DEVIL PHYSICS
THE BADDEST CLASS ON CAMPUS
IB PHYSICS

TSOKOS LESSON 1-2
UNCERTAINTIES AND ERRORS

Reading Activity Questions?

Essential Idea

- Scientists aim towards designing experiments that can give a “true value” from their measurements, but due to the limited precision in measuring devices, they often quote their results with some form of uncertainty.

Nature Of Science

- **Uncertainties**: "*All scientific knowledge is uncertain… if you have made up your mind already, you might not solve it. When the scientist tells you he does not know the answer, he is an ignorant man. When he tells you he has a hunch about how it is going to work, he is uncertain about it. When he is pretty sure of how it is going to work, and he tells you, 'This is the way it's going to work, I'll bet,' he still is in some doubt. And it is of paramount importance, in order to make progress, that we recognize this ignorance and this doubt. Because we have the doubt, we then propose looking in new directions for new ideas*." (3.4) *Feynman, Richard P. 1998. The Meaning of It All: Thoughts of a Citizen-Scientist. Reading, Massachusetts, USA. Perseus. P 13.*

Theory Of Knowledge

- "*One aim of the physical sciences has been to give an exact picture of the material world. One achievement of physics in the twentieth century has been to prove that this aim is unattainable*." – Jacob Bronowski.
- Can scientists ever be truly certain of their discoveries?

Understandings

- Random and systematic errors
- Absolute, fractional and percentage uncertainties
- Error bars
- Uncertainty of gradient and intercepts

Applications And Skills

- Explaining how random and systematic errors can be identified and reduced
- Collecting data that include absolute and/or fractional uncertainties and stating these as an uncertainty range (expressed as: best estimate ± uncertainty range)

Applications And Skills

- Propagating uncertainties through calculations involving addition, subtraction, multiplication, division and raising to a power
- Determining the uncertainty in gradients and intercepts

Guidance

- Analysis of uncertainties will not be expected for trigonometric or logarithmic functions in examinations

Data Booklet Reference

- If $y = a \pm b$
  - Then $\Delta y = \Delta a \pm \Delta b$
- If $y = \frac{ab}{c}$
  - Then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$
- If $y = a^n$
  - then $\frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$

Utilization

- Students studying more than one group 4 subject will be able to use these skills across all subjects

Aims

- **Aim 4:** it is important that students see scientific errors and uncertainties not only as the range of possible answers but as an integral part of the scientific process
- **Aim 9:** the process of using uncertainties in classical physics can be compared to the view of uncertainties in modern (and particularly quantum) physics

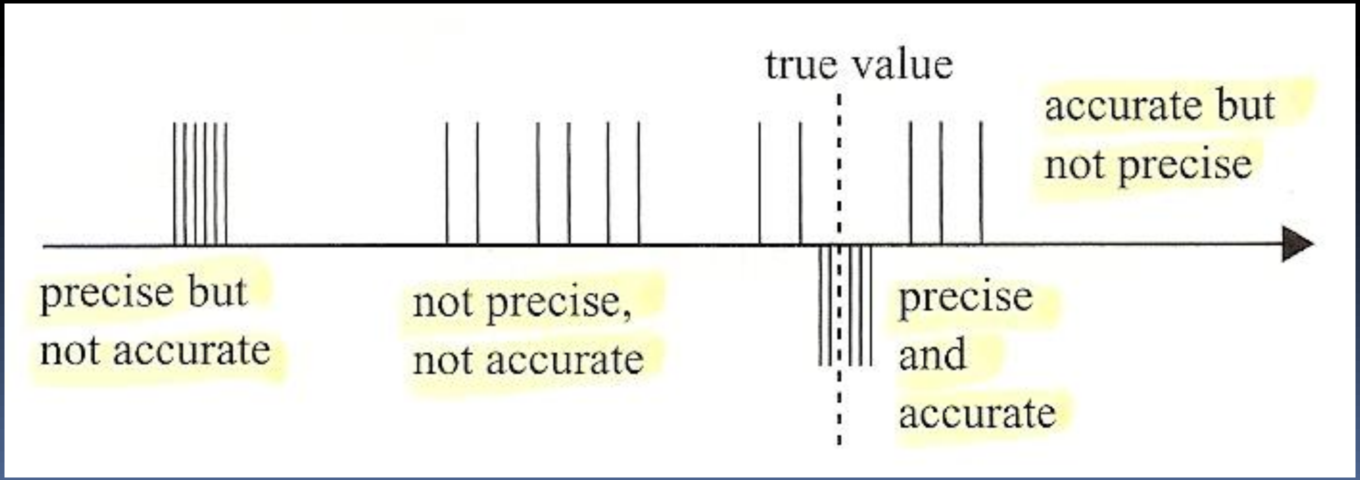
Accuracy and Precision Video

Accuracy and Precision

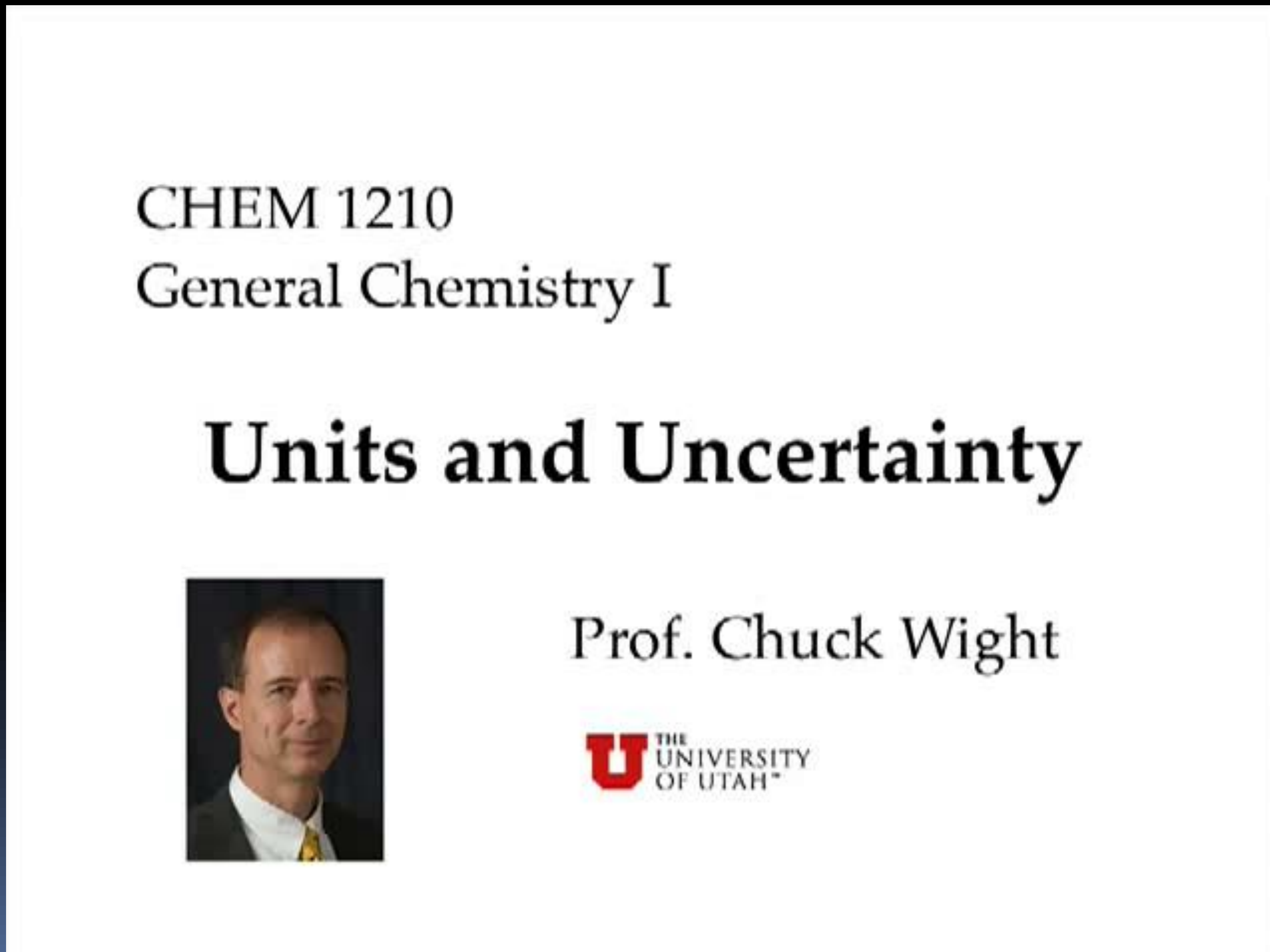
- Measurements can never be absolutely precise
- Because of this, there will always be some amount of uncertainty in a measurement
- ***As scientists, we must always strive to minimize uncertainty and to suggest ways to further minimize uncertainty in future experiments***
- ***As scientists, we must quantify and express the level of uncertainty in our data***

Accuracy and Precision

- Measurements are accurate if the systematic error is small
  - Individual deviations may be high, but mean is close to actual value
- Measurements are precise if random error is small
  - Individual deviations are low, but mean is significantly different from actual value



Accounting for Uncertainty



Errors of measurement

- Random
  - almost always the fault of the observer
  - human error
  - outside factors, uncontrolled variables
  - not predictable
  - assumed to be sometimes high, sometimes low
  - can be minimized by averaging over repeated measurements
  - What are some examples?

Errors of measurement

- Systematic
  - usually the fault of both the user and the instrument
  - error is in the same direction and in the same amount for each measurement
  - <u>cannot</u> be minimized by repeated measurements
  - What are some examples?

Errors of measurement

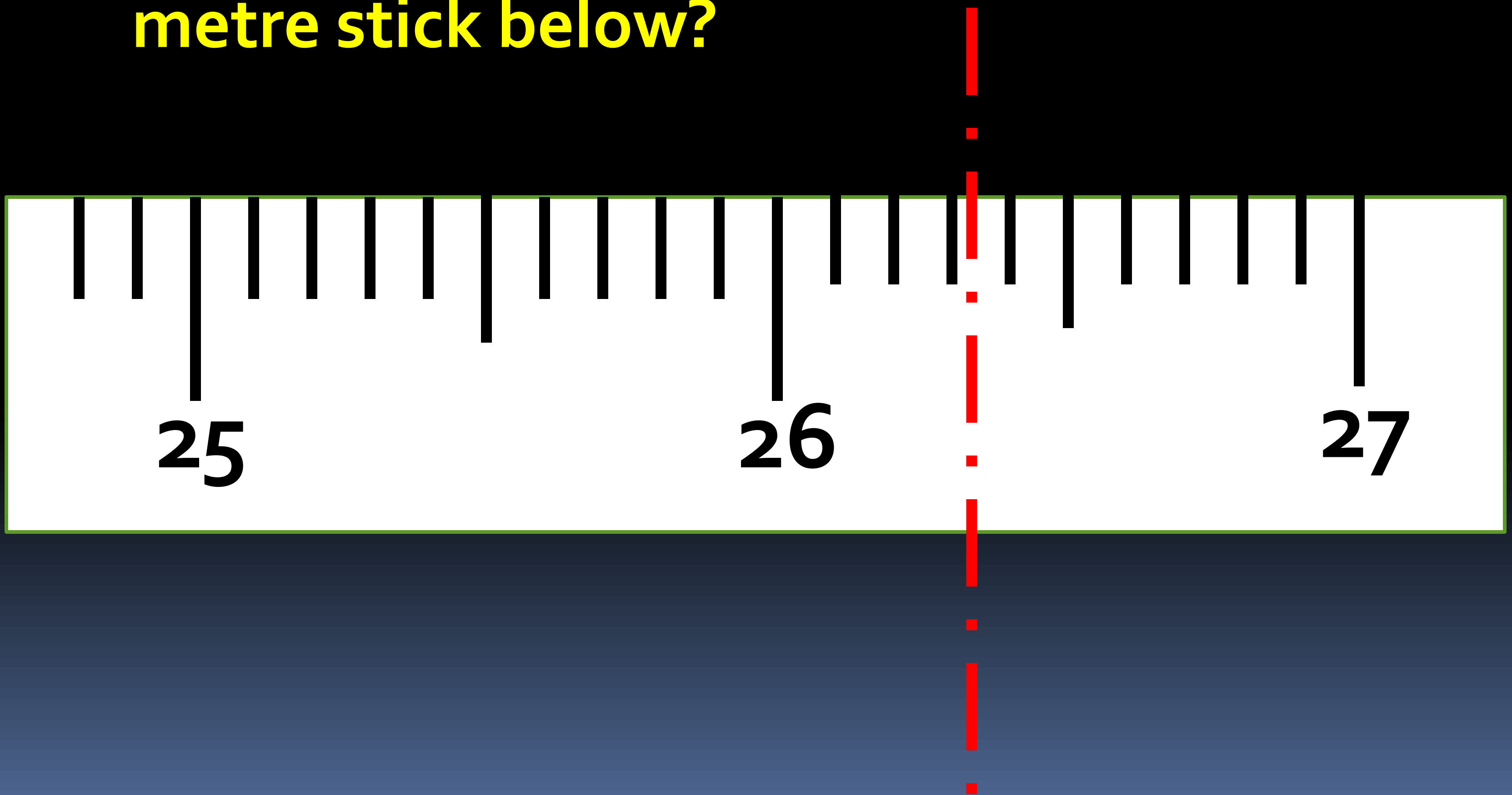
- Ball Bounce Lab
  - *Should you have an x- or y-intercept on the graph?*

Errors of measurement

- Reading or tool error
  - function of the precision of the instrument
  - assumed to be ± one half of the smallest division on an analog tool
    - If a ruler measures in mm, the reading/tool error is assumed to be ±0.5mm
    - Remember that you can estimate between divisions!
  - for digital instruments, the reading/tool error is assumed to be ± the smallest division
    - If a digital scale reads hundredths of a gram, the reading/ tool error is assumed to be ±0.01g

Errors of measurement

What is the measurement and error from the metre stick below?



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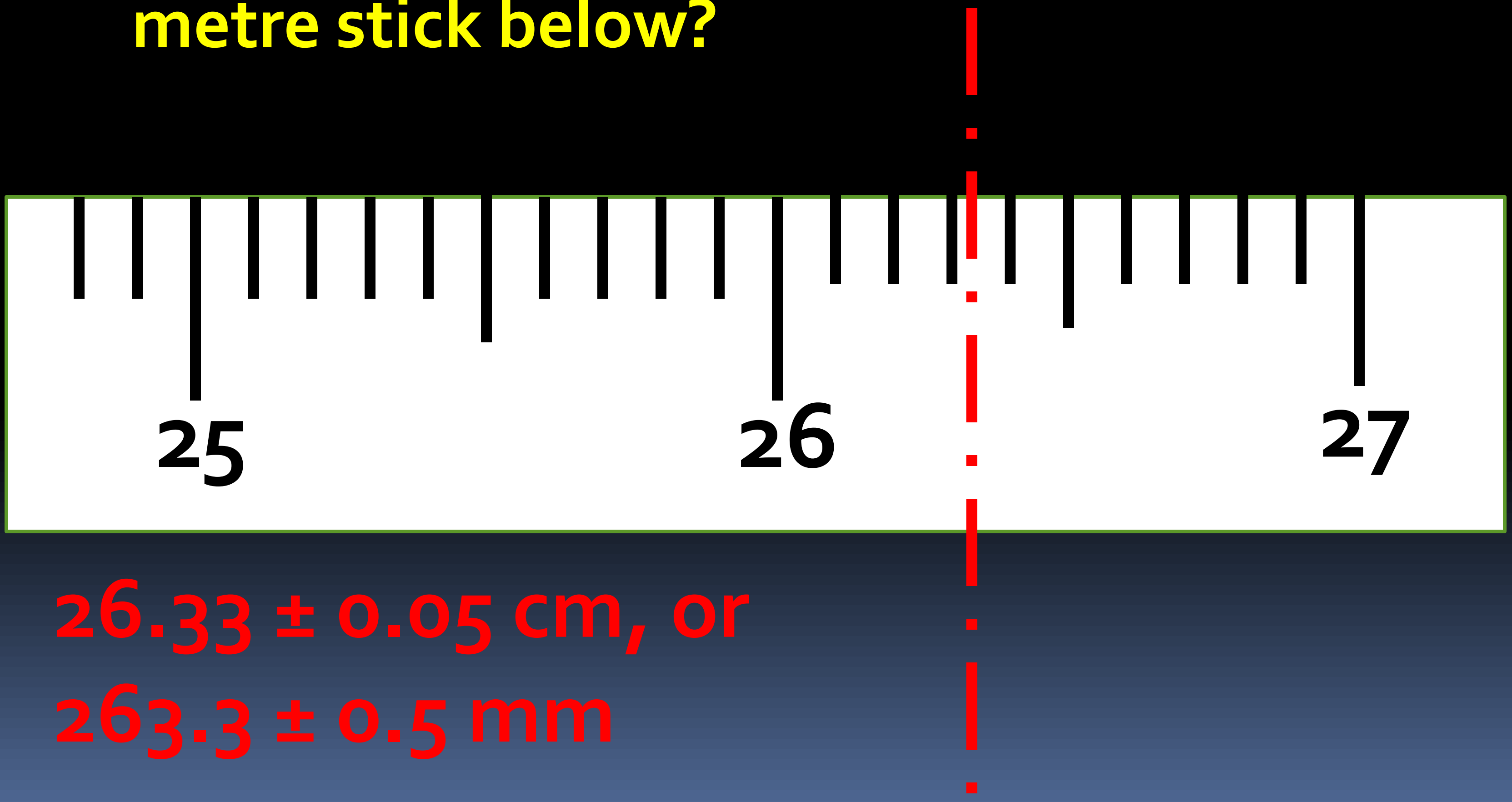
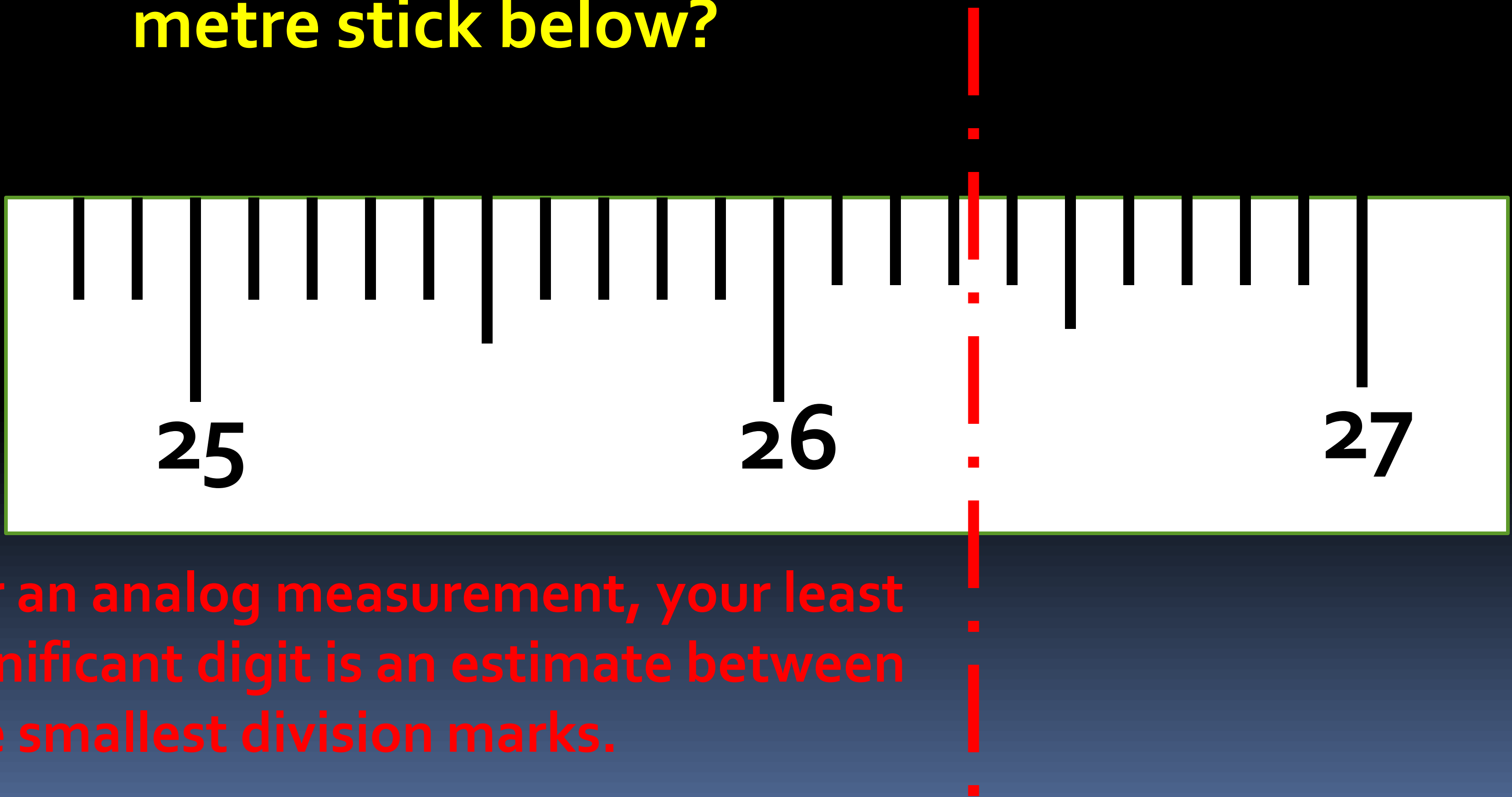


26.33 ± 0.05 cm, or

263.3 ± 0.5 mm

Errors of measurement

What is the measurement and error from the metre stick below?



For an analog measurement, your least significant digit is an estimate between the smallest division marks.

Accounting for Random Error

- A measurement repeated many times can be assumed to be too high as often as it is too low
- Random error is minimized by taking the average of many readings, the mean
- You then must consider the standard deviation from the mean – the average of the distance from the mean for all of the measurements

Accounting for Random Error

- Standard Deviation

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \bullet\bullet\bullet + (x_N - \bar{x})^2}{N - 1}}$$

  - Not evaluated on exams, good for IA

- Half the Max Difference

$$\Delta x = \frac{x_{\max} - x_{\min}}{2}$$

Accounting for Random Error

- If the standard deviation or half the max difference is greater than the reading error, you can use it as your uncertainty
- A final, and the most conservative, method is to simply take the highest deviation from the mean and use that as your uncertainty – assuming it is greater than your assumed reading error

Accounting for Random Error

- No right or wrong answer when it comes to method
- But, you *must* account for uncertainty
- Method chosen based on judgment
- Some judgments better than others, some open to questioning
  - Explain your reasoning

How Long Is Your Table

How Long Is Your Table . . . Really

- Measure each table
- Enter into spreadsheet
- What value will you use for the length of the table?
- What will you use for the uncertainty?

Table	Length	Deviation
1		#DIV/0!
2		#DIV/0!
3		#DIV/0!
4		#DIV/0!
5		#DIV/0!
6		#DIV/0!
7		#DIV/0!
Mean	#DIV/0!	
Std Dev	#DIV/0!	
Greatest Dev		

How Long Is Your Table . . . Really

- What error is involved in measuring the table?
- How can you minimize the error?
- What is the assumed tool error?
- What are other methods for considering uncertainty other than tool error?

How Long Is Your Table . . . Really

- What error is involved in measuring the table?
  - Random error, reading error
- How can you minimize the error?
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How Long Is Your Table . . . Really

- What error is involved in measuring the table?
  - Random error, reading error
- How can you minimize the error?
  - Take multiple measurements and average
- What is the assumed tool error?
- What are other methods for considering uncertainty other than tool error?

How Long Is Your Table . . . Really

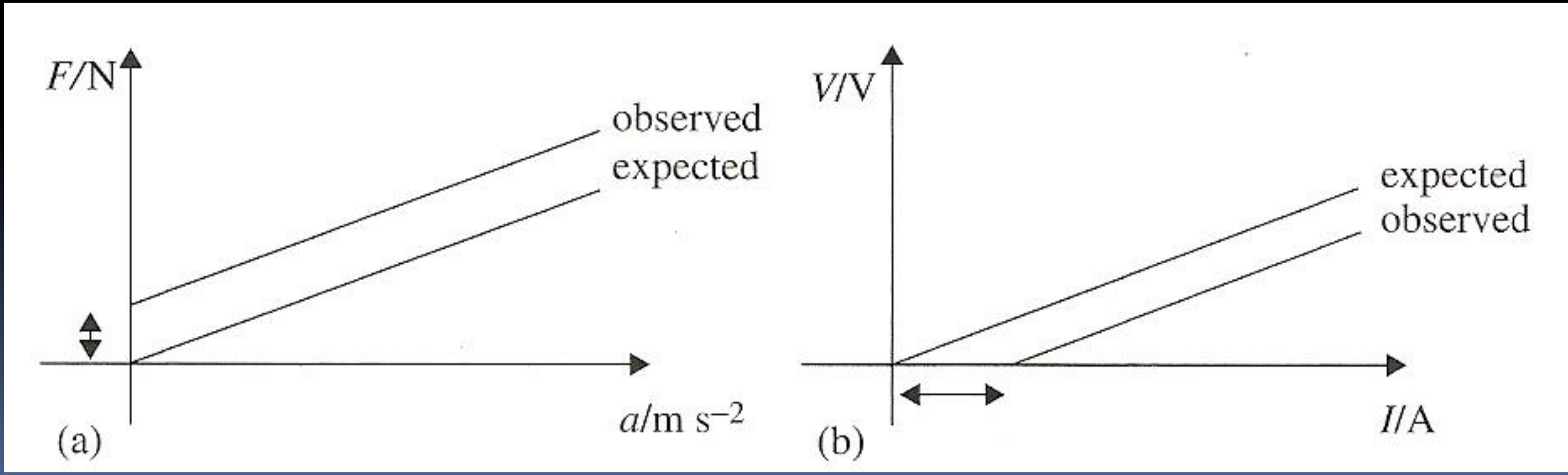
- What error is involved in measuring the table?
  - Random error, reading error
- How can you minimize the error?
  - Take multiple measurements and average
- What is the assumed tool error?
  - Metre stick, ± 0.5 mm or ± 0.05 cm
  - But how many times?
- What are other methods for considering uncertainty other than tool error?

How Long Is Your Table . . . Really

- What error is involved in measuring the table?
  - Random error, reading error
- How can you minimize the error?
  - Take multiple measurements and average
- What is the assumed tool error?
  - Metre stick, ± 0.5 mm or ± 0.05 cm
  - But how many times?
- What are other methods for considering uncertainty other than tool error?
  - Standard deviation of the mean
  - Half max difference
  - Using the greatest deviation from the mean

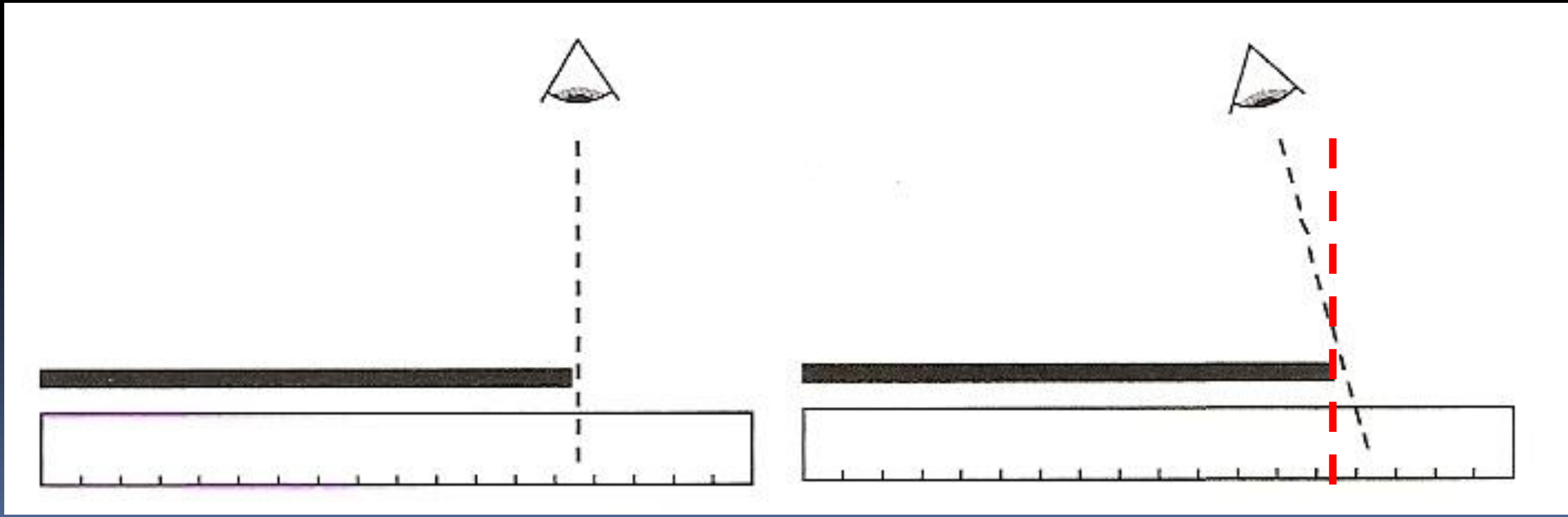
Systematic Error

- Typically in the same direction and the same amount for every measurement
- Calibration error
  - Zero error



Systematic Error

- Human-induced systematic error
  - Making the same error in reading an instrument each time She takes a measurement
  - Parallax is a good example
  - Tilted graduated cylinder is another



Total System Error

- Highest of the above?
- Sum of random and systematic?
- Sum of all the above?
- Based on your judgment
  - Be realistic
  - Be conservative, but not overly so
  - Explain your reasoning – graded on making a reasoned approach

Significant Digits in Uncertainties

- Just one, pretty much
- Customarily

Propagation of Errors

$$a = a_0 \pm \Delta a$$

- a = the “real” or “actual” value
- a_o = the measured value
- Δa = uncertainty or error
- $\Delta a / a_o$ = fractional uncertainty
- $\Delta a / a_o$ X 100% = percent uncertainty

Propagation of Errors

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- $\Delta a/a_o \times 100\%$ = percent uncertainty

- **Addition and Subtraction**
  - Absolute Error

$$Q = a + b$$

$$Q = a - b$$

$$Q = a + b - c$$

$$\Delta Q = \Delta a + \Delta b$$

$$\Delta Q = \Delta a + \Delta b$$

$$\Delta Q = \Delta a + \Delta b + \Delta c$$

Propagation of Errors

- a = the "real" or "actual" value
- a_o = the measured value
- Δa = uncertainty or error
- Δa/ a_o = fractional uncertainty
- Δa/ a_o X 100% = percent uncertainty

Assumes Δa, Δb, and Δc all have the same units!!!

- **Addition and Subtraction**
  - **Absolute Error**

$$Q = a + b$$

$$Q = a - b$$

$$Q = a + b - c$$

$$\Delta Q = \Delta a + \Delta b$$

$$\Delta Q = \Delta a + \Delta b$$

$$\Delta Q = \Delta a + \Delta b + \Delta c$$

Propagation of Errors

- **Multiplication and Division – Fractional Error**

$$Q = ab \quad \rightarrow \quad \frac{\Delta Q}{Q_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0}$$

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Propagation of Errors

- **Multiplication and Division – Fractional Error**

Units don't matter!!!

$$Q = ab \quad \Rightarrow \quad \frac{\Delta Q}{Q_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0}$$

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Propagation of Errors

- <u>**Multiplication and Division**</u>
  - **Absolute Uncertainty**

$$\frac{\Delta Q}{Q_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0}$$

$$\Delta Q = \left[\frac{\Delta a}{a_0} + \frac{\Delta b}{b_0} \right] Q_0$$

Propagation of Errors

- **Multiplication and Division**
  - **Absolute Uncertainty**

$$\frac{\Delta Q}{Q_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0}$$

$$\Delta Q = \left[\frac{\Delta a}{a_0} + \frac{\Delta b}{b_0}\right] Q_0$$

- **Example**
  - **W = F x d**

$$\frac{\Delta W}{W_0} = \frac{\Delta F}{F_0} + \frac{\Delta d}{d_0}$$

$$\Delta W = \left[\frac{\Delta F}{F_0} + \frac{\Delta d}{d_0}\right] W_0$$

Propagation of Errors

- **Powers and Roots**

$$Q = a^n \quad \rightarrow \quad \frac{\Delta Q}{Q_0} = |n| \frac{\Delta a}{a_0}$$

$$Q = \sqrt[n]{a} \quad \rightarrow \quad \frac{\Delta Q}{Q_0} = \frac{1}{n} \frac{\Delta a}{a_0}$$

Other Functions?

Other Functions

- Don’t worry about it

Now what

- You’ve got your data points
- You’ve propagated your uncertainties
- Now what?

Now what

- You’ve got your data points
- You’ve propagated your uncertainties
- Now what?
- Graphing – the ultimate data summary!

Uncertainties in Slope and Intercept

- You graph your points -- What's next?

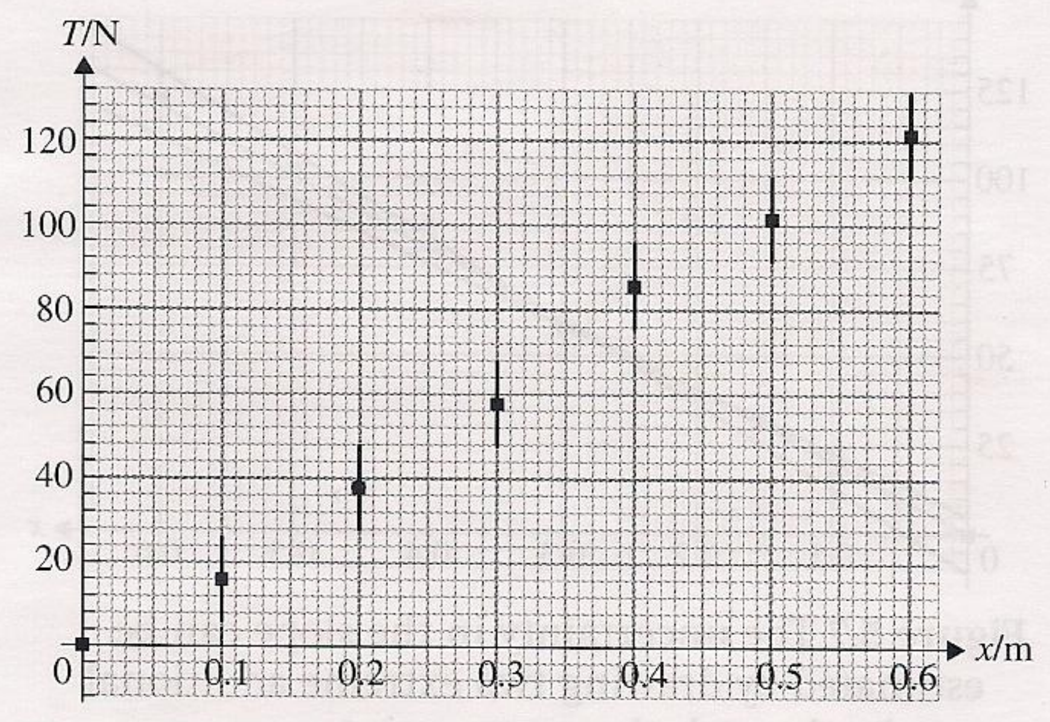


Figure 5.5 Data points plotted together with uncertainties in the values for the tension.

Uncertainties in Slope and Intercept

- You graph your points -- What's next?
  - Error bars
- What's next?

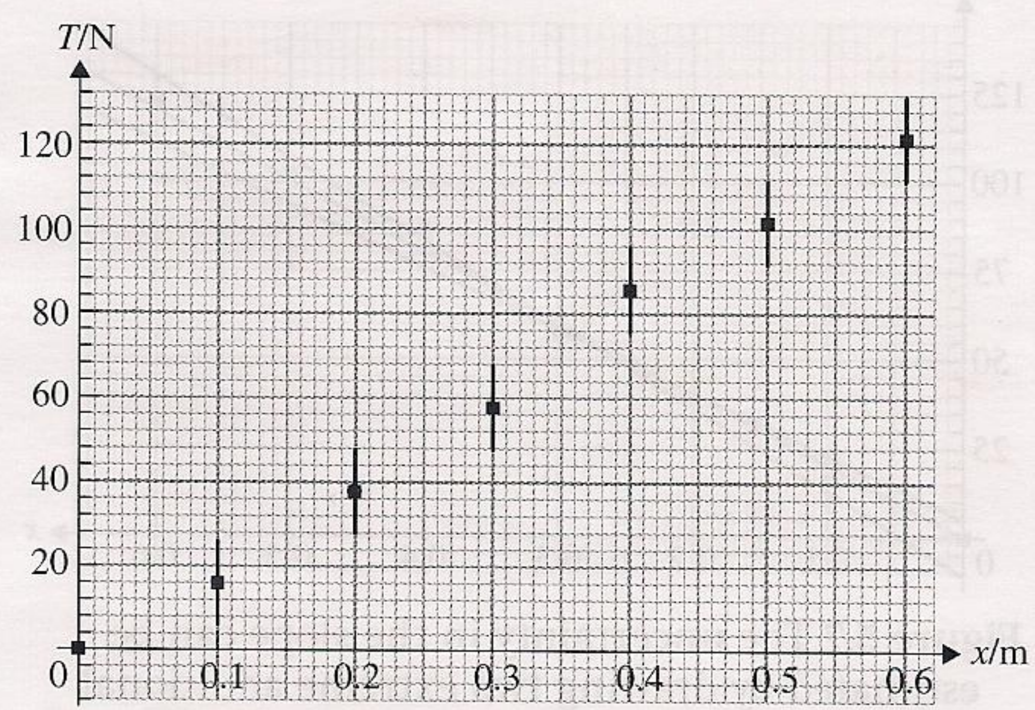


Figure 5.5 Data points plotted together with uncertainties in the values for the tension.

Uncertainties in Slope and Intercept

- You graph your points -- What's next?
  - Error bars
- What's next?
  - Line of Best Fit
- What's next?

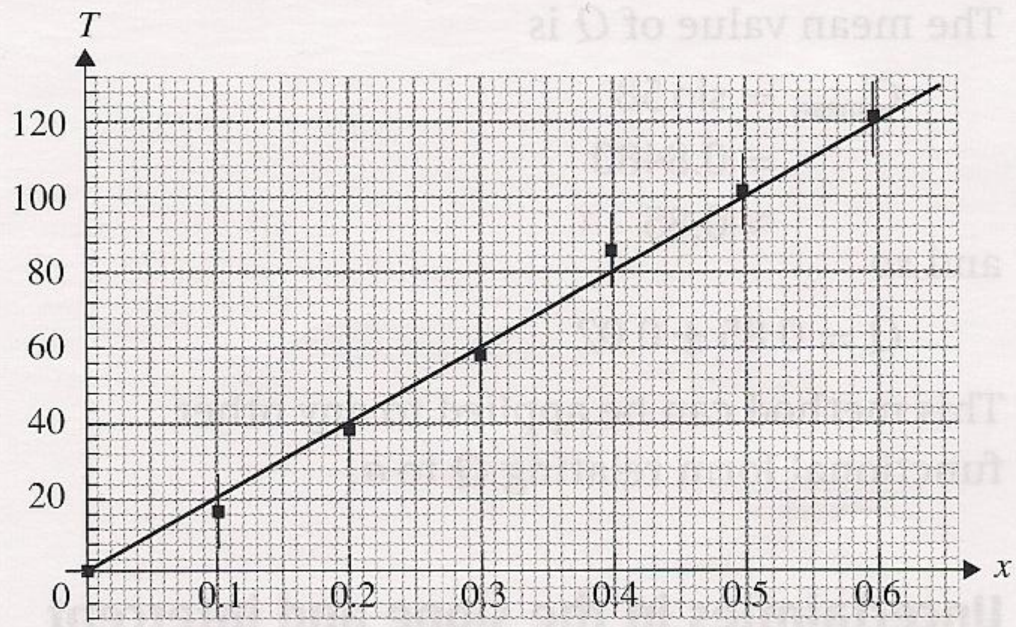


Figure 5.6 The line of best fit through the data points.

Uncertainties in Slope and Intercept

- You graph your points -- What's next?
  - Error bars
- What's next?
  - Line of Best Fit
- What's next?
  - Min/Max Slope
- What's next?

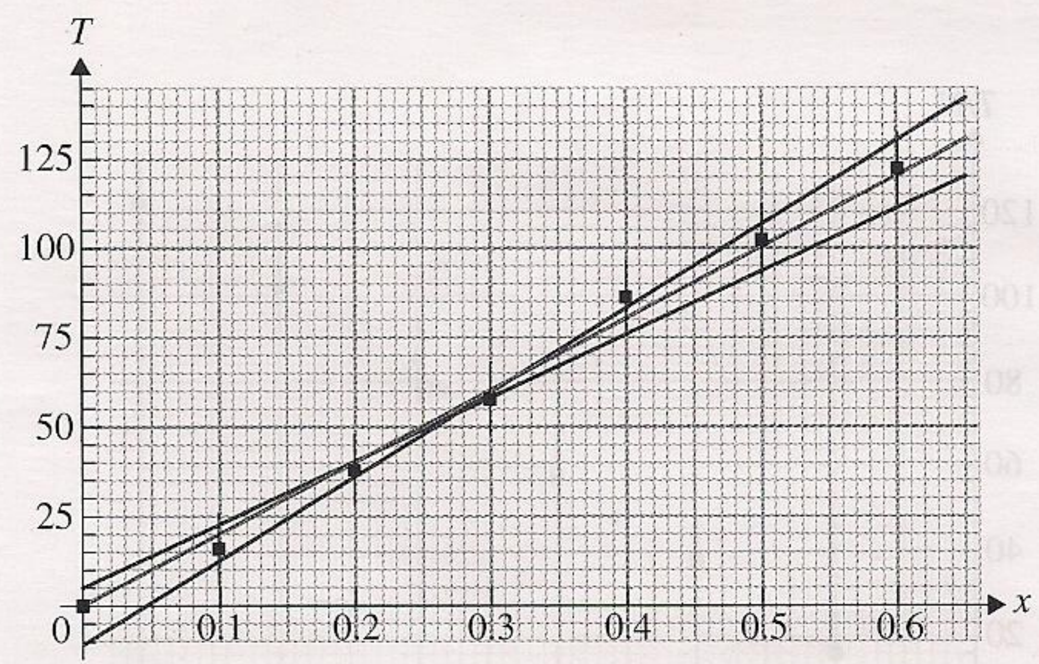


Figure 5.7 The uncertainty in the slope can be estimated by drawing two extreme additional graphs through the centre point.

Uncertainties in Slope and Intercept

- You graph your points -- What's next?
  - Error bars
- What's next?
  - Line of Best Fit
- What's next?
  - Min/Max Slope
- What's next?
  - Uncertainty of slope and intercept

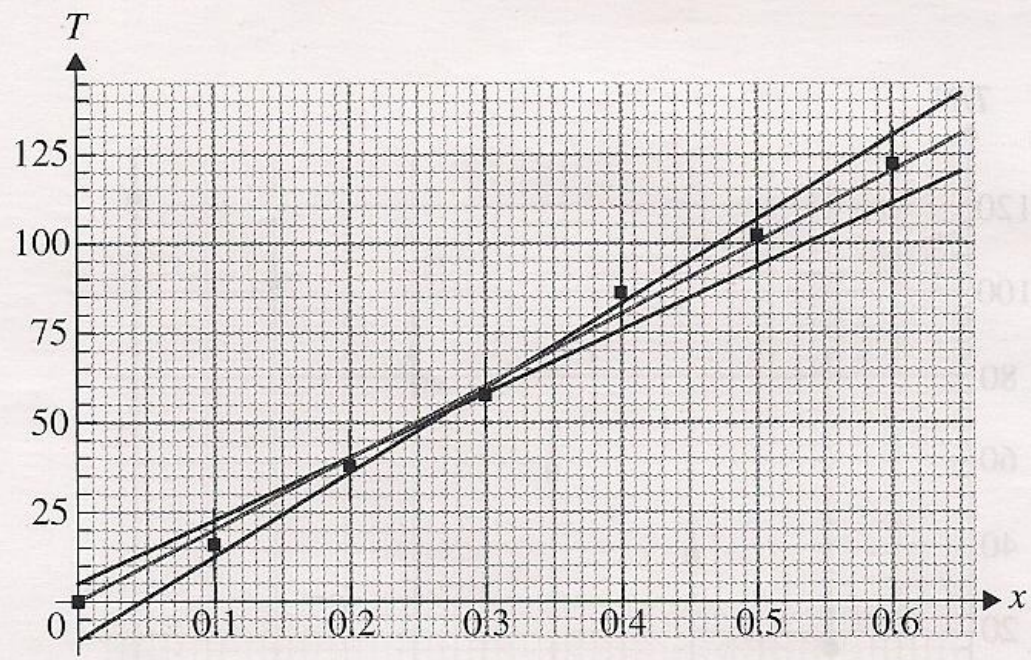
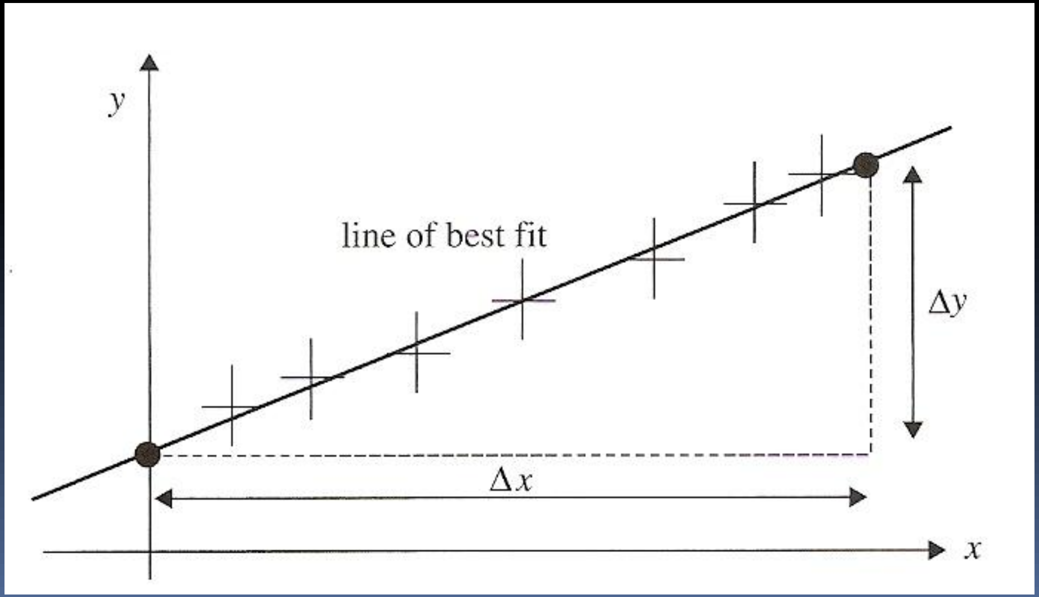


Figure 5.7 The uncertainty in the slope can be estimated by drawing two extreme additional graphs through the centre point.

Line of Best Fit Video

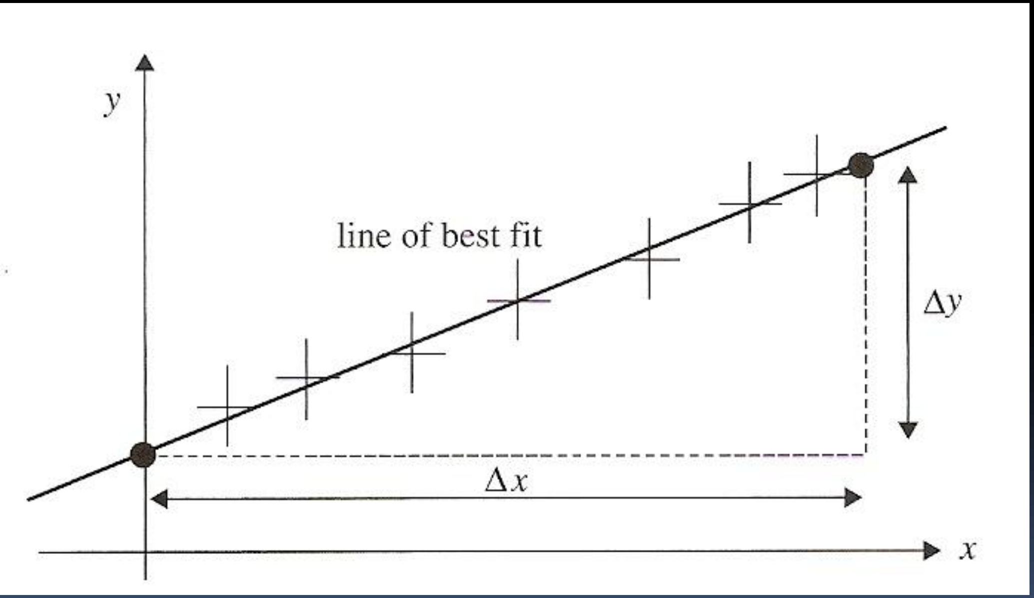
Line of Best Fit

- Choosing a line which goes through as many error bars as possible in such a way that the distances between the line and the points on one side of it are, on average, the same as the distances between the line and points on the other side of it.
- New



Line of Best Fit

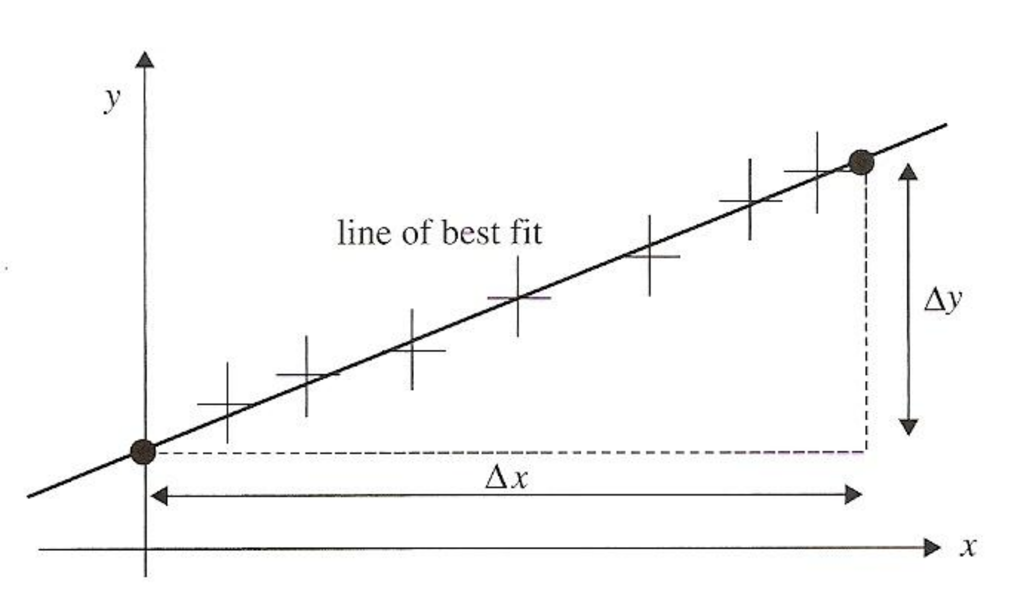
- The line of best fit will ideally pass through the vertical and horizontal error bars
- NEVER connect the dots with straight-line segments
- Let Excel do it
- Works on curves too
- Let Excel do it



Line of Best Fit - Slope

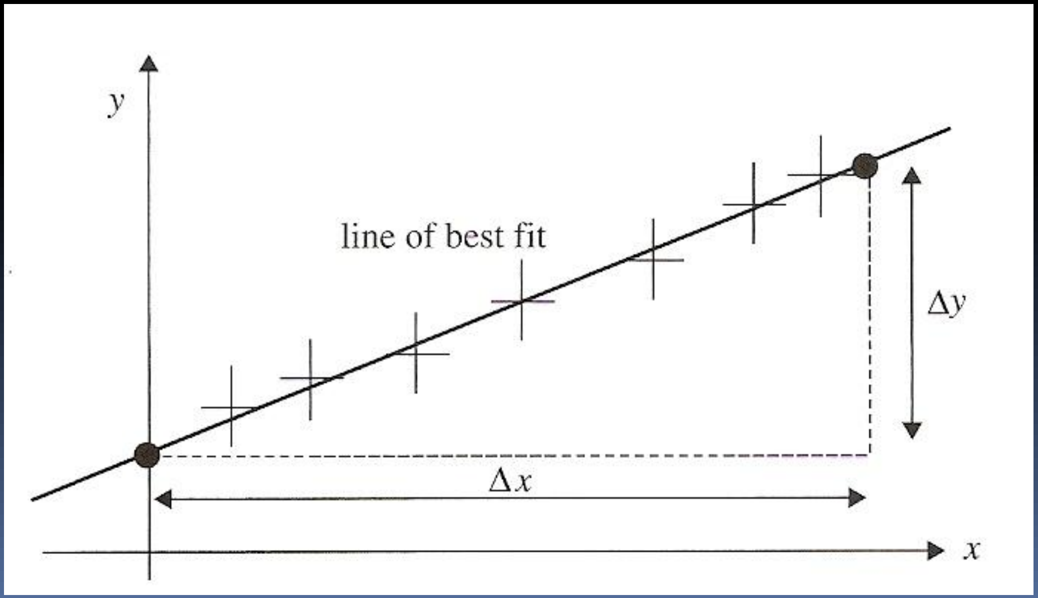
- The slope of the line of best fit gives you the best relationship between your two variables
- To find the slope,
  - Use points on line of best fit, ***NOT YOUR DATA POINTS!***
  - Take points as far away from each other as possible

$$m = \frac{\Delta y}{\Delta x}$$

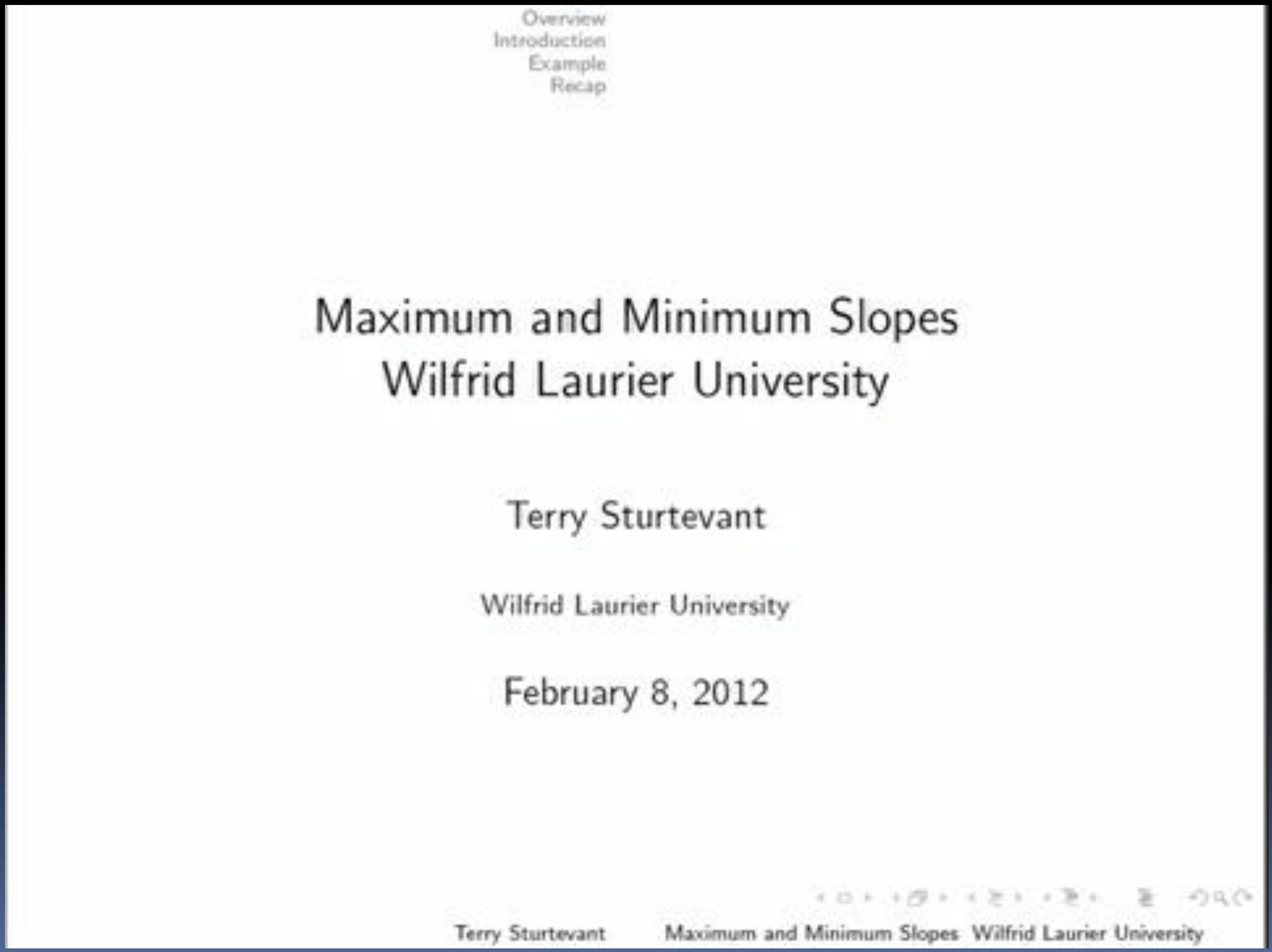


Line of Best Fit – Slope Uncertainty

- You've accounted for the uncertainty in your data points through error bars
- You've determined a relationship (slope) between two variables
- ***Now, what is the uncertainty in that relationship (slope)?***



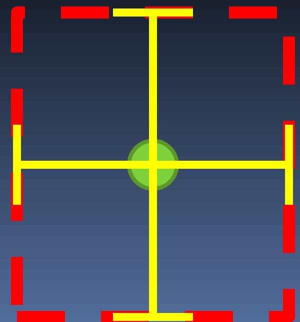
Line of Best Fit – Min/Max Slope - Video

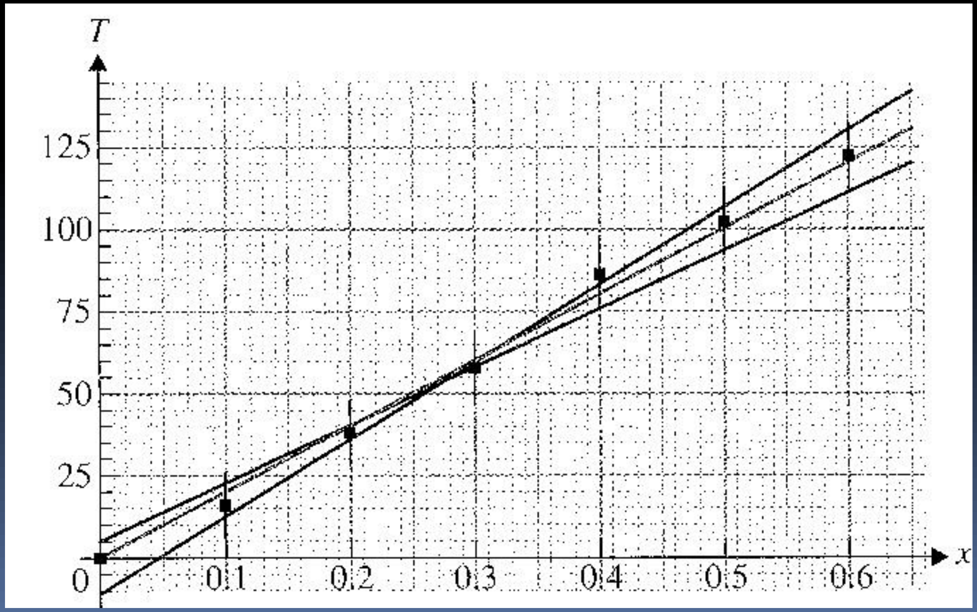


Line of Best Fit – Min/Max Slope

- Used to determine the uncertainty of your slope
- Min/Max lines are drawn to give you the least/greatest possible slopes while still passing through the error ‘area’ rectangles

$$m = \frac{\Delta y}{\Delta x}$$

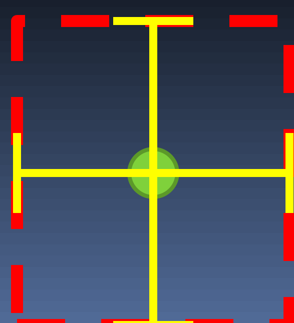


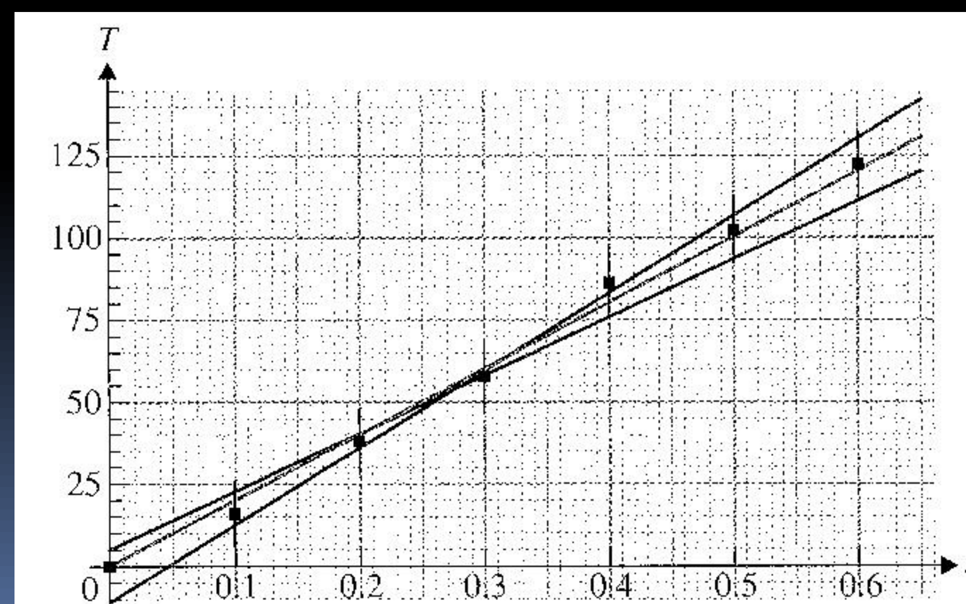


Line of Best Fit – Min/Max Slope

- You can determine the points for the min/max slope lines manually, or use guess-and-check in Excel

$$m = \frac{\Delta y}{\Delta x}$$

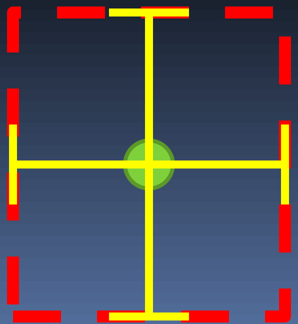


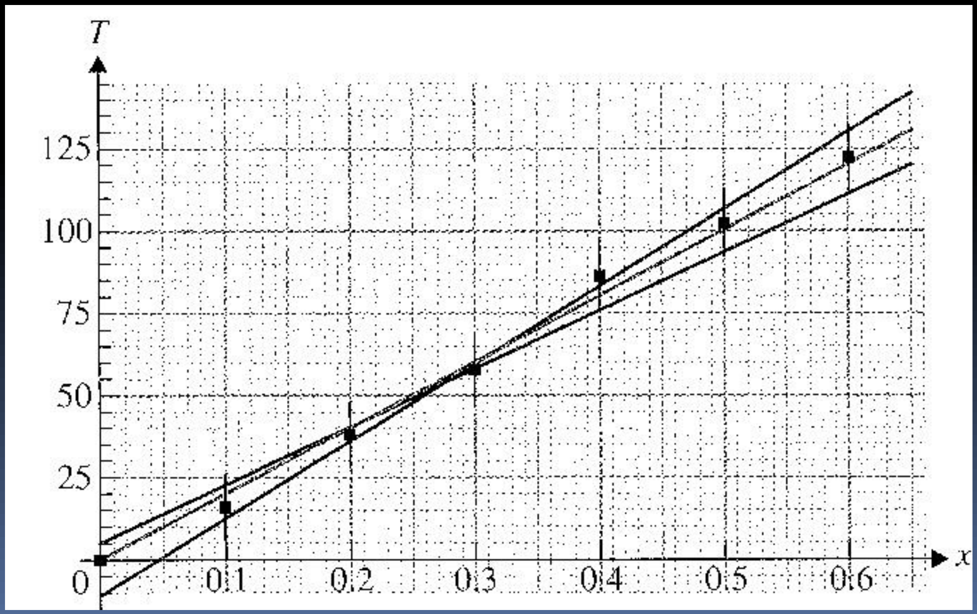


Line of Best Fit – Min/Max Slope

- Once you have the points for the min/max slope lines, use Excel to compute their slopes

$$m = \frac{\Delta y}{\Delta x}$$

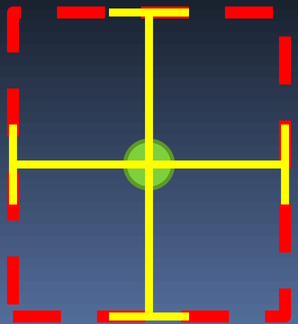


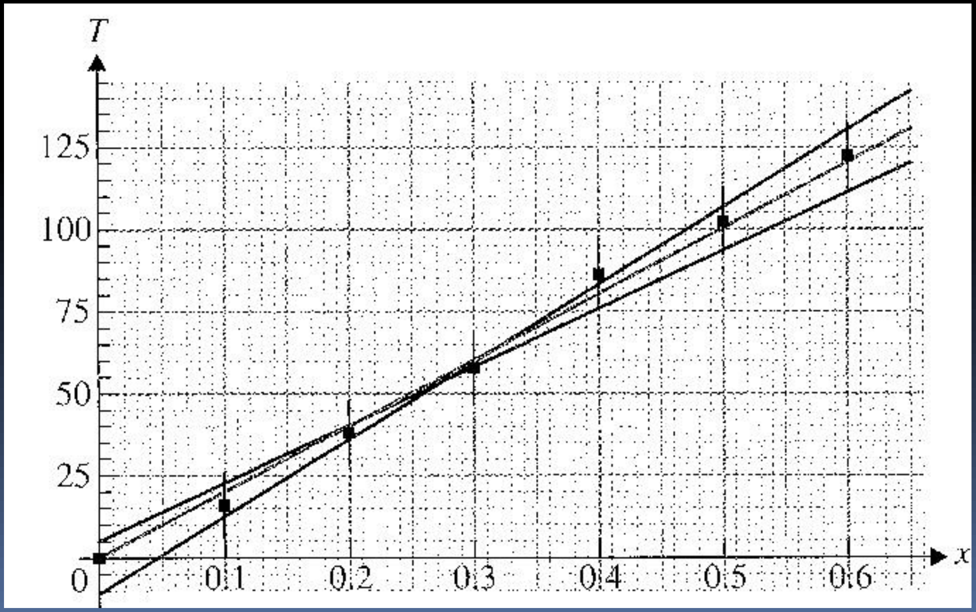


Line of Best Fit – Min/Max Slope

- The slope of your line defines the relationship between your *x* and *y* variables
- For example, if y = 7.63x, for every value of x, y will be 7.63 times x

$$m = \frac{\Delta y}{\Delta x}$$



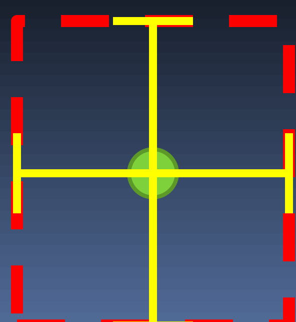


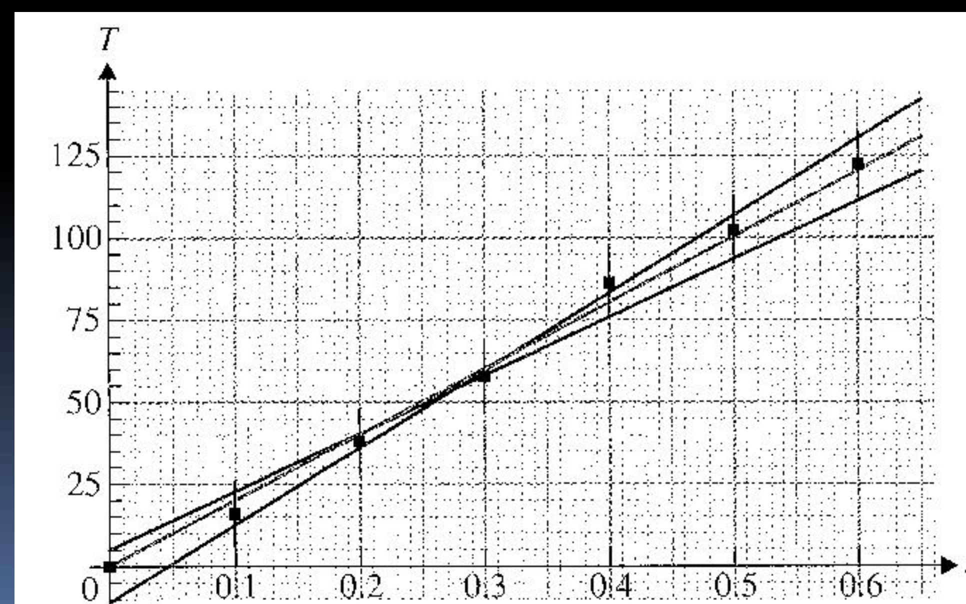
Line of Best Fit – Min/Max Slope

- Min/max slope gives you the uncertainty of the relationship, the range of possible ***actual*** values
- "My constant of proportionality (slope) is 7.63, but could be as high as 8.12 or as low as 7.36."
- 7.63, +0.49/-0.27

  OR
- 7.63 ±0.49

$$m = \frac{\Delta y}{\Delta x}$$





Uncertainties in Slope and Intercept – Your Choice

- Uncertainty of Slope
  - Slope +max/-min
  - Slope ± max difference
  - Slope ± average difference
- Uncertainty of Intercept
  - Intercept +max/-min intercept
  - Intercept ± max difference in intercepts
  - Intercept ± average difference of intercepts

Uncertainties in Slope – Propagation

- You also have the option of using standard propagation

$$m = \frac{\Delta y}{\Delta x}$$

$$\Delta m = \left(\frac{\Delta y}{y_0} + \frac{\Delta x}{x_0} \right) m_0$$

Uncertainties in Non-Linear Lines of Best Fit

Uncertainties in Non-Linear Lines of Best Fit

- Out of our league

Essential Idea

- Scientists aim towards designing experiments that can give a "true value" from their measurements, but due to the limited precision in measuring devices, they often quote their results with some form of uncertainty.

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- If $y = \dfrac{ab}{c}$
  - Then $\dfrac{\Delta y}{y} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} + \dfrac{\Delta c}{c}$
- If $y = a^n$
  - then $\dfrac{\Delta y}{y} = \left| n\dfrac{\Delta a}{a} \right|$

QUESTIONS?

Homework

 # 23-34

STOPPED HERE ON 8/27/14