DEVIL PHYSICS
THE BADDEST CLASS ON CAMPUS
IB PHYSICS

TSOKOS LESSON 4-1
SIMPLE HARMONIC MOTION

Introductory Video:

Simple Harmonic Motion

Essential Idea:

- A study of oscillations underpins many areas of physics with simple harmonic motion (SHM), a fundamental oscillation that appears in various natural phenomena.

Nature Of Science:

- Models: Oscillations play a great part in our lives, from the tides to the motion of the swinging pendulum that once governed our perception of time. General principles govern this area of physics, from water waves in the deep ocean or the oscillations of a car suspension system. This introduction to the topic reminds us that not all oscillations are isochronous. However, the simple harmonic oscillator is of great importance to physicists because all periodic oscillations can be described through the mathematics of simple harmonic motion.

International-Mindedness:

- Oscillations are used to define the time systems on which nations agree so that the world can be kept in synchronization.
- This impacts most areas of our lives including the provision of electricity, travel and location-determining devices and all microelectronics.

Theory Of Knowledge:

- The harmonic oscillator is a paradigm for modeling where a simple equation is used to describe a complex phenomenon.
- How do scientists know when a simple model is not detailed enough for their requirements?

Understandings:

- Simple harmonic oscillations
- Time period, frequency, amplitude, displacement and phase difference
- Conditions for simple harmonic motion

Applications And Skills:

- Qualitatively describing the energy changes taking place during one cycle of an oscillation
- Sketching and interpreting graphs of simple harmonic motion examples

Guidance:

- Graphs describing simple harmonic motion should include displacement–time, velocity–time, acceleration–time and acceleration–displacement
- Students are expected to understand the significance of the negative sign in the relationship:

$$a \propto -x$$

Data Booklet Reference:

$$T = \frac{1}{f}$$

Utilization:

- Isochronous oscillations can be used to measure time
- Many systems can approximate simple harmonic motion: mass on a spring, fluid in U-tube, models of icebergs oscillating vertically in the ocean, and motion of a sphere rolling in a concave mirror
- Simple harmonic motion is frequently found in the context of mechanics (see Physics topic 2)

Aims:

- Aim 6: experiments could include (but are not limited to): mass on a spring; simple pendulum; motion on a curved air track
- Aim 7: IT skills can be used to model the simple harmonic motion defining equation; this gives valuable insight into the meaning of the equation itself

Oscillation vs. Simple Harmonic Motion

- An <u>oscillation</u> is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion i.e. the motion repeats in some way.

Oscillation vs. Simple Harmonic Motion

- ***In <u>simple harmonic motion</u>, the displacement from an equilibrium position and the force/acceleration are proportional and opposite to each other.***
- ***There must be a restoring force in the direction of the equilibrium position***

Simple Harmonic Motion: Spring

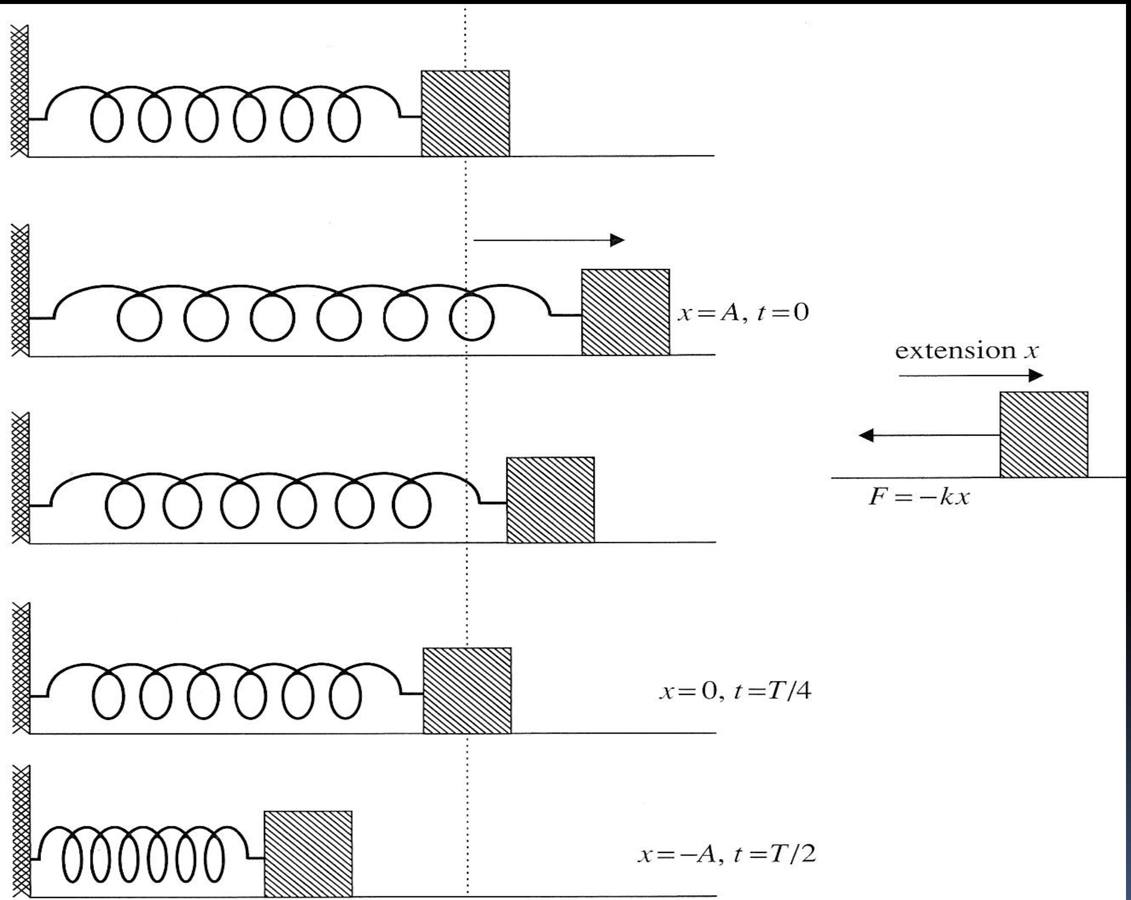
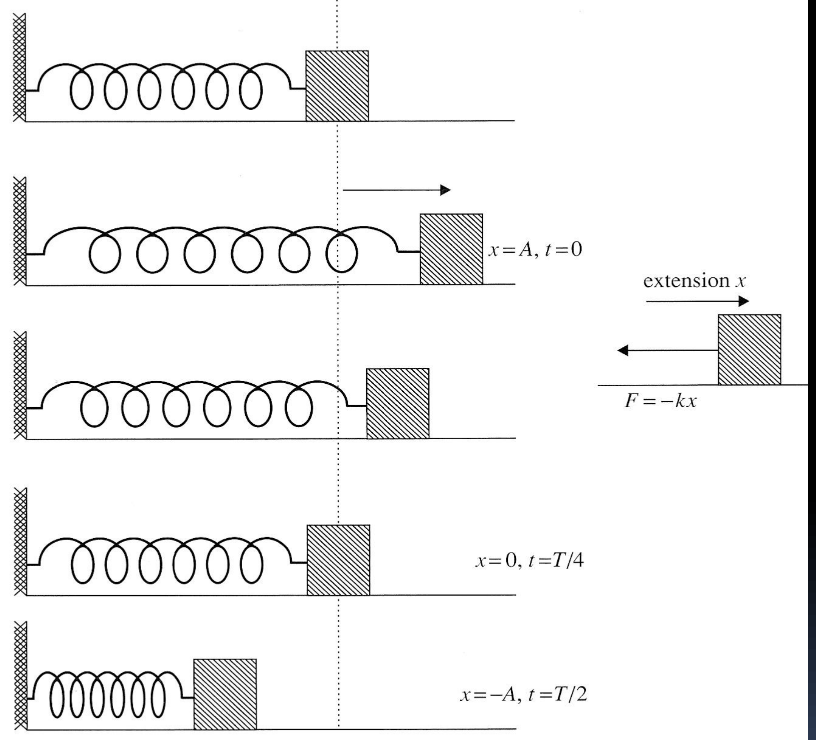


Figure 1.2 The mass–spring system. The net force on the body is proportional to the displacement and opposite to it.

Definitions

- ***Period*** – time to complete one full oscillation (time to return to starting point)
- ***Amplitude*** – maximum displacement from the equilibrium position



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Characteristics of SHM

- Period and amplitude are constant
- Period is independent of the amplitude
- Displacement, velocity, and acceleration are sine or cosine functions of time

Simple Harmonic Motion: Spring

- The spring possesses an intrinsic *restoring force* that attempts to bring the object back to equilibrium:

$$F = -kx$$

  - This is Hooke's Law
  - k is the spring constant (kg/s^2)
  - The negative sign is because the force acts in the direction opposite to the displacement -- *restoring force*

Simple Harmonic Motion: Spring

- Meanwhile, the inertia of the mass executes a force opposing the spring, F=ma:
  - spring executing force on mass

$$F = -kx$$

  - mass executing force on spring

$$F = ma$$

Simple Harmonic Motion: Spring

- These forces remain in balance throughout the motion:

$$ma = -kx$$

- The relationship between acceleration and displacement is thus,

$$a = -\frac{k}{m}x$$

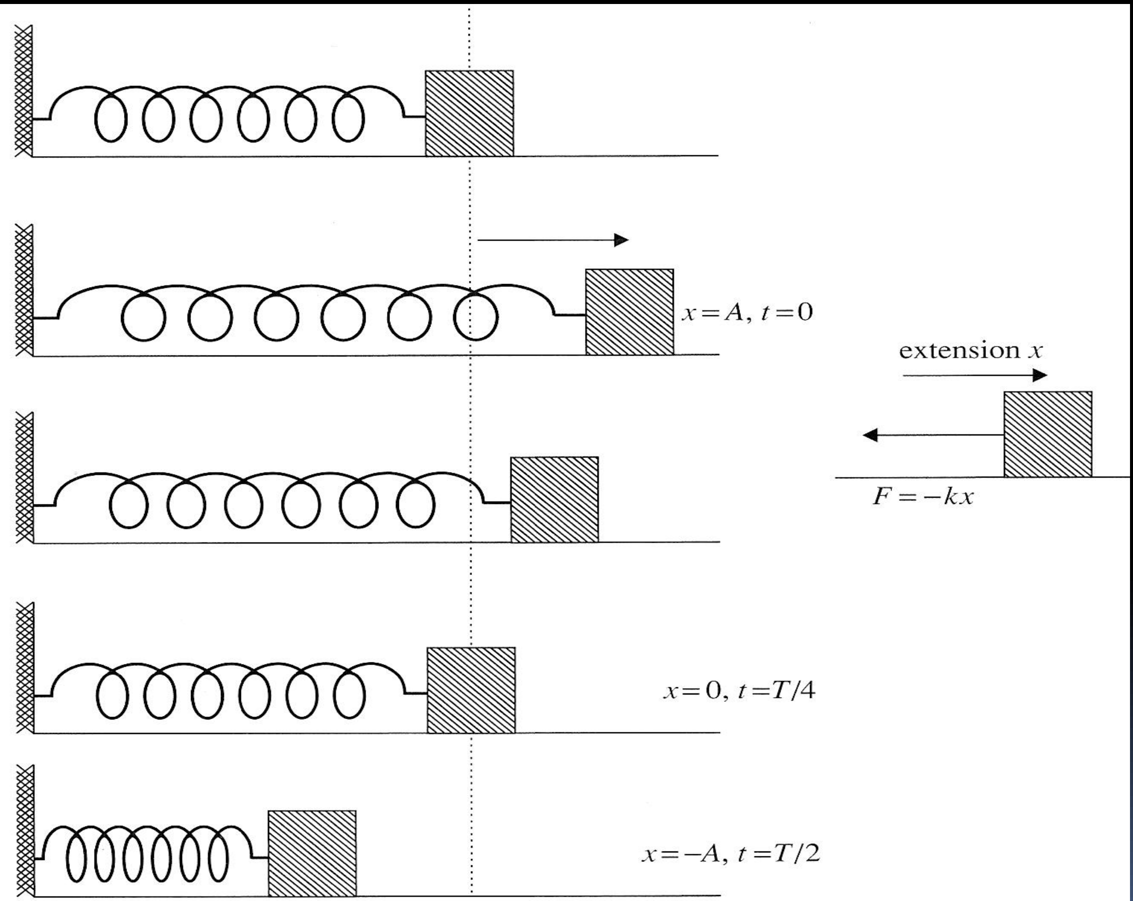
Simple Harmonic Motion: Spring

$$a = -\frac{k}{m}x$$

$$a \propto -x$$

- **Satisfies the** ***requirement for SHM that displacement from an equilibrium position and the force/acceleration are proportional and opposite to each other***

Simple Harmonic Motion: Spring



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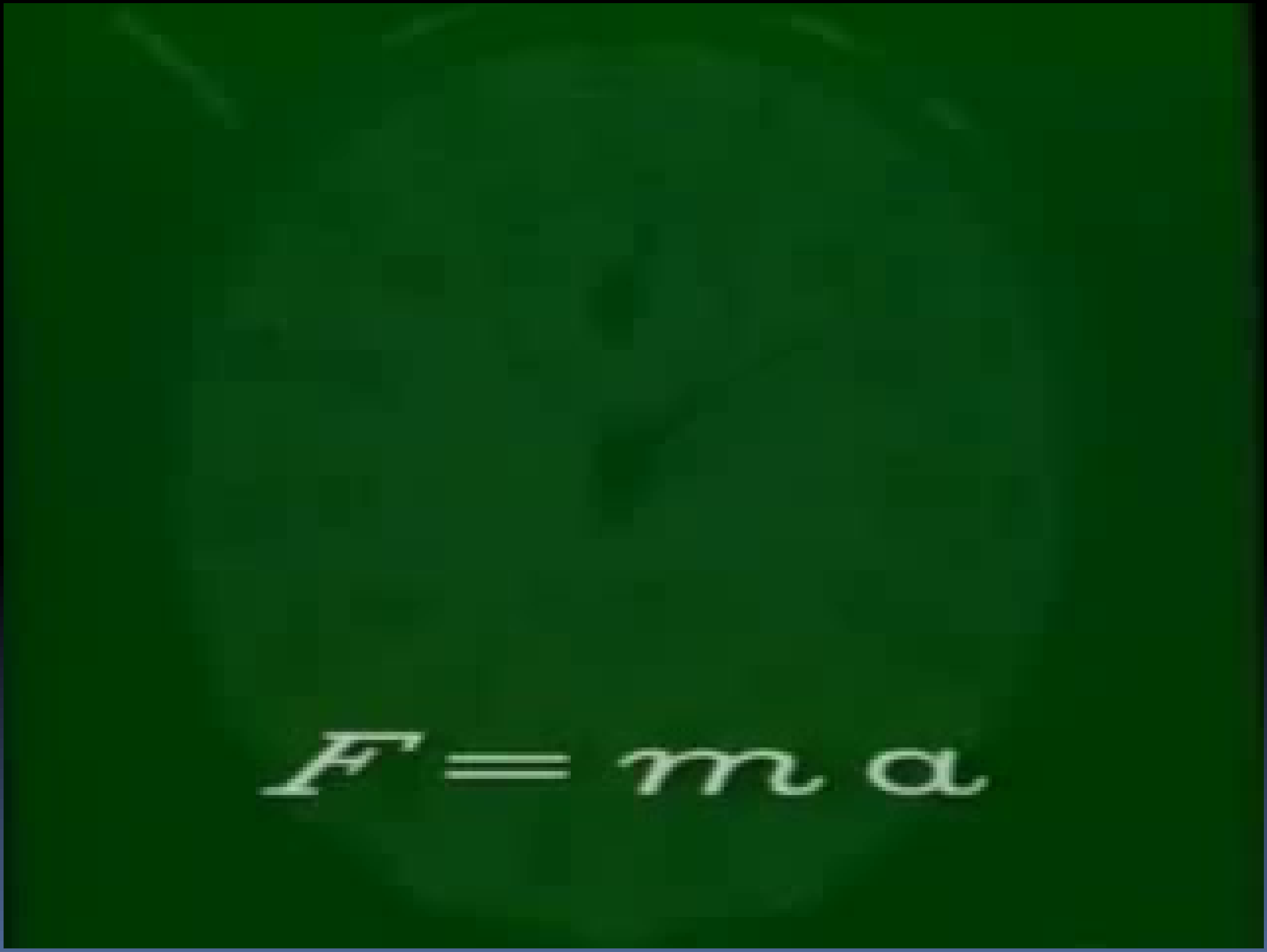
Relating SHM to Motion Around A Circle



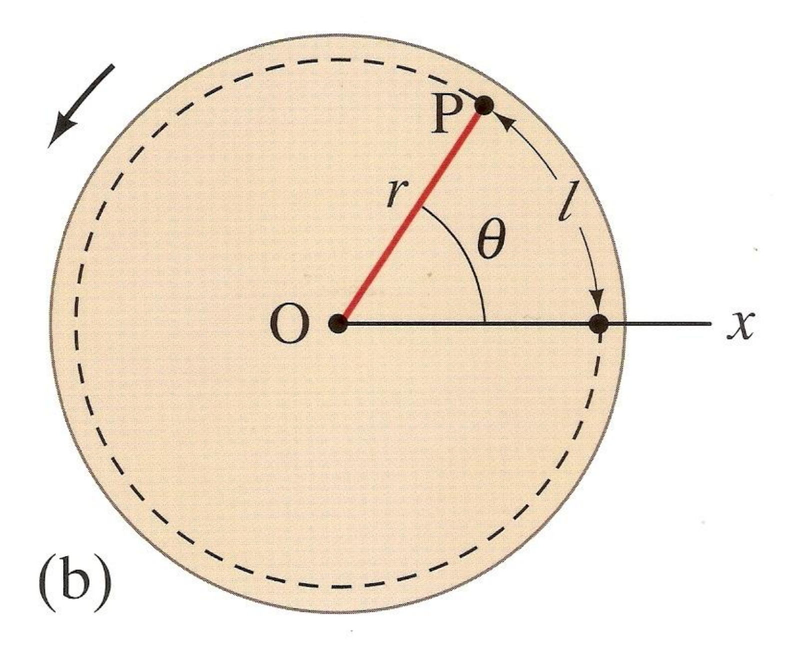
Radians

- One radian is defined as the angle subtended by an arc whose length is equal to the radius

$$\theta = \frac{l}{r}$$

$$l = r$$

$$\theta = 1$$



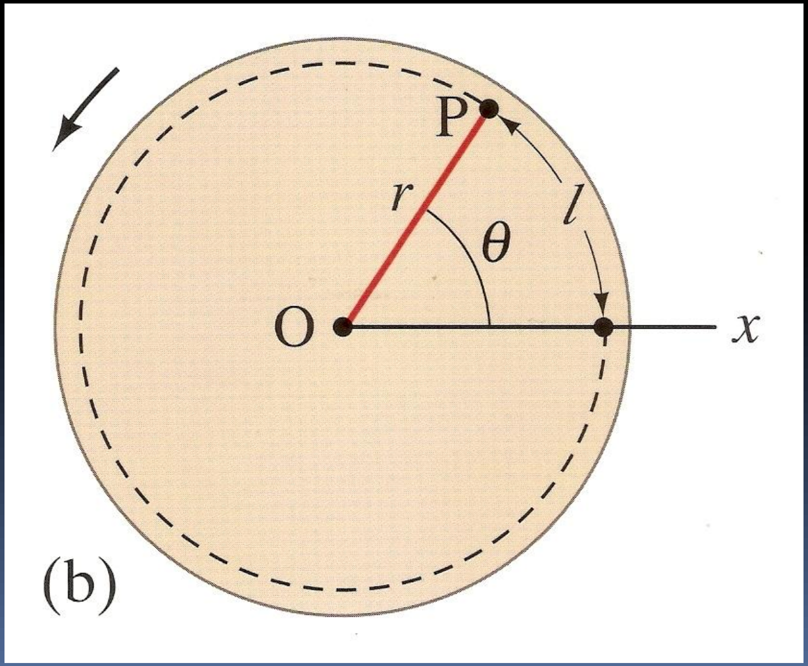
Radians

$$Circumference = 2\pi r$$

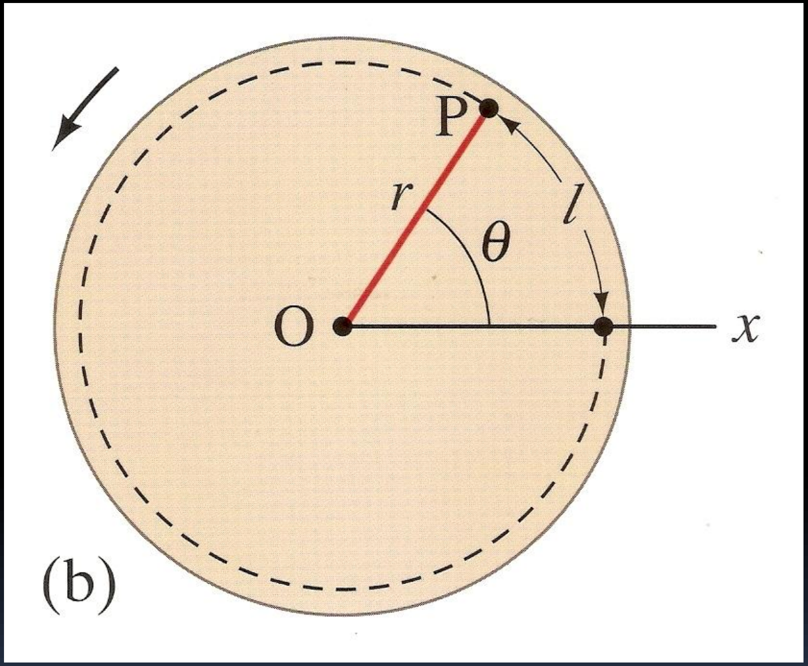
$$\theta = \frac{l}{r}$$

$$l = 2\pi r$$

$$Circumference = 2\pi(rad)$$



Angular Velocity



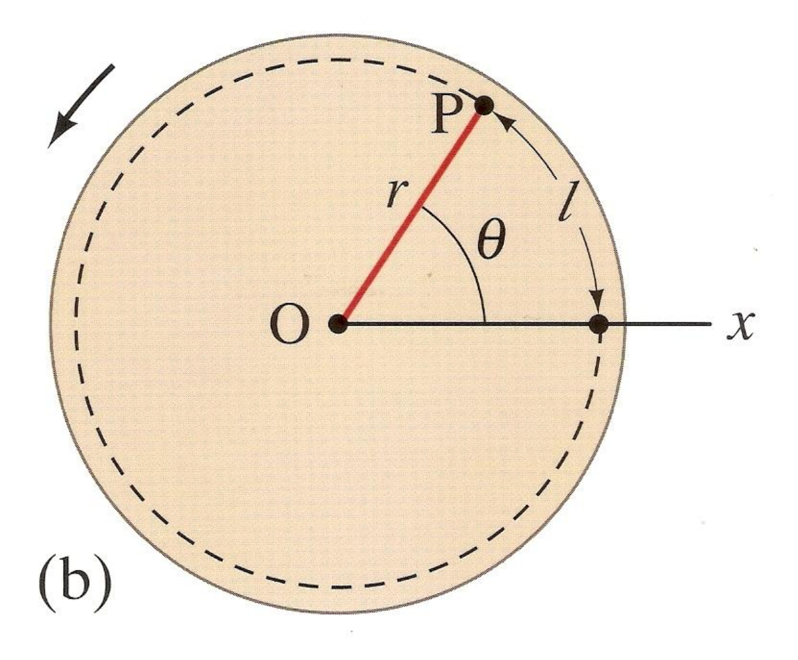
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v = \frac{\Delta l}{\Delta t}$$

$$\theta = \frac{l}{r}, l = r\theta$$

$$v = r\frac{\Delta\theta}{\Delta t} = r\omega$$

Angular Acceleration



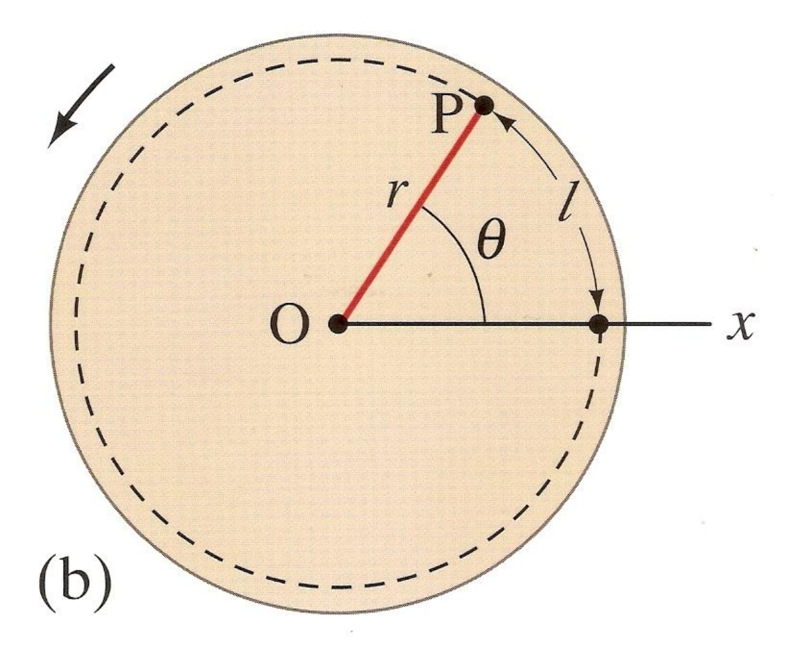
$$a = \frac{\Delta \omega}{\Delta t}$$

$$a_r = \frac{v^2}{r}$$

$$v = r\omega$$

$$a_r = \frac{(r\omega)^2}{r} = \omega^2 r$$

Period

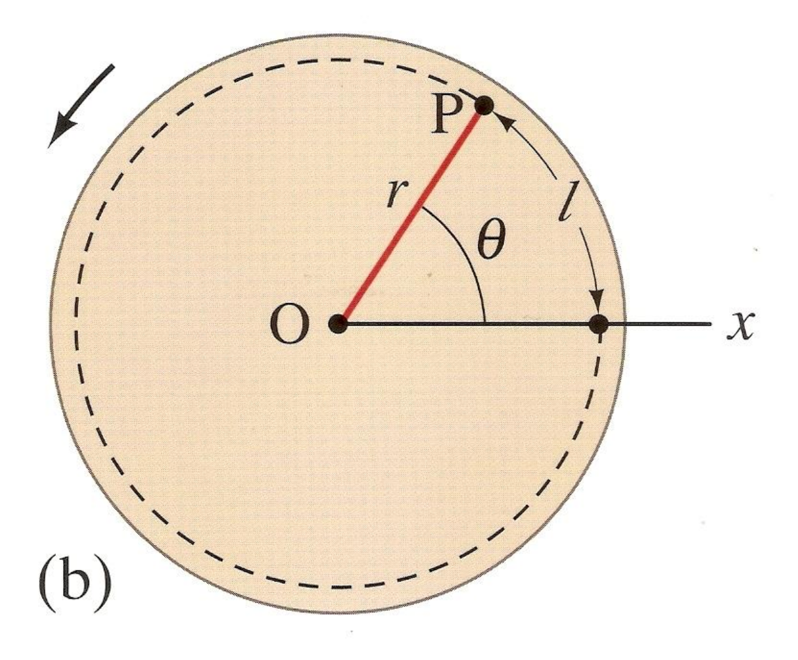


$$v_T = \frac{2\pi r}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Frequency



$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$

$$f = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

Relating SHM to Motion Around A Circle

- **The period in one complete oscillation of simple harmonic motion can be likened to the period of one complete revolution of a circle.**

$$\text{Time taken} = \frac{\text{angle swept}}{\text{angular speed } (\omega)}$$

$$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T}$$

Relating SHM to Motion Around A Circle

$$\Sigma F = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m}x$$

$$a = \omega^2 r$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$a = -\omega^2 x$$

Relating SHM to Motion Around A Circle

- Using,

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

- We then derive

$$x = A\cos(\omega t + \phi)$$

$$v = -\omega A\sin(\omega t + \phi)$$

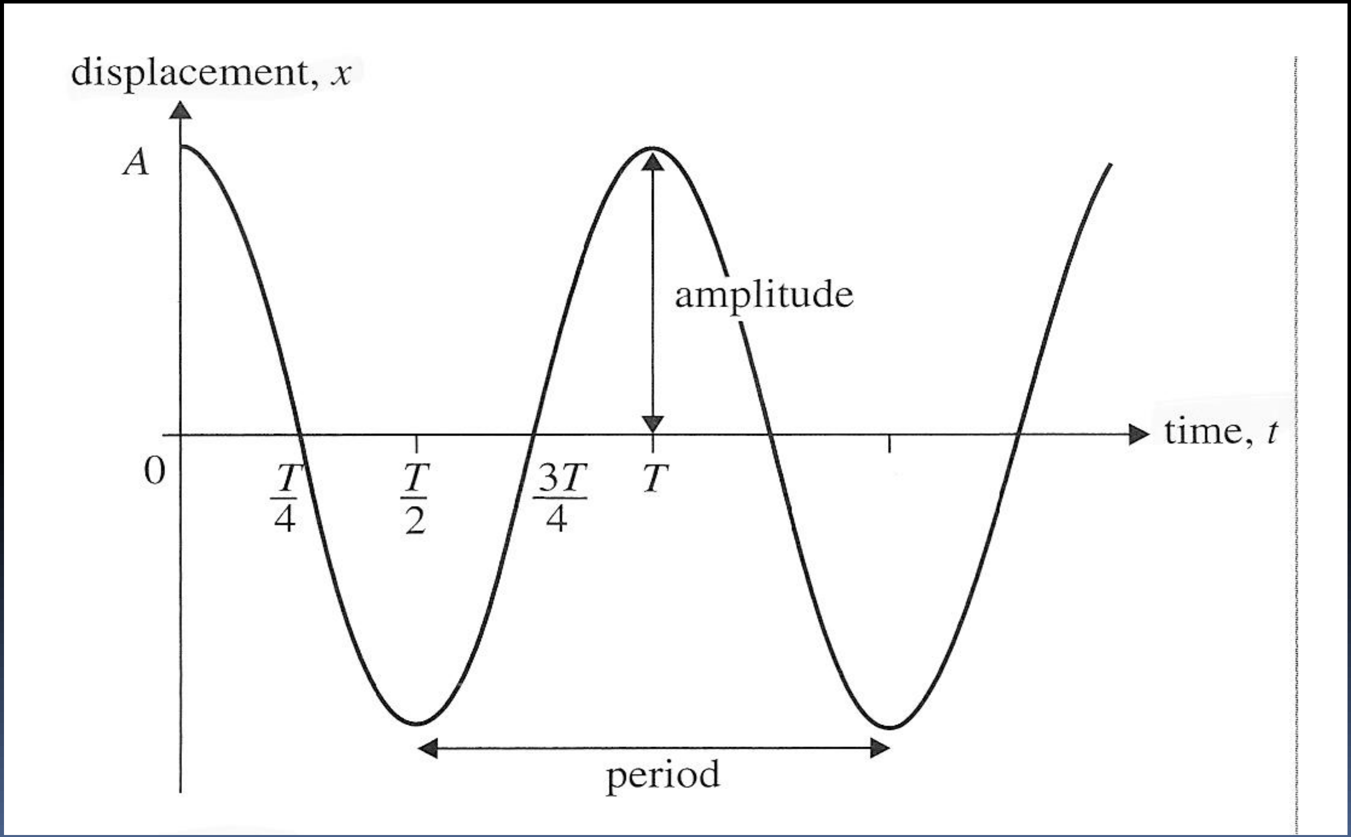
$$T = \frac{2\pi}{\omega}$$

Relating SHM to Motion Around A Circle

$$x = A\cos(\omega t)$$

$$v = -\omega A\sin(\omega t)$$

- These equations yield the following graphs



Relating SHM to Motion Around A Circle

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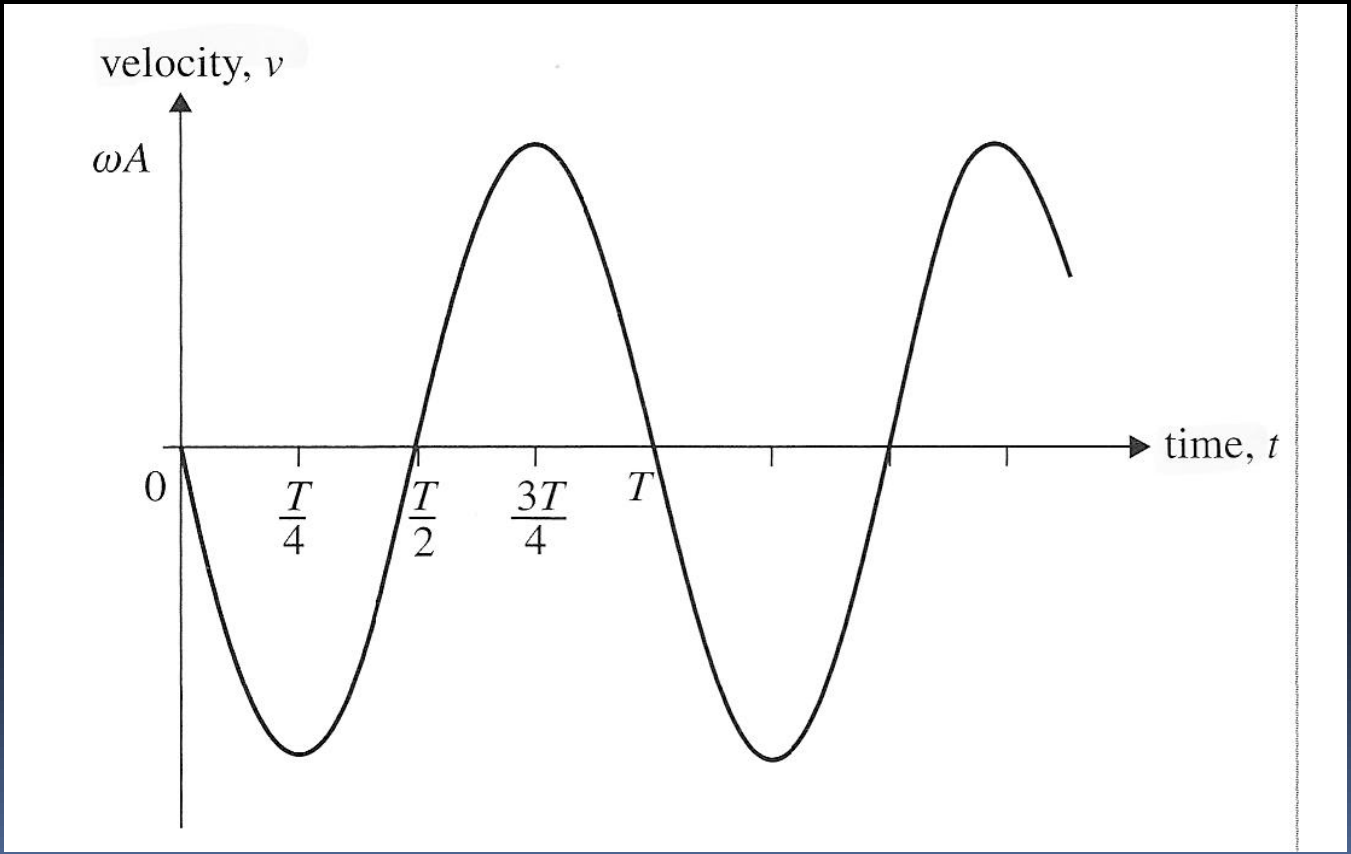

Relating SHM to Motion Around A Circle

$$x = A\cos(\omega t)$$

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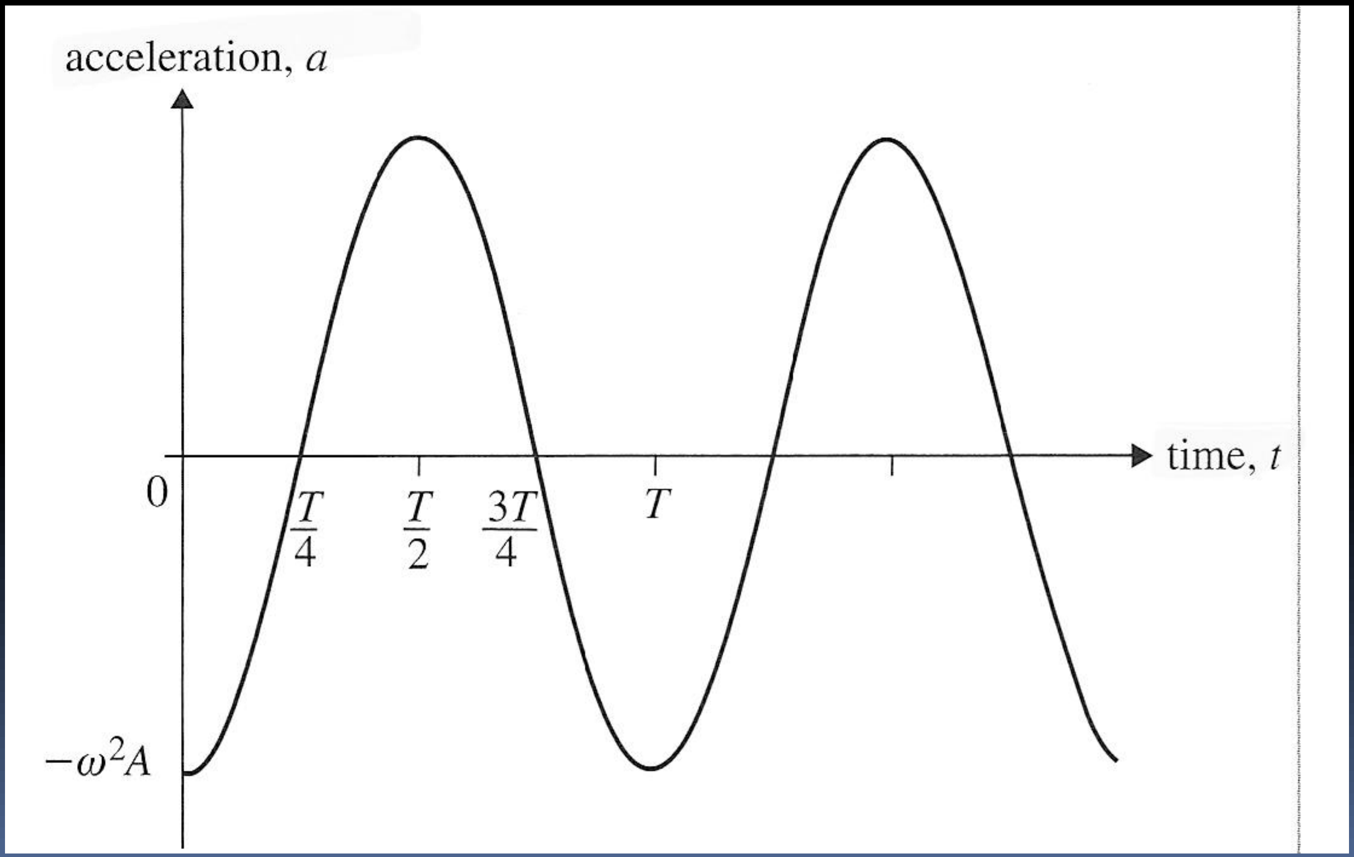
- These equations yield the following graphs



Relating SHM to Motion Around A Circle

- These equations yield the following graphs

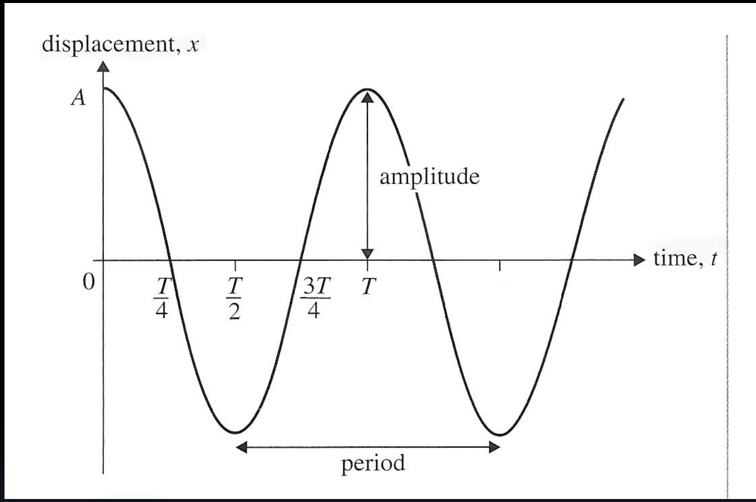
- ***What is the difference between***

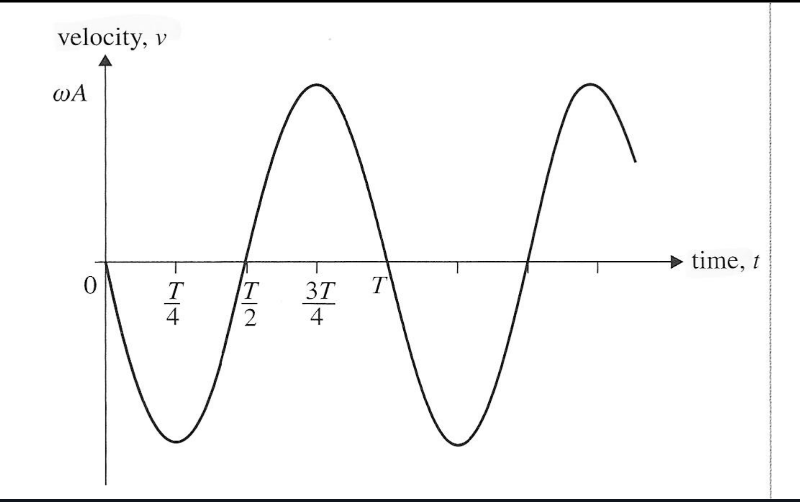
$$y = A\cos(\omega t)$$

$$y = -A\sin(\omega t)$$

Relating SHM to Motion Around A Circle

- These equations yield the following graphs





- ***What is the difference between***

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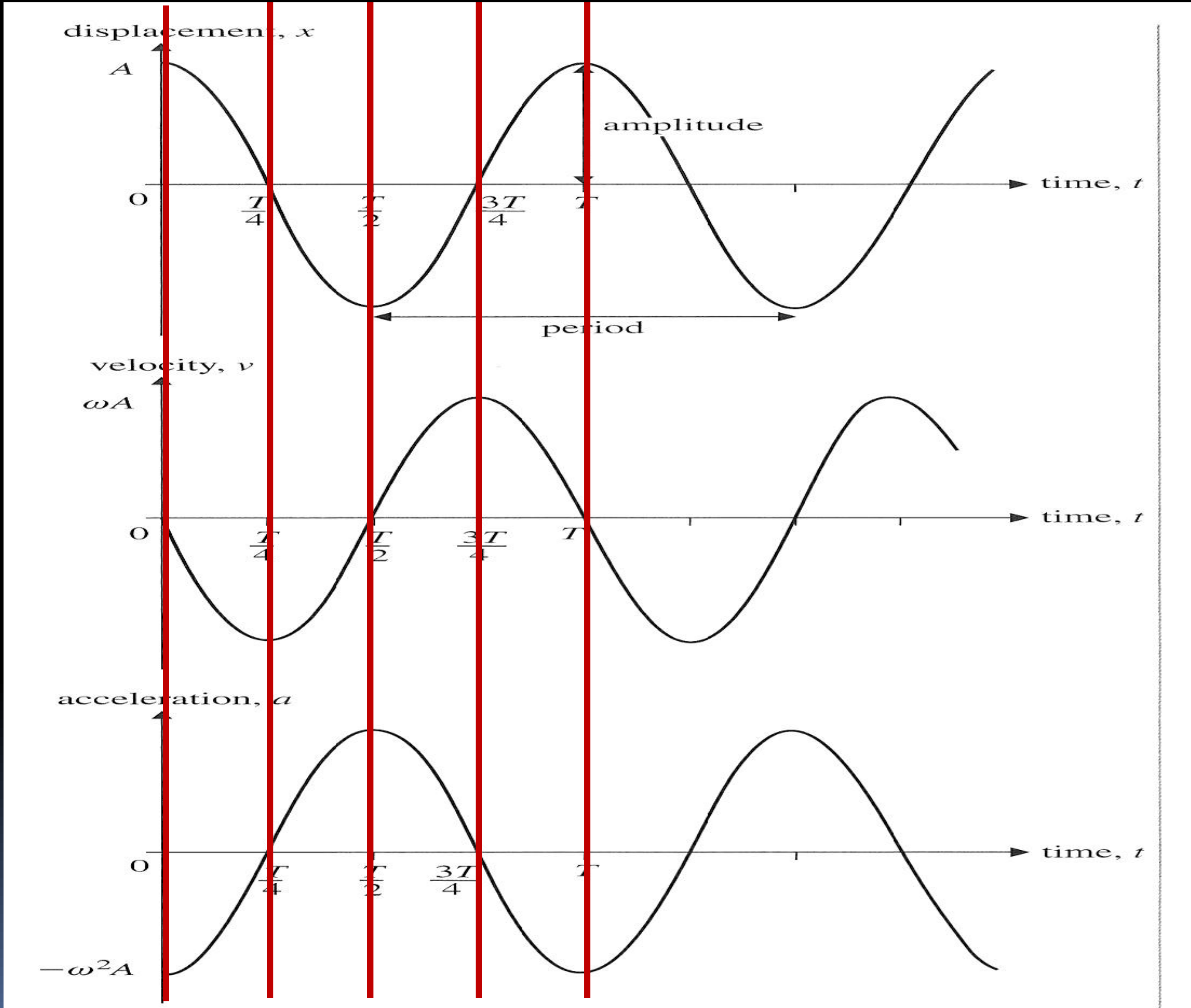
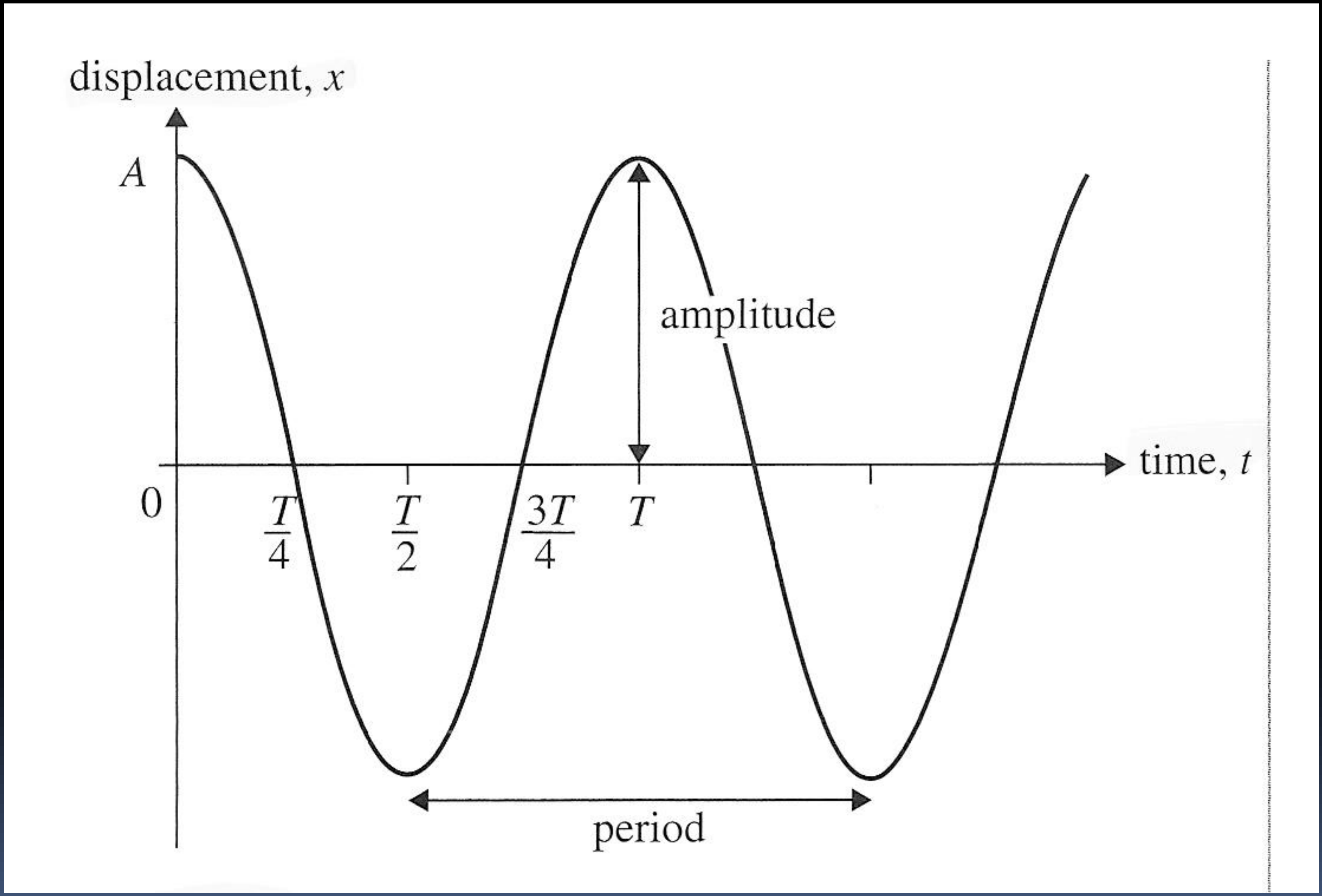


displacement, x
A
amplitude
time, t
0
T/4
T/2
3T/4
T
period
velocity, v
ωA
time, t
0
T/4
T/2
3T/4
T
acceleration, a
time, t
0
T/4
T/2
3T/4
T
−ω²A

Definitions

- Understand the terms *displacement, amplitude and period*
  - *displacement (x)* – distance from the equilibrium or zero point
  - *amplitude (A)* – maximum displacement from the equilibrium or zero point
  - *period (T)* – time it takes to complete one oscillation and return to starting point

Definitions



Definitions

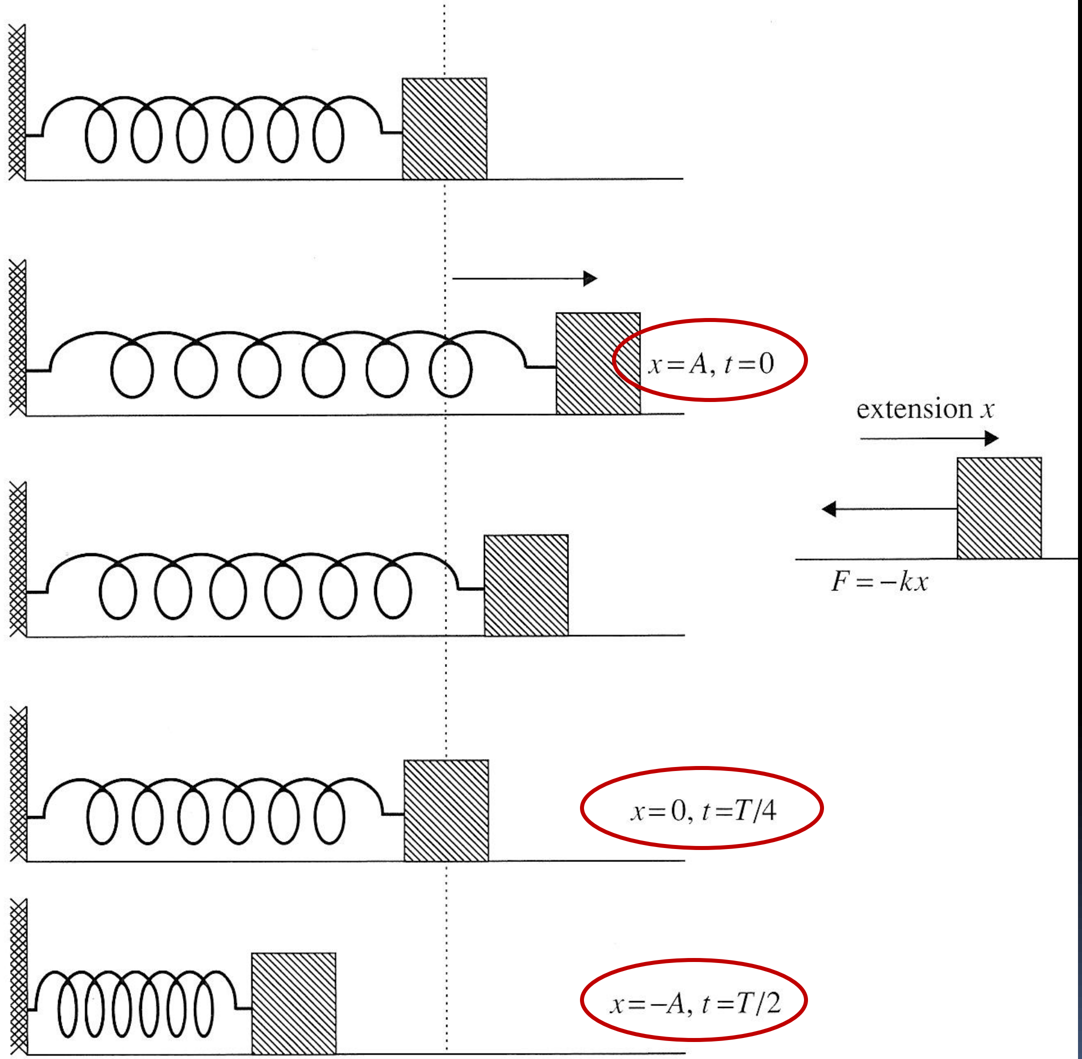


Figure 1.2 The mass–spring system. The net force on the body is proportional to the displacement and opposite to it.

Definitions

- **Understand the terms *period and frequency***
  - ***frequency (f)*** **– How many oscillations are completed in one second, equal to the inverse of the period**
  - ***period (T)*** **– Time for one complete oscillation, equal to the inverse of the frequency**

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{1}{f}$$

$$f = \frac{1}{T}$$

$$f = \frac{\omega}{2\pi}$$

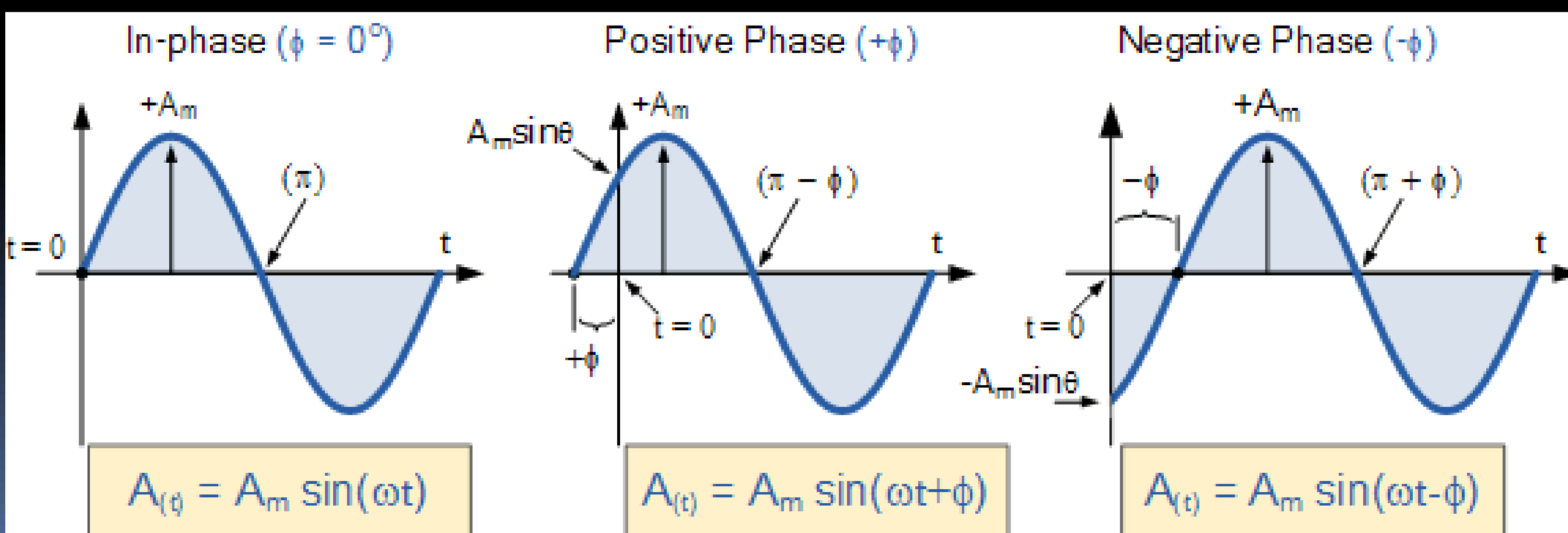
$$\omega = 2\pi f$$

Definitions

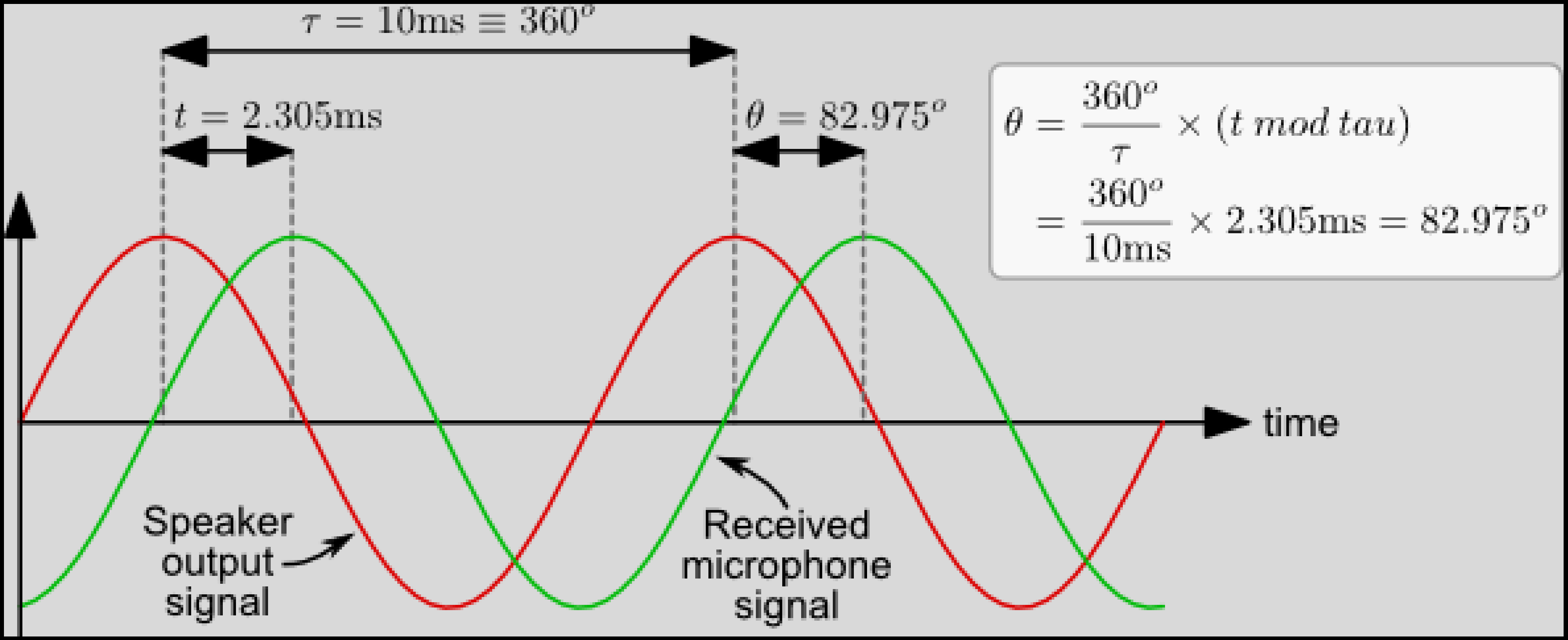
$$x = A\cos(\omega t + \phi)$$

$$v = -\omega A\sin(\omega t + \phi)$$

- **Understand the term *phase;***
  - ***phase (φ)* – the difference between the actual position of a sine wave at t=o and zero. The value of φ determines the displacement at t=o**



Phase Shift



$$\phi = \frac{\Delta t}{T} x360°$$

$$\phi = \frac{\Delta t}{T} x2\pi$$

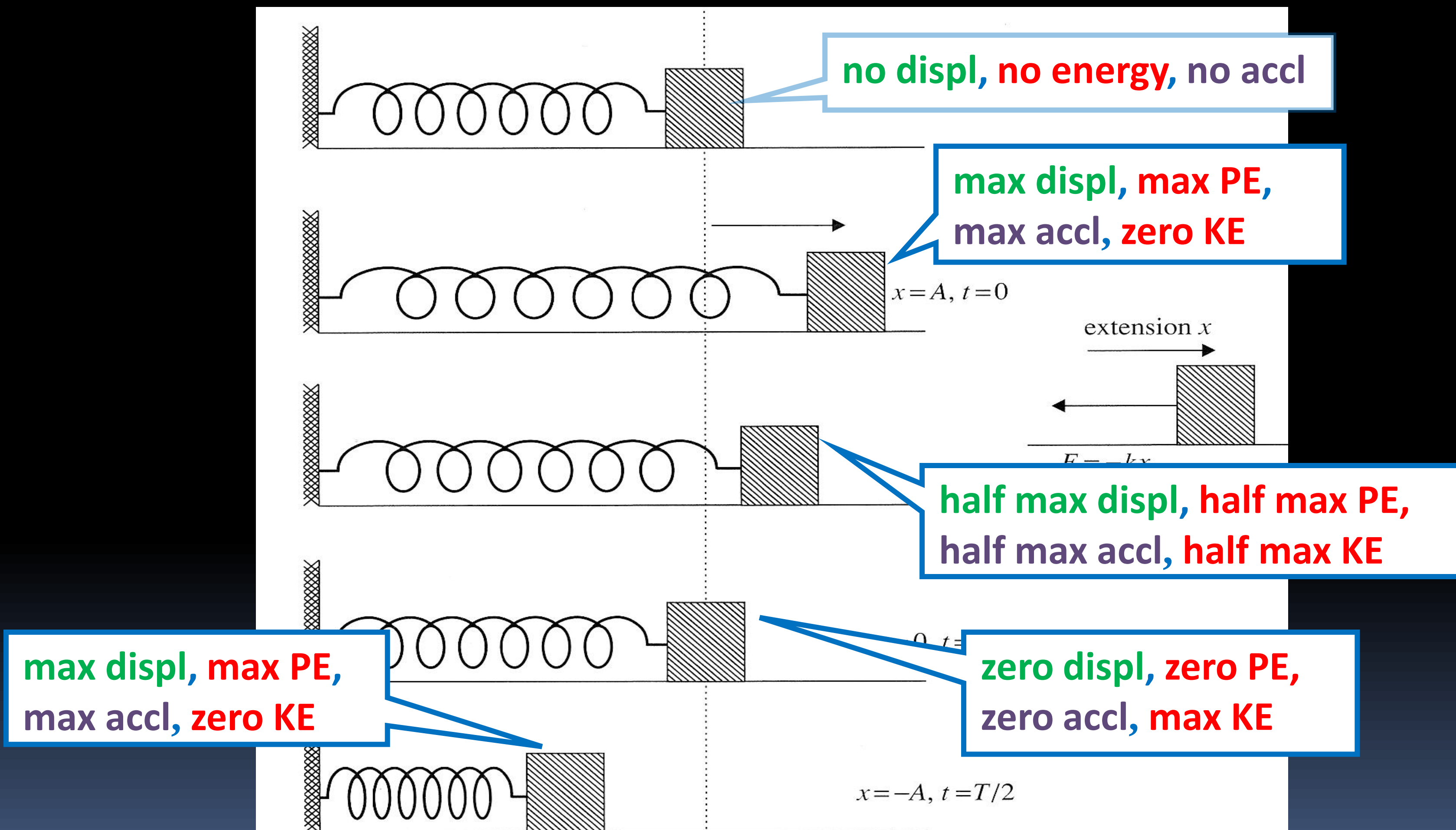
$$\phi = \frac{\Delta x}{\lambda} x360°$$

$$\phi = \frac{\Delta x}{\lambda} x2\pi$$

Energy in Simple Harmonic Motion

- Conservation of Energy (assuming no dissipative forces
- In simple harmonic motion there is *continuous transformation of energy* from kinetic energy into elastic potential energy and vice versa

Simple Harmonic Motion: Spring



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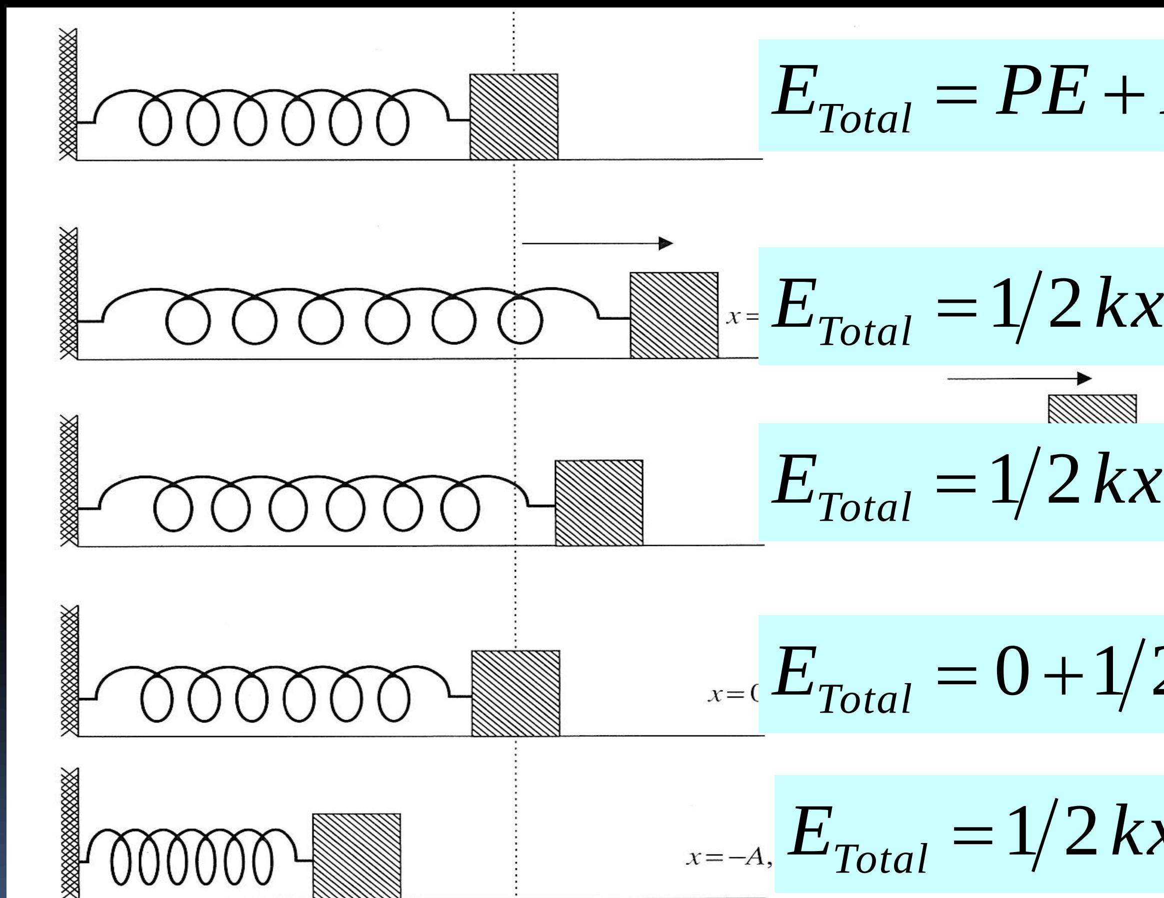

$$E_{Total} = PE + KE = 0 + 0$$

$$E_{Total} = 1/2\, kx^2 + 0$$

$$E_{Total} = 1/2\, kx^2 + 1/2\, mv^2$$

$$E_{Total} = 0 + 1/2\, mv^2$$

$$E_{Total} = 1/2\, kx^2 + 0$$

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Energy in SHM

$$E = PE + KE = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = cons\tan t$$

$$E_{\max} = PE_{\max} + 0 = 0 + KE_{\max}$$

$$E_{\max} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$$

Understandings:

- Simple harmonic oscillations
- Time period, frequency, amplitude, displacement and phase difference
- Conditions for simple harmonic motion

Data Booklet Reference:

$$T = \frac{1}{f}$$

Applications And Skills:

- Qualitatively describing the energy changes taking place during one cycle of an oscillation
- Sketching and interpreting graphs of simple harmonic motion examples

Utilization:

- Isochronous oscillations can be used to measure time
- Many systems can approximate simple harmonic motion: mass on a spring, fluid in U-tube, models of icebergs oscillating vertically in the ocean, and motion of a sphere rolling in a concave mirror
- Simple harmonic motion is frequently found in the context of mechanics (see Physics topic 2)

Essential Idea:

- A study of oscillations underpins many areas of physics with simple harmonic motion (SHM), a fundamental oscillation that appears in various natural phenomena.

Tacoma Narrows Bridge Collapse

Homework

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Beautiful

Resonance

Stopped Here 2/21/2013