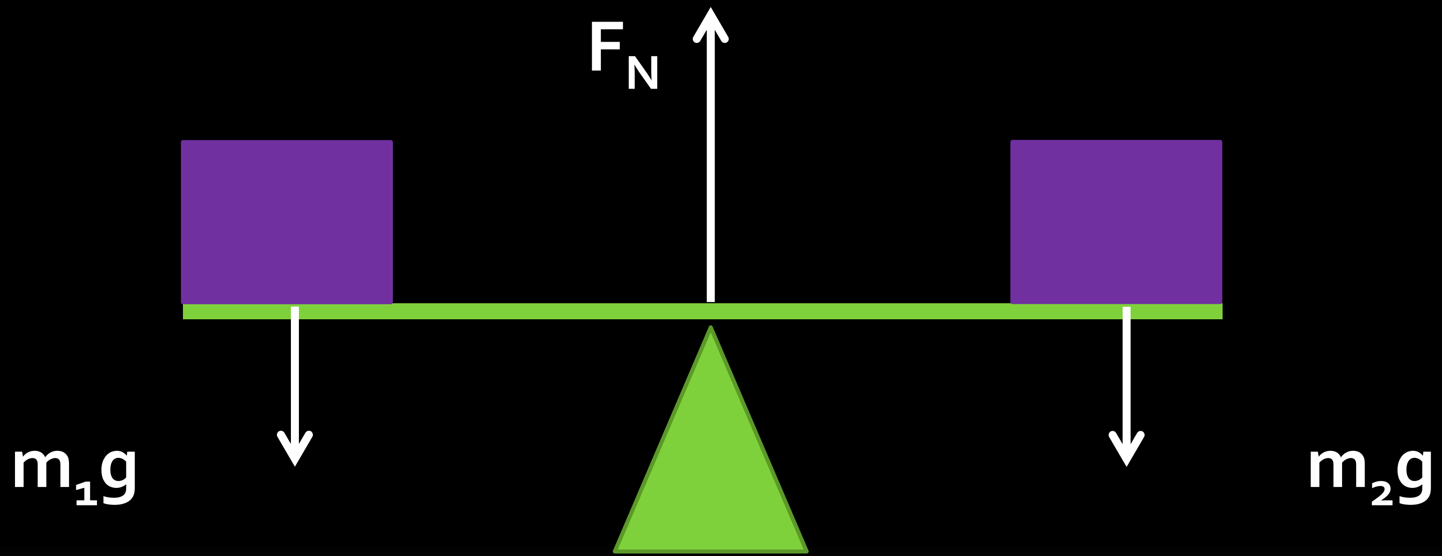
Equilibrium

- Translational
  - No movement in any direction or movement in a constant direction at constant velocity
  - The net ***force*** on the body treated as a point mass is equal to zero.
  - The net force acting on the center of mass is zero.

$$\Sigma F = F_{net} = 0$$

Equilibrium

- Translational



$$\Sigma F_y = 0$$

$$F_N = m_1 g + m_2 g$$

$$\Sigma F_x = 0$$

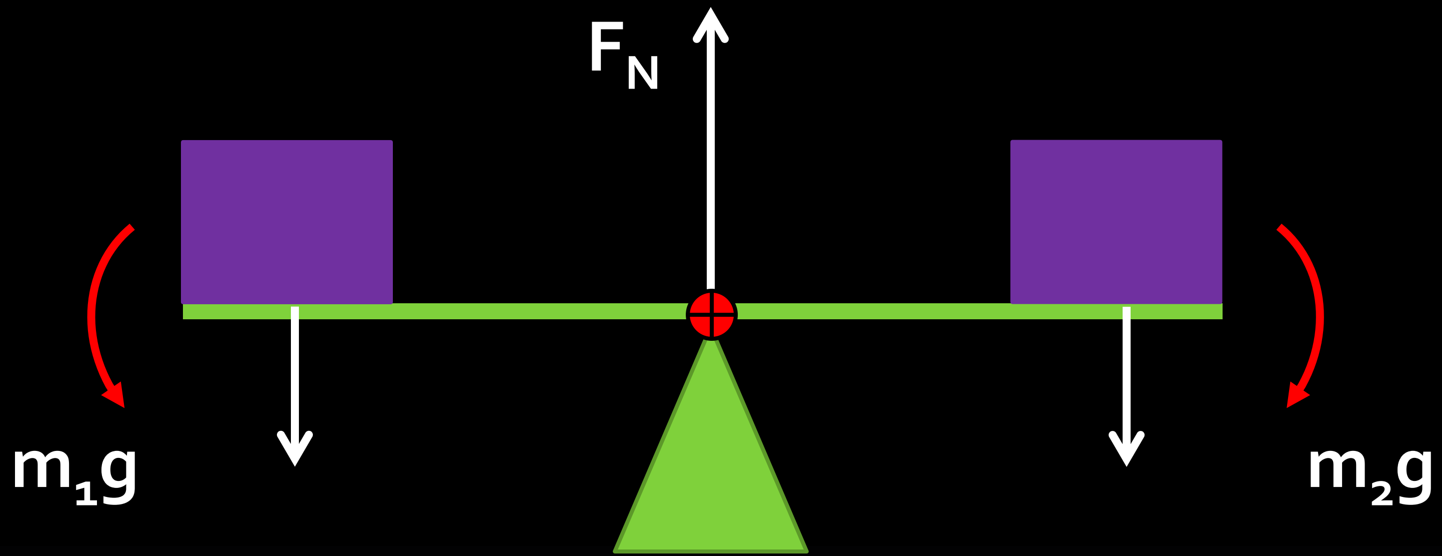
Equilibrium

- Rotational
  - No rotation of the body
  - The net ***torque*** on the body is equal to zero.

$$\Sigma\Gamma = \Gamma_{net} = 0$$

Equilibrium

- Rotational



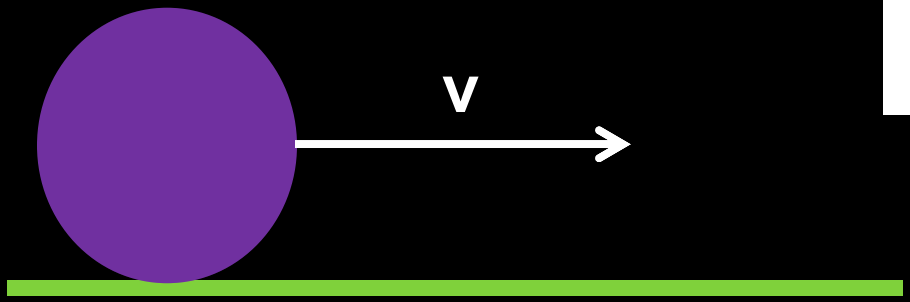
$$\Sigma\Gamma = 0$$

$$m_1 g r_1 = m_2 g r_2$$

Kinetic Energy of a Body

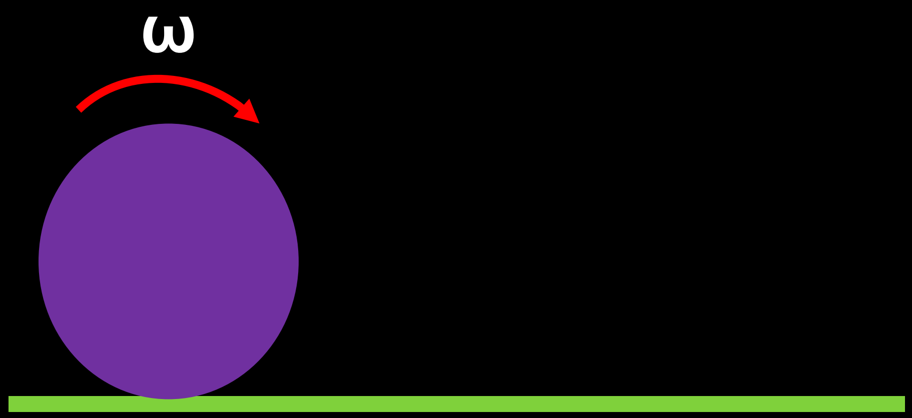
- Translational

$$KE = \frac{1}{2}mv^2$$



Kinetic Energy of a Body

- Rotational



$$KE = \frac{1}{2}mv^2$$

$$v = \omega r$$

$$KE = \frac{1}{2}m\omega^2 r^2$$

- ***Not that easy***
  - ***What mass?***
  - ***What r?***

Kinetic Energy of a Body

- Rotational

$$KE = \frac{1}{2} m_1 \omega^2 r_1{}^2 + \frac{1}{2} m_1 \omega^2 r_2{}^2 + \ldots$$

$$KE = \frac{1}{2}\left(\Sigma m_i r_i{}^2\right)\omega^2$$

- ***There has to be a better way***

Moment of Inertia

- *The moment of inertia, otherwise known as the angular mass or rotational inertia, of a rigid body determines the torque needed for a desired angular acceleration about a rotational axis. It depends on the body's mass distribution and the axis chosen, with larger moments requiring more torque to change the body's rotation. It is an extensive (additive) property: the moment of inertia of a composite system is the sum of the moments of inertia of its component subsystems (all taken about the same axis).*

Moment of Inertia

$$I = \Sigma m_i r_i^2$$

- This works for a single point mass, but not much else

Moment of Inertia

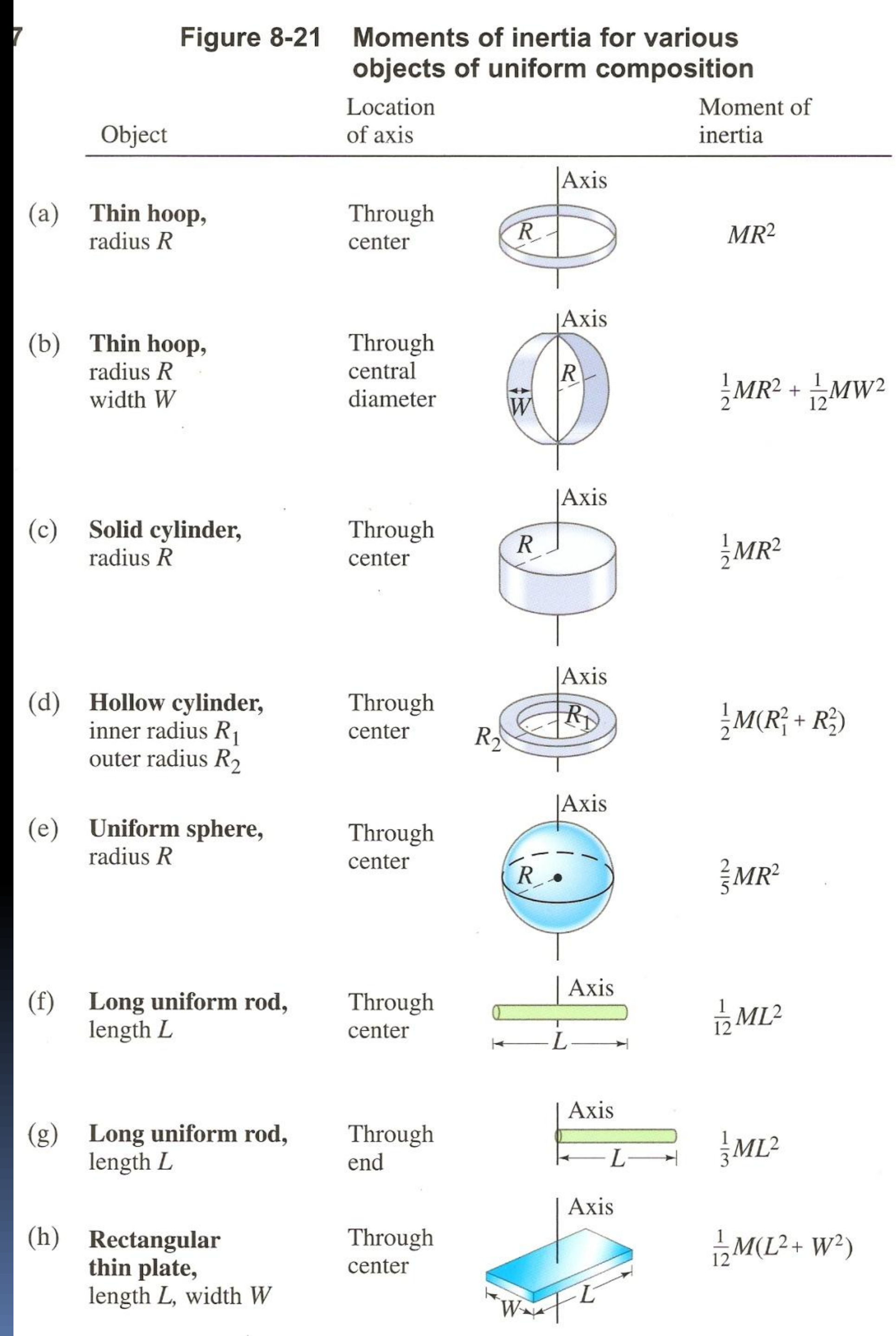


Figure 8-21 Moments of inertia for various objects of uniform composition

	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R	Through center	MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter	$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center	$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center	$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center	$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end	$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center	$\frac{1}{12}M(L^2 + W^2)$

Guidance:

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- Graphs will be limited to angular displacement–time, angular velocity–time and torque–time

Moment of Inertia

$$I = \Sigma m_i r_i^2$$

- This makes rotational kinetic energy much easier

$$KE = \frac{1}{2}\left(\Sigma m_i r_i^2\right)\omega^2$$

$$KE = \frac{1}{2} I\omega^2$$

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Questions?

HOMEWORK

#1-7