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BAODEST GLASSON GAXPTIS
gis prousses

## LESSON 2-1C

PROJECTILE MOTION FLUID RESISTANCE

## Introduction Videos

## - Projectile Motion 1

- Useful Applications of Projectile Motion


## Essential Idea:

- Motion may be described and analyzed by the use of graphs and equations.


## Nature Of Science:

- Observations: The ideas of motion are fundamental to many areas of physics, providing a link to the consideration of forces and their implication. The kinematic equations for uniform acceleration were developed through careful observations of the natural world.


## International-Mindedness:

- International cooperation is needed for tracking shipping, land-based transport, aircraft and objects in space.


## Theory Of Knowledge:

- The independence of horizontal and vertical motion in projectile motion seems to be counter-intuitive.
- How do scientists work around their intuitions?
- How do scientists make use of their intuitions?


## Understandings:

- Projectile motion
- Fluid resistance and terminal speed


## Applications And Skills:

- Analyzing projectile motion, including the resolution of vertical and horizontal components of acceleration, velocity and displacement.
- Qualitatively describing the effect of fluid resistance on falling objects or projectiles, including reaching terminal speed.


## Guidance:

- Calculations will be restricted to those neglecting air resistance.
- Projectile motion will only involve problems using a constant value of g close to the surface of the Earth.
- The equation of the path of a projectile will not be required.


## Data Booklet Reference:

$$
\begin{aligned}
& v=u+a t \\
& s=u t+1 / 2 a t^{2} \\
& v^{2}=u^{2}+2 a s \\
& s=\frac{(v+u) t}{2}
\end{aligned}
$$

## Utilization:

- Diving, parachuting and similar activities where fluid resistance affects.
- The accurate use of ballistics requires careful analysis.
- Quadratic functions (see Mathematics HL sub-topic 2.6; Mathematics SL sub-topic 2.4; Mathematical studies SL sub-topic 6.3).
- The kinematic equations are treated in calculus form in Mathematics HL sub-topic 6.6 and Mathematics SL sub-topic 6.6.


## Aims:

- Aim 2: much of the development of classical physics has been built on the advances in kinematics


## Aims:

- Aim 6: experiments, including use of data logging, could include (but are not limited to): determination of g , estimating speed using travel timetables, analyzing projectile motion, and investigating motion through a fluid


## Aims:

- Aim 7: technology has allowed for more accurate and precise measurements of motion, including video analysis of real-life projectiles and modeling/simulations of terminal velocity

One Dimensional Motion

## WHERE WE'VE BEEN

## Horizontal Motion



## Kinematic Equations for Horizontal Motion

$$
\begin{array}{ll}
v=v_{0}+a t & {[a=\text { constant }]} \\
x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} & {[a=\text { constant }]} \\
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) & {[a=\text { constant }]} \\
\bar{v}=\frac{v+v_{0}}{2} . & {[a=\text { constant }]}
\end{array}
$$

## Vertical Motion Drop Problems



## Vertical Motion with Vertical Velocity



## Kinematic Equations

## Horizontal

## Vertical

$$
v_{x}=v_{x 0}+a_{x} t
$$

$$
x=x_{0}+v_{x 0} t+1 / 2 a_{x} t^{2}
$$

$$
v_{x}^{2}=v_{x 0}^{2}+2 a_{x}\left(x-x_{0}\right)
$$

$$
\begin{aligned}
& v_{y}=v_{y 0}+a_{y} t \\
& y=y_{0}+v_{y 0} t+1 / 2 a_{y} t^{2} \\
& v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right)
\end{aligned}
$$

## Kinematic Equations

- IB Style

$$
\begin{aligned}
& v=u+a t \\
& s=u t+1 / 2 a t^{2} \\
& v^{2}=u^{2}+2 a s \\
& s=\frac{(v+u) t}{2}
\end{aligned}
$$

## Vectors <br> WHERE WE'VE BEEN

## Vectors and Scalars

Figure 3-3 A person walks 10.0 km east and then 5.0 km north


South

## Vector Addition Graphically



## Vector Subtraction Graphically

$$
-\vec{V}_{1}
$$

$$
\vec{V}_{2} /-
$$

$$
=\vec{V}_{2} /
$$



$$
=\overrightarrow{\mathrm{V}}_{2}-\overrightarrow{\mathrm{V}}_{1} \sqrt{\overrightarrow{\mathrm{~V}}_{2}}
$$

## Breaking

 Vectors Down Into Components
## Adding Vectors Using Components



## PUTTING IT ALL TOGETHER

## Two Dimensional Projectile Motion Case 1



## Two Dimensional Projectile Motion - Case 2



## Thanks, Galileo



## One Key Finding

- The time it takes for an object to fall from a given height is the same whether it is simply dropped or if it begins with a horizontal velocity.
- Demonstration



## Kinematic Equations

## Horizontal

## Vertical

$$
v_{x}=v_{x 0}+a_{x} t
$$

$$
x=x_{0}+v_{x 0} t+1 / 2 a_{x} t^{2}
$$

$$
v_{x}^{2}=v_{x 0}^{2}+2 a_{x}\left(x-x_{0}\right)
$$

$$
\begin{aligned}
& v_{y}=v_{y 0}+a_{y} t \\
& y=y_{0}+v_{y 0} t+1 / 2 a_{y} t^{2} \\
& v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right)
\end{aligned}
$$

## Assumptions

- We consider motion only after it has been projected and is moving freely through the air
- We don't consider the acceleration it took to reach that velocity
- We consider air resistance to be negligible
- When the object is moving through the air, both horizontally and vertically, it doesn't slow down due to air resistance


## Make the Math Easier

$$
\begin{array}{l|l}
v_{x}=v_{x 0}+a_{x} t & v_{y}=v_{y 0}+a_{y} t \\
x=x_{0}+v_{x 0} t+1 / 2 a_{x} t^{2} & y=y_{0}+v_{y 0} t+1 / 2 a_{y} t^{2} \\
v_{x}^{2}=v_{x 0}^{2}+2 a_{x}\left(x-x_{0}\right) & v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right)
\end{array}
$$

- No horizontal acceleration, horizontal velocity remains constant
- Y-axis positive up, gravity negative down
- Acceleration in Parabolic Motion
$v_{x}=v_{x 0}+a_{x} t$
$x=x_{0}+v_{x 0} t+1 / 2 a_{x} t^{2}$
$v_{x}^{2}=v_{x 0}^{2}+2 a_{x}\left(x-x_{0}\right)$
$v_{y}=v_{y 0}+a_{y} t$
$y=y_{0}+v_{y 0} t+1 / 2 a_{y} t^{2}$
$v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right)$

Kinematic Equations for Projectile Motion

$$
\begin{aligned}
& v_{x}=v_{x 0} \\
& x=x_{0}+v_{x 0} t
\end{aligned}
$$

$$
\begin{aligned}
& v_{y}=v_{y 0}-g t \\
& y=y_{0}+v_{y 0} t-1 / 2 g t^{2} \\
& v_{y}^{2}=v_{y 0}^{2}-2 g y
\end{aligned}
$$

Kinematic Equations for
Projectile Motion

$$
\begin{aligned}
& v_{x}=v_{x 0} \\
& x=x_{0}+v_{x 0} t
\end{aligned}
$$

$v=u$

$$
s=u t
$$

$$
v=u+g t
$$

$$
\begin{aligned}
& s=u t+1 / 2 g t^{2} \\
& v^{2}=u^{2}+2 g s \\
& s=\frac{(v+u) t}{2}
\end{aligned}
$$

## Problem Solving Process - Extra

## Steps

1. Read the problem carefully and draw a picture
2. Choose origin and $x$ - $y$ coordinate system
3. If given an initial velocity, resolve it into $x$ and $y$-components.
4. Analyze horizontal ( $x$ ) and vertical ( $y$ ) motion separately
5. Continue with problem solving process for kinematic equations

## MANIPULATION OF VARIABLES PROJECTILE MOTION SIMULATOR

## Fluid Resistance

- Fluid resistance, or drag force, acts opposite to the direction of motion.
- The force due to drag is given by the equation (not testable)
- For our purposes, fluid resistance is proportional to velocity at low speeds and velocity squared at high speeds

$$
\begin{aligned}
& F_{D}=k v \\
& F_{D}=k v^{2}
\end{aligned}
$$

## Effect of Fluid Resistance



## Fluid Resistance

- Sequence of events:
- An object starts falling under the force of gravity with acceleration of $9.81 \mathrm{~m} / \mathrm{s}^{2}$ and no resistance
- As velocity increases, fluid resistance increases and acceleration decreases
- Eventually, the object reaches a speed where fluid resistance equals the force of gravity, acceleration has decreased to zero, and velocity is constant
This constant velocity is known as terminal velocity or terminal speed


## Terminal Velocity

$$
F_{D}=\frac{1}{2} \rho v^{2} C_{D} A
$$

- The equation for terminal velocity based on the drag force is given to the right

$$
F_{D}=m g
$$

$$
v_{T}=\sqrt{\frac{2 m g}{\rho C_{D} A}}
$$

- For our purposes, we will assume low speed ( $F_{\mathrm{D}} \approx v$ ) and terminal velocity will be

$$
\begin{aligned}
& v_{T}=\frac{2 m g}{\rho C_{D} A} \\
& k=1 / 2 \rho C_{D} A
\end{aligned}
$$

$$
v_{T}=\frac{m g}{k}
$$

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$$
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## Applications And Skills:

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QUESTIONS?

## Homework

## Pg. 56-57, \#25-33

## Weightlessness

- On Skis
- On A Motorcycle
- Sphere of Death

