DEVIL PHYSICS
THE BADDEST CLASS ON CAMPUS
IB PHYSICS

TOPIC 9
WAVE PHENOMENA

TSOKOS LESSON 9-1
SIMPLE HARMONIC MOTION

Essential Idea:

- The solution of the harmonic oscillator can be framed around the variation of kinetic and potential energy in the system.

Nature Of Science:

- Insights:
  - The equation for simple harmonic motion (SHM) can be solved analytically and numerically.
  - Physicists use such solutions to help them to visualize the behavior of the oscillator.
  - The use of the equations is very powerful as any oscillation can be described in terms of a combination of harmonic oscillators.
  - Numerical modeling of oscillators is important in the design of electrical circuits.

Understandings:

- The defining equation of SHM
- Energy changes

Applications And Skills:

- Solving problems involving acceleration, velocity and displacement during simple harmonic motion, both graphically and algebraically
- Describing the interchange of kinetic and potential energy during simple harmonic motion
- Solving problems involving energy transfer during simple harmonic motion, both graphically and algebraically

Guidance:

- Contexts for this sub-topic include the simple pendulum and a mass-spring system.

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum: T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass-spring: T = 2\pi\sqrt{\frac{m}{k}}$$

Utilization:

- Fourier analysis allows us to describe all periodic oscillations in terms of simple harmonic oscillators. The mathematics of simple harmonic motion is crucial to any areas of science and technology where oscillations occur
- The interchange of energies in oscillation is important in electrical phenomena

Utilization:

- Quadratic functions (see Mathematics HL sub-topic 2.6; Mathematics SL sub-topic 2.4; Mathematical studies SL sub-topic 6.3)
- Trigonometric functions (see Mathematics SL sub-topic 3.4)

Aims:

- Aim 4: students can use this topic to develop their ability to synthesize complex and diverse scientific information
- Aim 7: the observation of simple harmonic motion and the variables affected can be easily followed in computer simulations

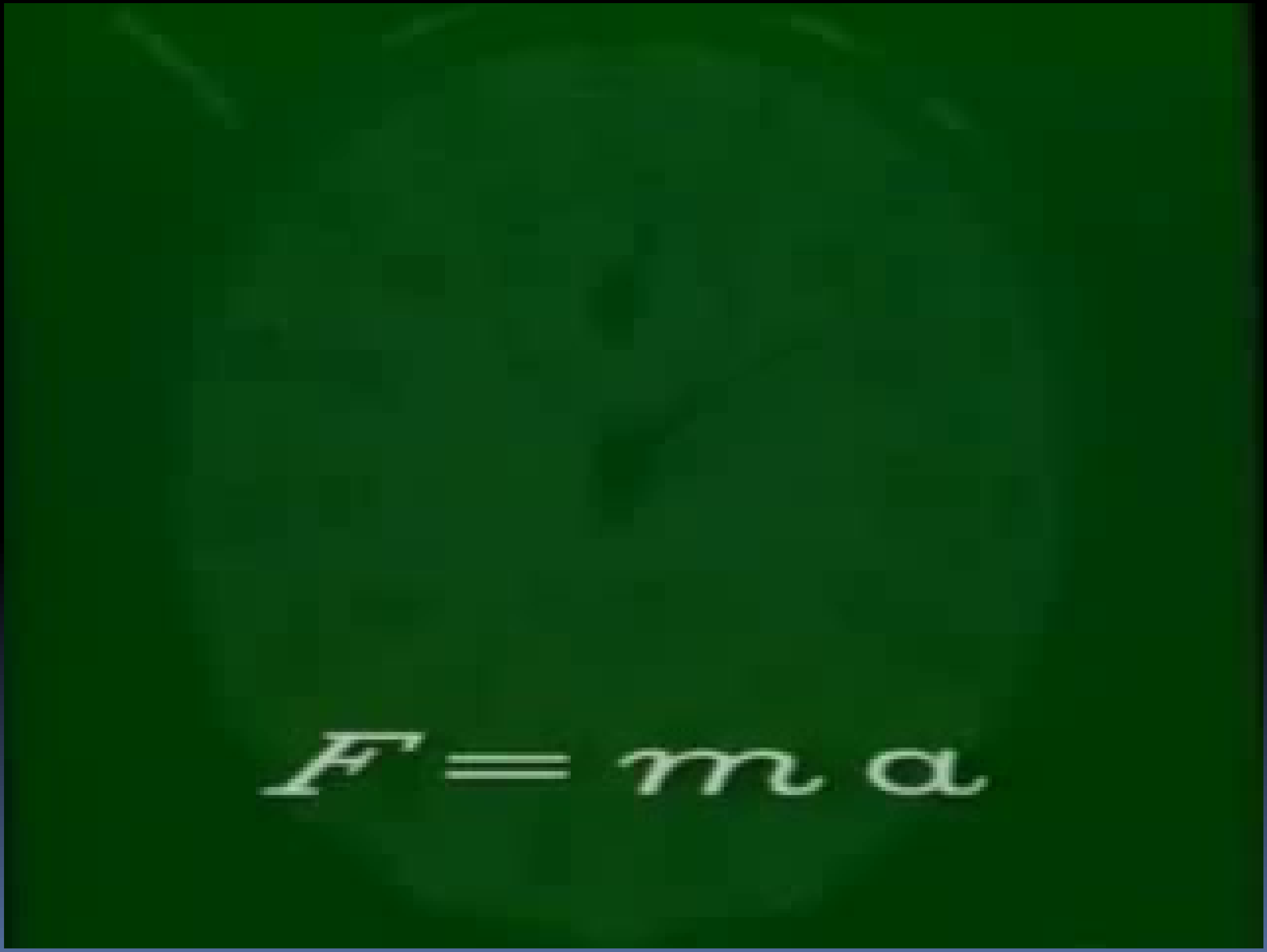
Aims:

- Aim 6: experiments could include (but are not limited to): investigation of simple or torsional pendulums; measuring the vibrations of a tuning fork; further extensions of the experiments conducted in sub-topic 4.1. By using the force law, a student can, with iteration, determine the behaviour of an object under simple harmonic motion. The iterative approach (numerical solution), with given initial conditions, applies basic uniform acceleration equations in successive small time increments. At each increment, final values become the following initial conditions.

Oscillation vs. Simple Harmonic Motion

- An <u>oscillation</u> is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion i.e. the motion repeats in some way.
- *In <u>simple harmonic motion</u>, the displacement from an equilibrium position and the force/acceleration are proportional and opposite to each other.*

Relating SHM to Motion Around A Circle



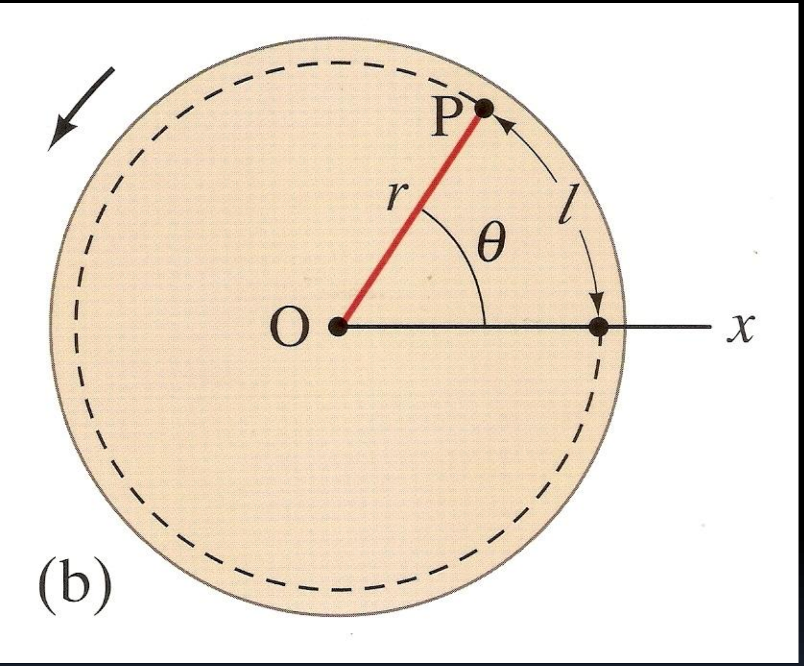
Radians

- One radian is defined as the angle subtended by an arc whose length is equal to the radius

$$\theta = \frac{l}{r}$$

$$l = r$$

$$\theta = 1$$



Radians

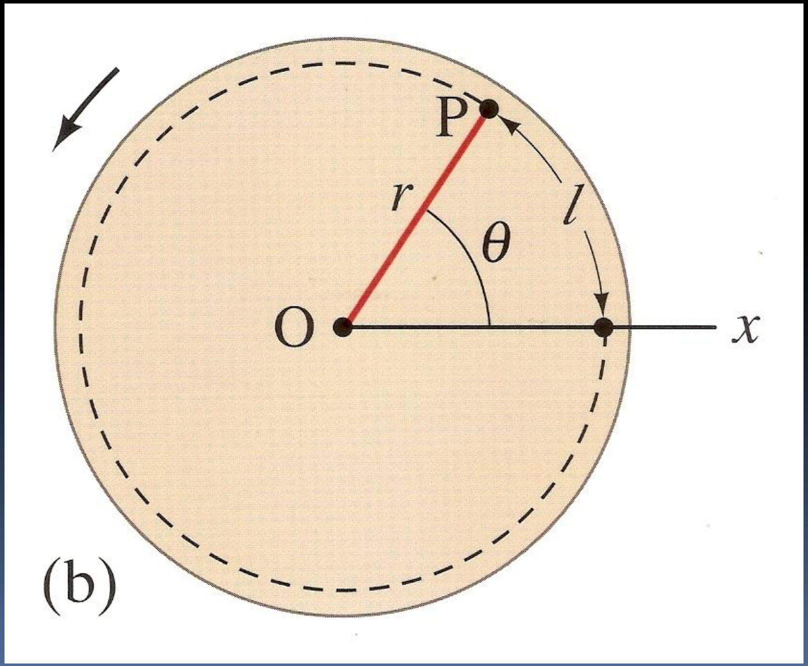
$$Circumference = 2\pi r$$

$$\theta = \frac{l}{r}$$

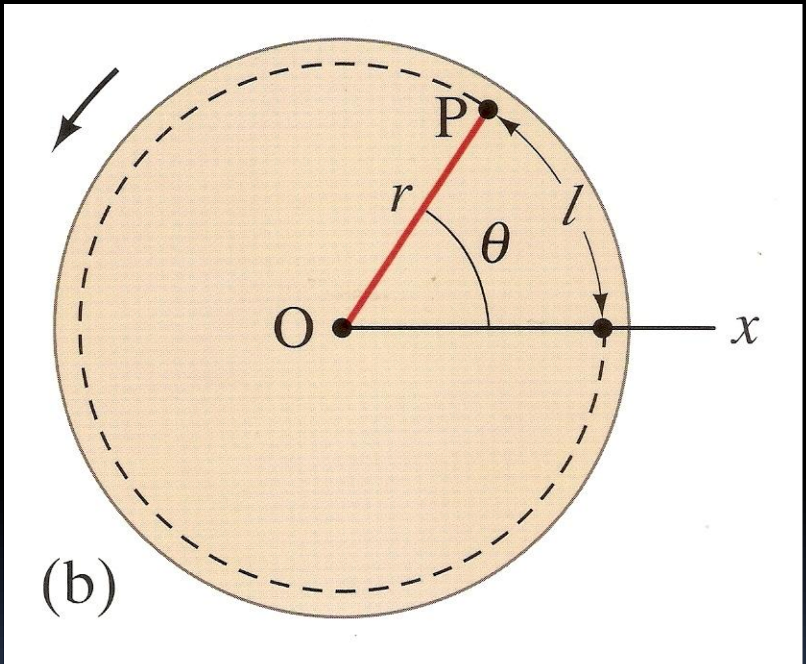
$$l = 2\pi r$$

$$Circumference = 2\pi(rad)$$

$$360° = 2\pi(rad)$$



Angular Velocity



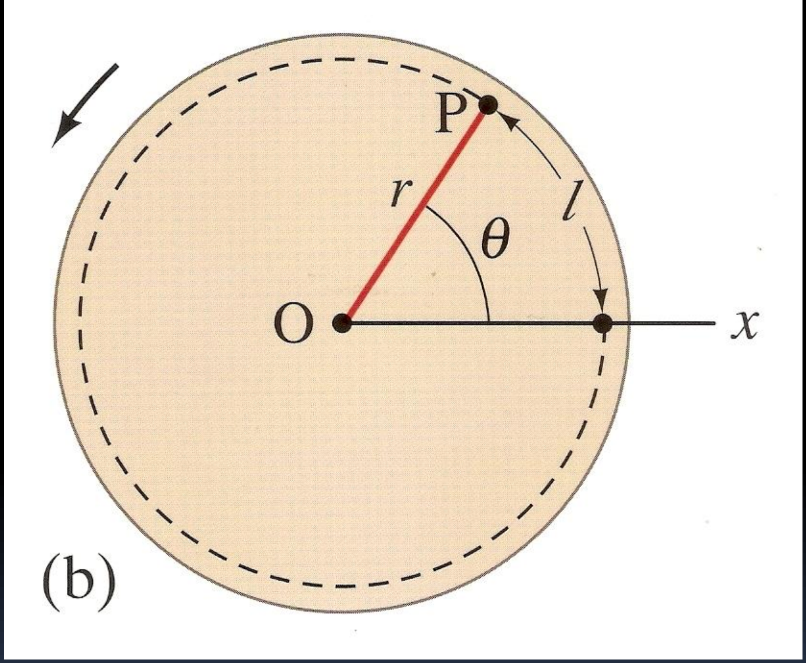
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v = \frac{\Delta l}{\Delta t}$$

$$\theta = \frac{l}{r}, l = r\theta$$

$$v = r\frac{\Delta\theta}{\Delta t} = r\omega$$

Angular Acceleration



$$a = \frac{\Delta\omega}{\Delta t}$$

$$a_r = \frac{v^2}{r}$$

$$v = r\omega$$

$$a = \frac{(r\omega)^2}{r} = \omega^2 r$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

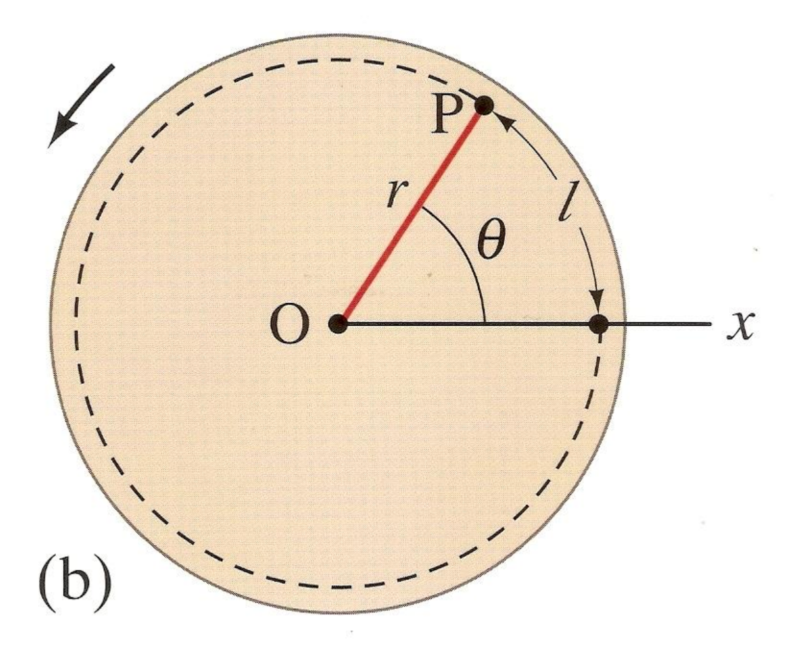
$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Period



$$v_T = \frac{2\pi r}{T}$$

$$v_T = r\omega$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

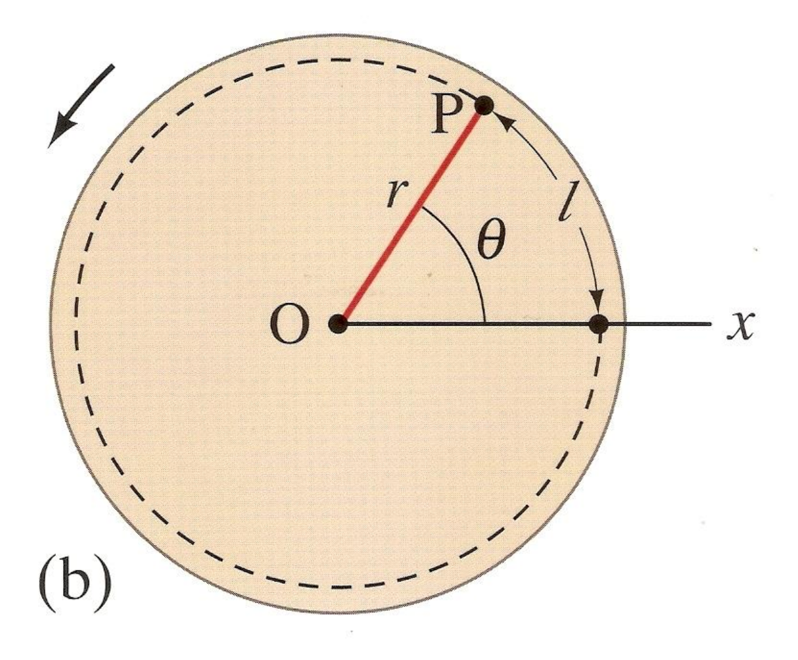
$$E_K = \frac{1}{2} m\omega^2\left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum: T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring: T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency



$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$

$$f = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

Relating SHM to Motion Around A Circle

- **The period in one complete oscillation of simple harmonic motion can be likened to the period of one complete revolution of a circle.**

$$\text{Time taken} = \frac{\text{angle swept}}{\text{angular speed } (\omega)}$$

$$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T}$$

Mass-Spring System

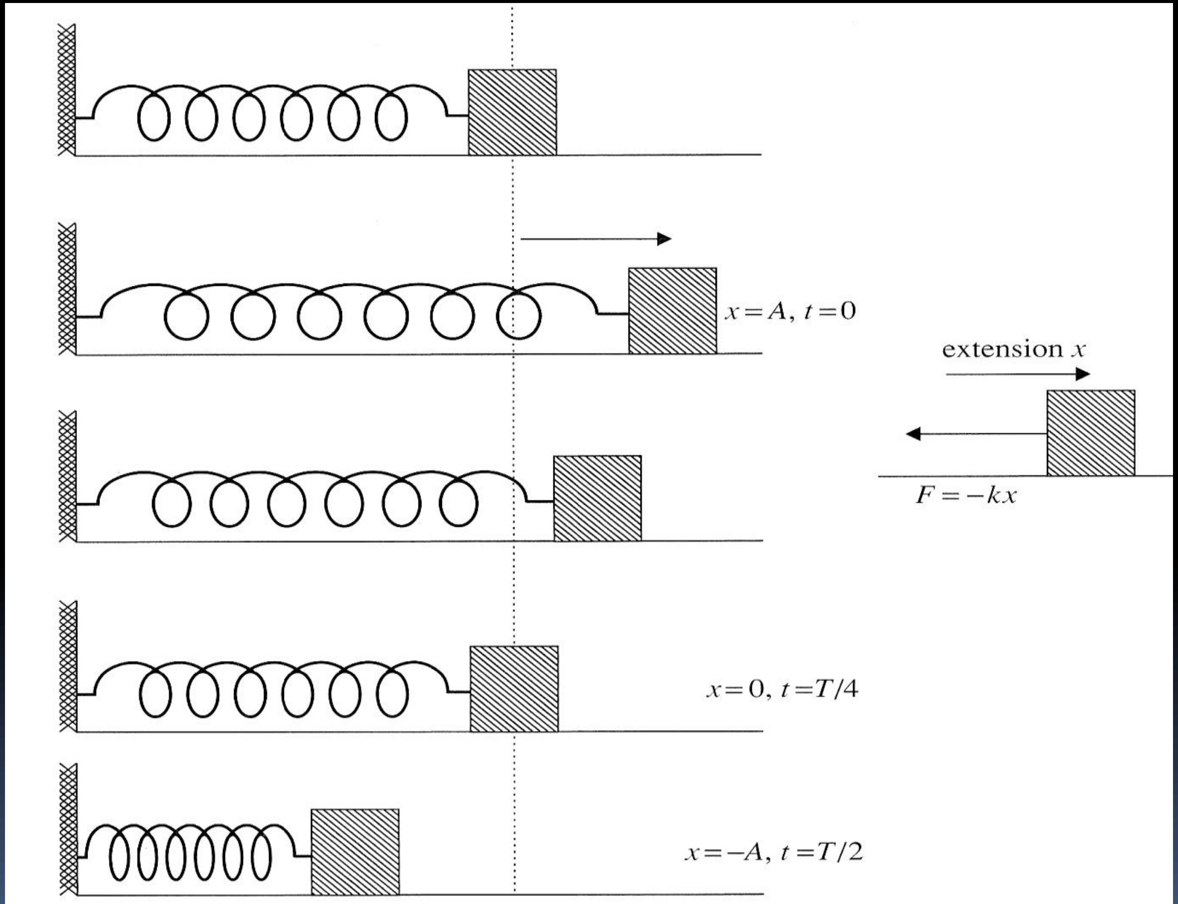


Figure 1.2 The mass–spring system. The net force on the body is proportional to the displacement and opposite to it.

Relating SHM to Motion Around A Circle, Mass-Spring

$$\Sigma F = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m}x$$

$$a = -\omega^2 r$$

$$\omega^2 r = \frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Relating SHM to Motion Around A Circle, Mass-Spring

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

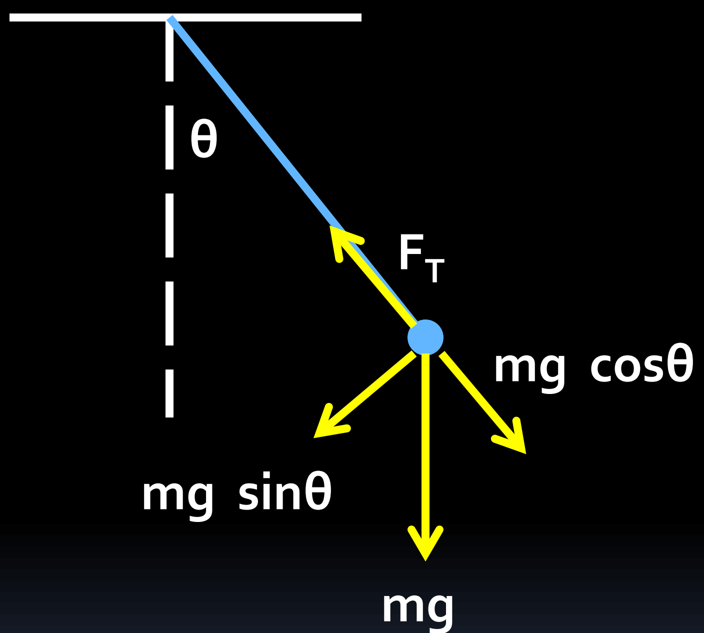
$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum: T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring: T = 2\pi\sqrt{\frac{m}{k}}$$

Relating SHM to Motion Around A Circle, Pendulum



$$ma = -mg\sin\theta$$

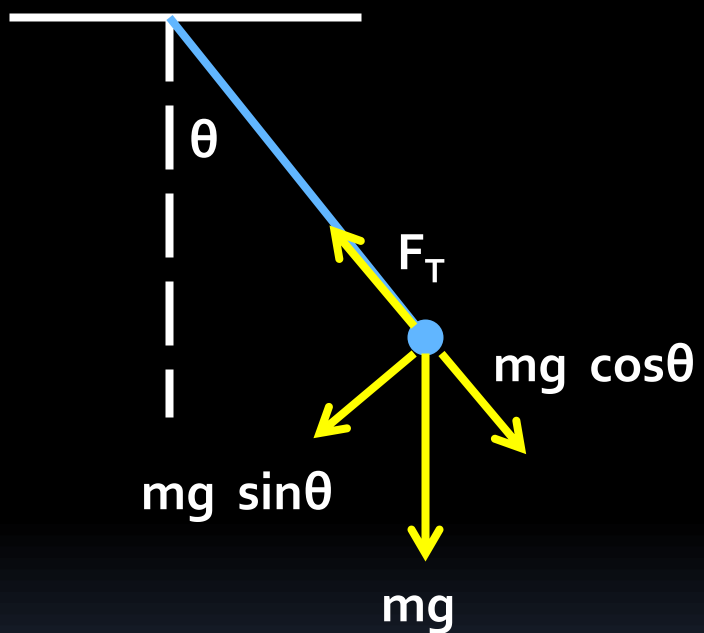
$$a = -g\sin\theta$$

Displacement

$$x = L\theta$$

$$\theta = \frac{x}{L}$$

Relating SHM to Motion Around A Circle, Pendulum



$$a = -g \sin \theta$$

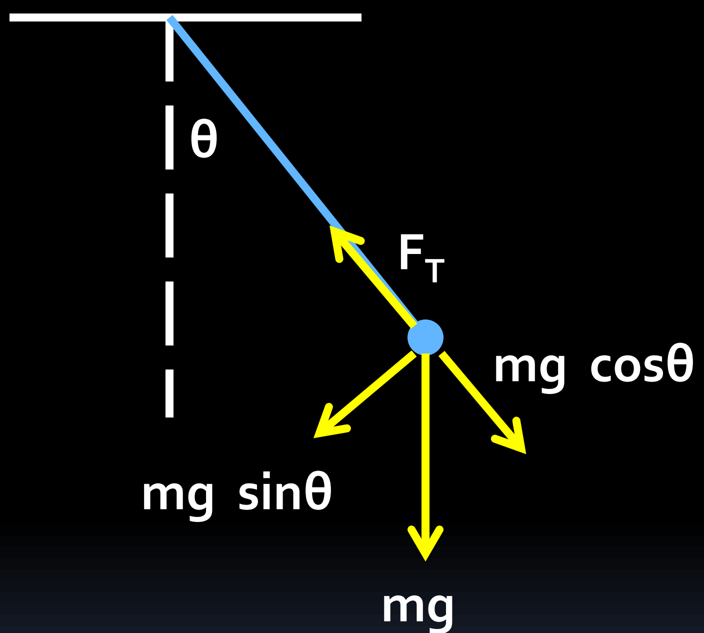
$$\theta = \frac{x}{L}$$

$$a = -g \sin \frac{x}{L}$$

$$Small\,\theta$$

$$\sin\left(\frac{x}{L}\right) \approx \frac{x}{L}$$

Relating SHM to Motion Around A Circle, Pendulum



$$a = -g\sin\theta$$

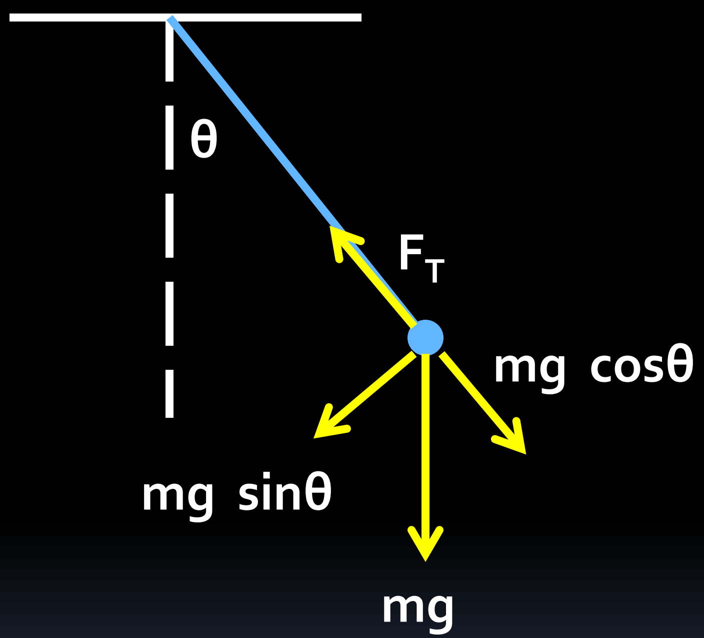
$$\sin\left(\frac{x}{L}\right) = \frac{x}{L}$$

$$a = -g\frac{x}{L} = -\frac{g}{L}x$$

$$a = -\omega^2 x$$

$$\omega^2 = \frac{g}{L}$$

Relating SHM to Motion Around A Circle, Pendulum



$$\omega^2 = \frac{g}{L}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

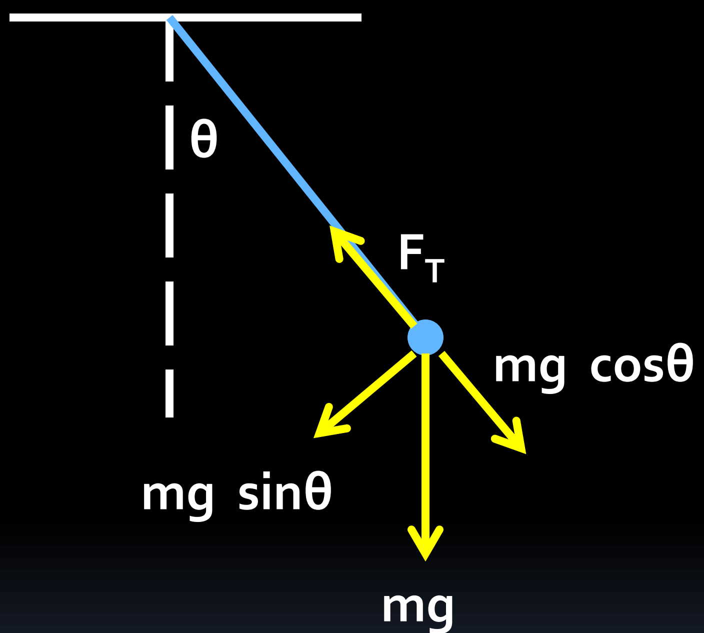
$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Relating SHM to Motion Around A Circle, Pendulum



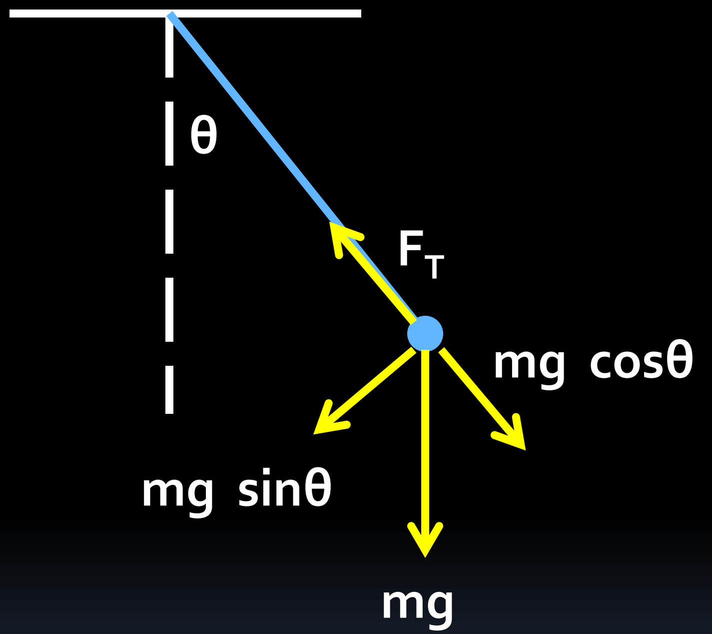
$$a = -\omega^2 x$$

$$a = \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Relating SHM to Motion Around A Circle, Pendulum



$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Differential Equation Machine

$$x = x_0 \cos(\omega t)$$
$$x = x_0 \sin(\omega t)$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

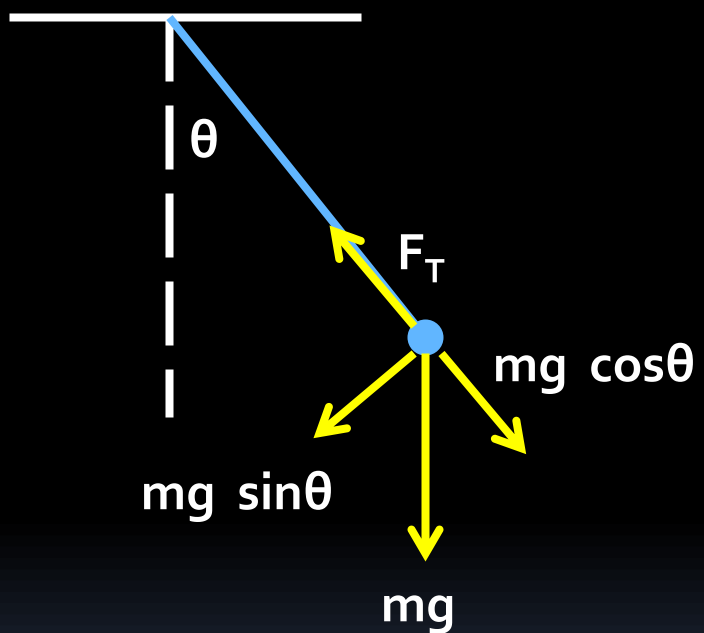
$$E_K = \frac{1}{2} m\omega^2\left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Relating SHM to Motion Around A Circle, Pendulum



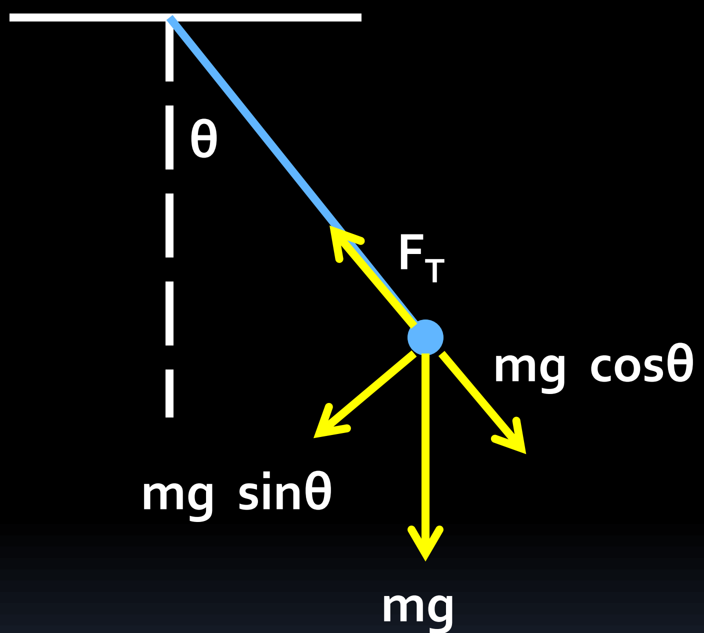
$$x = x_0 \cos(\omega t)$$

Max Displacement

$$x = x_0 \sin(\omega t)$$

Zero Displacement

Relating SHM to Motion Around A Circle, Pendulum



$$x = x_0 \cos(\omega t)$$

$$x = x_0 \sin(\omega t)$$

$$v = \frac{dx}{dt} = \omega x_0 \cos(\omega t)$$

$$v = \frac{dx}{dt} = -\omega x_0 \sin(\omega t)$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

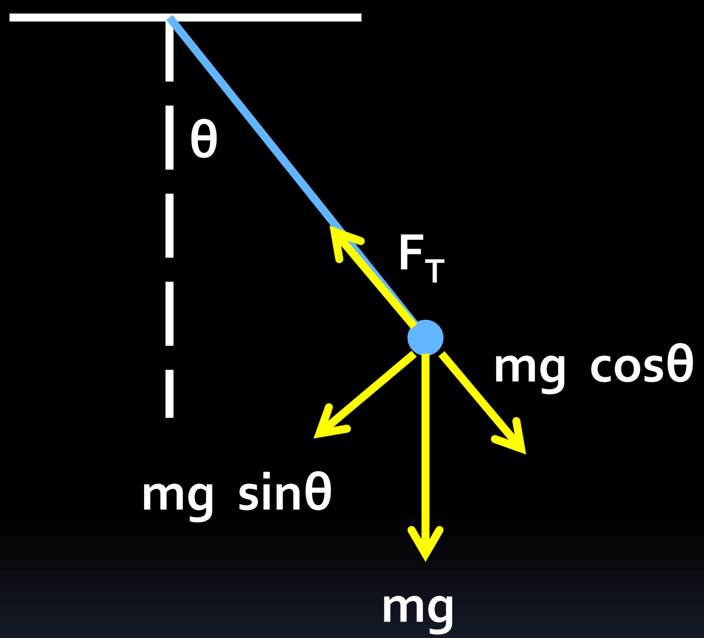
$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum: T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass-spring: T = 2\pi\sqrt{\frac{m}{k}}$$

Relating SHM to Motion Around A Circle, Pendulum



$$x = x_0 \cos(\omega t)$$

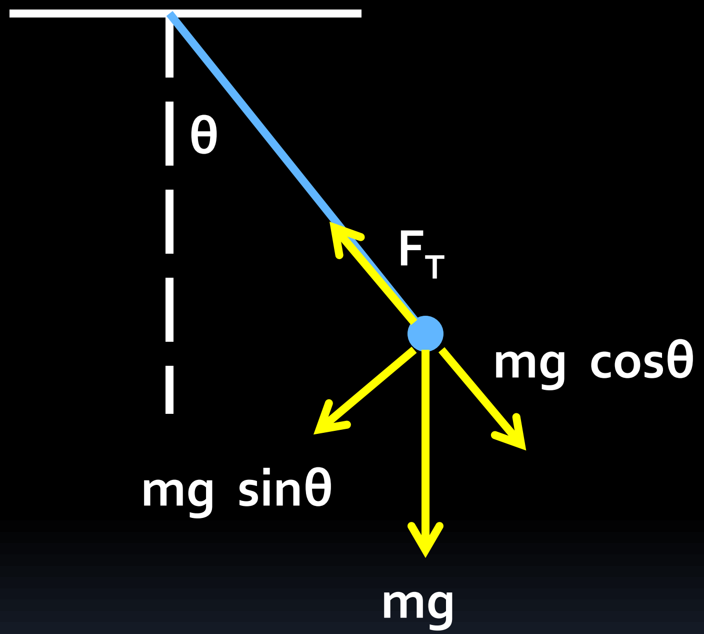
$$x = x_0 \sin(\omega t)$$

$$v = \frac{dx}{dt} = \omega x_0 \cos(\omega t)$$

$$v = \frac{dx}{dt} = -\omega x_0 \sin(\omega t)$$

So what is x_o?

Relating SHM to Motion Around A Circle, Pendulum



$$x = x_0 \cos(\omega t)$$

$$x = x_0 \sin(\omega t)$$

$$v = \frac{dx}{dt} = \omega x_0 \cos(\omega t)$$

$$v = \frac{dx}{dt} = -\omega x_0 \sin(\omega t)$$

So what is x_o?

Amplitude

Equation Summary

- Defining Equation: $a = -\omega^2 x$
- Angular Frequency: $\omega = \frac{2\pi}{T} = 2\pi f$
- Period: $T = \frac{2\pi}{\omega}$

Equation Summary

$$x = x_0 \cos(\omega t)$$
$$v = -\omega x_0 \sin(\omega t)$$
$$a = -\omega^2 x_0 \cos(\omega t)$$

$$x = x_0 \sin(\omega t)$$
$$v = \omega x_0 \cos(\omega t)$$
$$a = -\omega^2 x_0 \sin(\omega t)$$

Equation Summary

- Maximum speed: $v_0 = \omega x_0$
- Maximum acceleration: $a = \omega^2 x_0$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Energy in SHM

$$E_K = \frac{1}{2}mv^2$$

$$v = -\omega x_0 \sin(\omega t)$$

$$E_K = \frac{1}{2}m\omega^2 x_0{}^2 \sin^2(\omega t)$$

Energy in SHM

$$E_K = \frac{1}{2} m\omega^2 {x_0}^2 \sin^2(\omega t)$$

$$v_{\max} = \omega x_0$$

$$E_{K_{\max}} = \frac{1}{2} m\omega^2 {x_0}^2$$

Energy in SHM

$$E_{K_{\max}} = \frac{1}{2} m\omega^2 {x_0}^2$$

$$E_T = E_{K_{\max}} = E_P + E_K$$

$$E_P = E_T - E_K$$

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2 - \frac{1}{2} m\omega^2 {x_0}^2 \sin^2(\omega t)$$

Energy in SHM

$$E_P = \frac{1}{2} m\omega^2 x_0{}^2 - \frac{1}{2} m\omega^2 x_0{}^2 \sin^2(\omega t)$$

$$E_P = \frac{1}{2} m\omega^2 x_0{}^2 \left(1 - \sin^2(\omega t)\right)$$

$$E_P = \frac{1}{2} m\omega^2 x_0{}^2 \left(\cos^2(\omega t)\right)$$

Energy in SHM

$$E_P = \frac{1}{2} m\omega^2 \left[x_0{}^2 \left(\cos^2(\omega t) \right) \right]$$

$$x = x_0 \cos(\omega t)$$

$$x^2 = x_0{}^2 \cos^2(\omega t)$$

$$E_P = \frac{1}{2} m\omega^2 x^2$$

Energy in SHM

$$v = -\omega x_0 \sin(\omega t)$$

$$v^2 = \omega^2 {x_0}^2 \sin^2(\omega t)$$

$$v^2 = \omega^2 {x_0}^2 \left(1 - \cos^2(\omega t)\right)$$

$$v^2 = \omega^2 {x_0}^2 - \omega^2 \left[{x_0}^2 \cos^2(\omega t)\right]$$

$$v^2 = \omega^2 \left({x_0}^2 - x^2\right)$$

$$v = \pm \omega \sqrt{{x_0}^2 - x^2}$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

$$E_K = \frac{1}{2} m\omega^2\left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Energy in SHM

$$E_K = \frac{1}{2}mv^2$$

$$v = \pm\omega\sqrt{{x_0}^2 - x^2}$$

$$E_K = \frac{1}{2}m\omega^2\left({x_0}^2 - x^2\right)$$

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t$$

$$x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$v = \pm \omega \sqrt{\left(x_o^2 - x^2\right)}$$

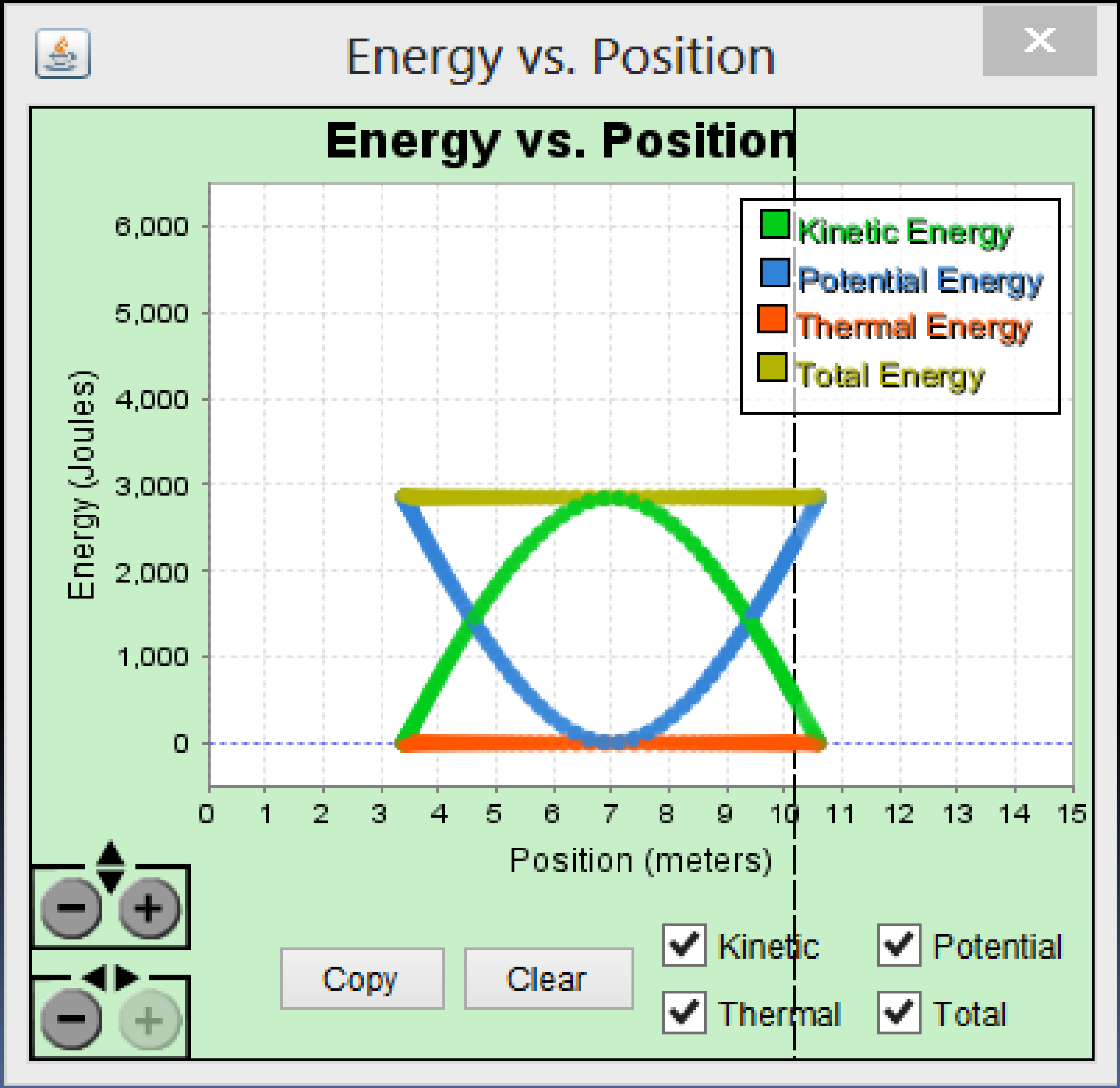
$$E_K = \frac{1}{2} m \omega^2 \left(x_o^2 - x^2\right)$$

$$E_T = \frac{1}{2} m \omega^2 x_o^2$$

$$Pendulum : T = 2\pi \sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi \sqrt{\frac{m}{k}}$$

End Result



Energy vs. Position
Energy vs. Position
Kinetic Energy
Potential Energy
Thermal Energy
Total Energy
6,000
5,000
4,000
3,000
2,000
1,000
0
Energy (Joules)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Position (meters)
Copy
Clear
Kinetic
Potential
Thermal
Total

Section Summary:

$x = x_0 \cos(\omega t)$

$v = -\omega x_0 \sin(\omega t)$

$x = x_0 \sin(\omega t)$

$E_{K_{max}} = \frac{1}{2} m \omega^2 x_0^2$

$\omega = \frac{2\pi}{T}$

$E_T = \frac{1}{2} m v^2$

$v^2 = \omega^2 x_0^2 \sin^2(\omega t)$

$E_K = \frac{1}{2} m \omega^2 (x_o^2 - x^2)$

$T = \frac{2\pi}{\omega}$

$E_T = E_P + E_K$

$v = \frac{dx}{dt} = \omega x_0 \cos(\omega t)$

$\omega = \frac{\Delta \theta}{\Delta t}$

$a = \omega^2 x$

$v^2 = \omega^2 x_0^2 (1 - \cos^2(\omega t))$

$E_T = \frac{1}{2} m \omega^2 x_o^2$

$v = \pm \omega \sqrt{x_0^2 - x^2}$

$v^2 = \omega^2 (x_0^2 - x^2)$

$x^2 = x_0^2 \cos^2(\omega t)$

$f = \frac{\omega}{2\pi}$

$Pendulum: T = 2\pi \sqrt{\frac{l}{g}}$

$Mass-spring: T = 2\pi \sqrt{\frac{m}{k}}$

$v = -\omega x_0 \sin \omega t$

$x = x_0 \cos \omega t$

$v = \omega x_0 \cos \omega t$

$E_P = \frac{1}{2} m \omega^2 x_0^2$

$\theta = \frac{l}{r}, l = r\theta$

$v = r \frac{\Delta \theta}{\Delta t} = r\omega$

$v = \pm \omega \sqrt{(x_o^2 - x^2)}$

$E_P = \frac{1}{2} m \omega^2 x^2$

$\omega = 2\pi f$

Essential Idea:

- The solution of the harmonic oscillator can be framed around the variation of kinetic and potential energy in the system.

Understandings:

- The defining equation of SHM
- Energy changes

Applications And Skills:

- Solving problems involving acceleration, velocity and displacement during simple harmonic motion, both graphically and algebraically
- Describing the interchange of kinetic and potential energy during simple harmonic motion
- Solving problems involving energy transfer during simple harmonic motion, both graphically and algebraically

Data Booklet Reference:

$$\omega = \frac{2\pi}{T}$$

$$a = \omega^2 x$$

$$x = x_0 \sin \omega t; x = x_0 \cos \omega t$$

$$v = \omega x_0 \cos \omega t; x = -\omega x_0 \sin \omega t$$

$$v = \pm\omega\sqrt{\left(x_o^2 - x^2\right)}$$

$$E_K = \frac{1}{2} m\omega^2 \left(x_o^2 - x^2\right)$$

$$E_K = \frac{1}{2} m\omega^2 x_o^2$$

$$Pendulum : T = 2\pi\sqrt{\frac{l}{g}}$$

$$Mass - spring : T = 2\pi\sqrt{\frac{m}{k}}$$

Aims:

- Aim 4: students can use this topic to develop their ability to synthesize complex and diverse scientific information
- Aim 7: the observation of simple harmonic motion and the variables affected can be easily followed in computer simulations

Aims:

- Aim 6: experiments could include (but are not limited to): investigation of simple or torsional pendulums; measuring the vibrations of a tuning fork; further extensions of the experiments conducted in sub-topic 4.1. By using the force law, a student can, with iteration, determine the behaviour of an object under simple harmonic motion. The iterative approach (numerical solution), with given initial conditions, applies basic uniform acceleration equations in successive small time increments. At each increment, final values become the following initial conditions.

Homework

#1-13