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International Baccalaureate

Extended essay cover

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Candidate session number	0	0	0	2	5	0		
Candidate name								
School number	0	0	0	2	5	0		
School name	St. Petersburg High School							
Examination session (May or November)	May		Year	2013				

Diploma Programme subject in which this extended essay is registered: Physics

(For an extended essay in the area of languages, state the language and whether it is group 1 or group 2.)

Title of the extended essay: The Validity of the Newton's Law of Cooling Model

Candidate's declaration

This declaration must be signed by the candidate; otherwise a grade may not be issued.

The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

I have acknowledged each use of the words, graphics or ideas of another person, whether written, oral or visual.

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Assessment form (for examiner use only)

Candidate session number	0	0	0	2	5	0		

Achievement level

Criteria	Examiner 1	maximum	Examiner 2	maximum	~~Examiner 3~~ Mr. Smith
A research question	1	2	1	2	
B introduction	1	2	1	2	
C investigation	2	4	2	4	
D knowledge and understanding	2	4	2	4	
E reasoned argument	3	4	3	4	
F analysis and evaluation	3	4	3	4	
G use of subject language	3	4	1	4	
H conclusion	2	2	2	2	
I formal presentation	2	4	2	4	
J abstract	2	2	2	2	
K holistic judgment	2	4	2	4	
Total out of 36	23		21		

Lost Grade Form – Predicted an A

Name of examiner 1: FERDi KAYA Examiner number: 088164
(CAPITAL letters)

Name of examiner 2: D. SHORT Examiner number: 1282
(CAPITAL letters)

Name of examiner 3: ______ Examiner number: ______
(CAPITAL letters)

IB Cardiff use only: **B:** ✓

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The supervisor must complete this report, sign the declaration and then give the final version of the extended essay, with this cover attached, to the Diploma Programme coordinator.

Name of supervisor (CAPITAL letters) KYLE J. SMITH

Please comment, as appropriate, on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome (see page 13 of the extended essay guide). The concluding interview (viva voce) may provide useful information. These comments can help the examiner award a level for criterion K (holistic judgment). Do not comment on any adverse personal circumstances that may have affected the candidate. If the amount of time spent with the candidate was zero, you must explain this, in particular how it was then possible to authenticate the essay as the candidate's own work. You may attach an additional sheet if there is insufficient space here.

Ted originally started with a Math extended essay. His advisor suggested that his topic was better suited for a physics essay and he changed topic areas. The extensive mathematical analysis is a byproduct of this circumstance. Because of the change in topic areas, he was behind his classmates in completing the assignment. The result is that a few areas could be more 'polished' in the way they are written. In spite of this, this is the best extended essay that I have read. He demonstrates a thorough understanding of the topic, incorporates several different topic areas from our coursework, as well as in-depth outside research.

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I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent 6.0 hours with the candidate discussing the progress of the extended essay.

Supervisor's signature: Kyle J. Smith Date: 2/26/13

International Baccalaureate

Extended Essay: Physics

The Validity of the Newton's Law of Cooling Model

Candidate Number:

St. Petersburg High School

Word Count: 3902

Acknowledgements

I would like to thank everyone who helped make my experiment possible. First and foremost, I'd like to thank my parents who provided me with transportation, bought materials, and allowed me to turn a room into a laboratory for a day. I'd like to thank the wonderful employees of Alro Steel Corporation who provided me with the aluminum I used for this experiment free of charge. I'd also like to thank my physics teacher and extended essay advisor Mr. Smith for his patience, understanding, and faith in me. Finally, I'd like to thank my best friend, Troy Koser, who spent an entire day helping me gather data for this experiment.

Abstract

This extended essay aims to investigate heat transfer because of its wide applicability to engineering. The research question is: **to what extent does the Newton's Law of Cooling model accurately approximate the temperature of a solid piece of aluminum?** → R. Q

Five blocks of aluminum metal with surface area to volume ratios of 2062 ± 67, 375 ± 9, 239 ± 4, 150 ± 2, and 117 ± 1 were heated in an oven and placed in air of 28 °C on wooden supports. Temperature measurements of the metals were taken with times. These were plotted, graphed, and an empirical approach was used to determine the coefficient of decay (k-value) in the Newton's Law of Cooling model by determining which k-value gave the highest R^2 value. The average k-values were then used to determine the average convective heat transfer coefficient for each metal. Finally, a linear relationship was found between the "k-value" and the surface area to volume ratio of an aluminum block.

The Newton's Law of Cooling model was able to match the empirical data for every metal and every trial to at least R^2=.99. This is a good indicator that the model is valid to some extent. It was further found that all the metals had a fairly similar convective heat transfer coefficient (with the average being 14.7 ± 0.3 W m^{-2} K^{-1} which is consistent with the theory behind the model. Most importantly it was determined that the "k-value" for aluminum under the same atmospheric conditions can be reliable found by the following formula: $k = 6 \times 10^{-6} \frac{(Surface\ Area)}{(Volume)} + 0.0001$ with R^2=.9998. This can be used to predict the cooling curve of any piece of aluminum under the same conditions. Therefore, the Newton's Law of Cooling model is extremely accurate at approximating the temperature of a cooling piece of aluminum.

Word Count: 297

mathematically biased

Table of Contents

NOT PART OF ESSAY

Abuse of Appendix

1 Introduction

1.1 Research Question

To what extent does the Newton's Law of Cooling model accurately approximate the temperature of a solid piece of aluminum?

1.2 Introduction

Heat transfer is a common problem faced by engineers in many fields and applications. Sometimes, lives depend on the successful application of heat transfer such as in the case of the space shuttle's re-entry shield or a nuclear power plant's cooling system.[1] In other cases, sensitive electronics can be damaged if heat is not properly controlled. Consumer electronics, like desktop computers, have components with fans and heat-sinks, components with just heat-sinks, and even components simply exposed to the surrounding air to allow them to maintain safe operating temperature. Engineers design these features to prevent overheating and damage.

To determine what cooling features a component must have, engineers must understand how heat moves within the specific component and out of it. There are multiple methods to approach the modeling of heat transfer for this type of application, the most recent and salient being an advanced computer model and simulation. This is probably the most accurate model, but it comes from the knowledge of other models of heat transfer. Sophisticated models, such as the transient conduction model rely on Fourier's Law of Conduction and approximate the temperature distribution throughout the object.[1] Under some circumstances, it is possible to simplify the model into a lumped capacitance model like Newton's Law of Cooling. An investigation of the accuracy of the Newton's Law of Cooling model follows.

[1] Jiji, Latif M., *Heat Conduction - Third Edition*, Berlin: Springer, 2009, Print, Pages 1-4

2 Theory

2.1 Newton's Law of Cooling

it should have been defined with its formula.

Newton's Law of Cooling is a lumped capacitance model. It simplifies a thermal system into a single "lump" which is uniform at all points. This lump then loses thermal energy to the environment through conduction, convection, and radiation at a rate that is proportional to the temperature difference between the lump and the environment. In the case of Newton's Law of Cooling, the body must have high internal thermal conductivity and be immersed in a fluid bath that is not highly conductive. This experiment investigates this type of situation by using aluminum metal, which has a very high conductivity of 235 W m^{-1} K^{-1} and placing it in still air which has a low convective heat transfer coefficient.[2][3] This introduces the concept of the Biot number.

No definition of the law

2.2 Biot Number

The biot number provides a ratio of the rate of heat transfer at the surface of a body to the rate of heat transfer within the body. It is found by multiply the convective heat transfer coefficient by the characteristic length of the body and dividing by the thermal conductivity of the body. The characteristic length is found by dividing the volume of the body by its surface area.[4] define

The lower the biot number, the more uniform is the distribution of temperature within the body and therefore it can be treated as a whole of a single temperature and not as a complicated system with temperature gradients. A biot number of less than 0.1 is considered to be low enough for the simplification to a lumped capacitance model.[4] In other words, Newton's Law of Cooling should theoretically be most valid when the biot number is very low. All of the biot numbers were found to be significantly under 0.1 with the highest being 0.00056. How?

[2] Dayah, Michael. Dynamic Periodic Table. 1 October 1997. 21 February 2012 <http://www.ptable.com>.

[3] "Convective Heat Transfer," *The Engineering Toolbox*, N.p., n.d. Web, 21 Feb, 2013, <http://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html>.

[4] Jiji, Latif M., *Heat Conduction - Third Edition*, Berlin: Springer, 2009, Print, Pages 120-121

2.3 IR Thermometer

It was determined that the use of the infrared thermometer would be most appropriate for these types of experiments because other methods of measuring temperature would prove dangerous or unreliable. The electronic temperature probes and regular thermometers can suffer permanent damage when exposed to temperatures above 150°C and are usually designed to work in a fluid, not by touching a solid.[5]

The IR thermometer takes accurate measurements of the temperature of a surface through the concept of black body radiation. All bodies radiate energy as electromagnetic waves. The peak wavelength, λ, of the spectrum of emitted radiation is related to temperature, T, according to Wein's law[6]:

$$\lambda = \frac{2.90 \times 10^{-3}\, m\, K}{T}$$

The thermometer uses this concept, to determine the temperature of an object. The thermometer assumes an emissivity of e=.95. Aluminum, however, naturally has a high albedo and low emissivity and must therefore be spray painted black.

[5] "Stainless Steel Temperature Probe," *Vernier Software & Technology*, N.p., n.d. Web, 21 Feb. 2013, <http://www.vernier.com/products/sensors/temperature-sensors/tmp-bta/>.
[6] Tsokos, K. A. *Physics for the IB Diploma*, Cambridge: Cambridge UP, 2008, Print, Page 436.

3 Methodology of Experiment

3.1 Goal

To investigate the extent to which the Newton's Law of Cooling model is valid for hot pieces of aluminum metal of various surface area to volume ratios placed in room temperature air.

3.2 Hypothesis

The Newton's Law of Cooling model will have a high level of validity and applicability in the case of aluminum metal because of aluminum's high thermal conductivity, which will maintain relatively similar temperatures throughout the metal.

3.3 Variables

Independent Variable

The surface area to volume ratio of the five aluminum metal blocks.

Dependent Variable

The temperatures of the metals at a certain time after the initial temperature measurement is taken and their relation to the temperatures predicted by the Newton's Law of Cooling model.

Constants

- Temperature of the room (was measured to be 28 ± 1°C)
- Measurement devices
- Thermal and emissive properties of the metals
- Set-up for holding the metal blocks in the air
- Humidity of the room (72 ± 1%)

3.4 Materials

list not required

- Base (a wood pallet in this case) that is flat, sturdy, and will not be damaged if a hot aluminum bar were to be dropped on it. Note: this does not affect the outcome of the experiment and is only used as a safety measure.
- 10 house bricks (about eight inches, by five inches, by four inches)
- 10 pieces of wood shaped as rectangular prisms of about one inch, by two inches, by six inches
- Exetech IR thermometer (-20 to 332 ± 0.1°C)
- Small Convection Oven
- Tongs (with rubber handles)
- Indoor thermometer
- Black Spray paint (for metal) Note: this is only one thin layer of spray paint which should not affect the outcome of the experiment significantly because the heat does not have to conduct across a thick layer of paint.
- Scale (±0.001 kg)
- Ruler (±0.00025 m)
- 5 aluminum bars:
    - Bar one: LxWxH = 0.20200±0.00025m x 0.07800±0.00025m x 0.00100±0.00025m
    - Bar two: LxWxH = 0.14350±0.00025m x 0.03825±0.00025m x 0.00650±0.00025m
    - Bar three: LxWxH = 0.10850±0.00025m x 0.06450±0.00025m x 0.01050±0.00025m
    - Bar four: LxWxH = 0.08850±0.00025m x 0.08825±0.00025m x 0.01925±0.00025m
    - Bar five: LxWxH = 0.07925±0.00025m x 0.05075±0.00025m x 0.03800±0.00025m

too many digits.

tabulate

style written like an IA

3.5 Procedure

Setup

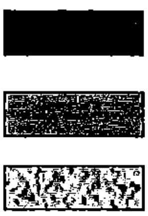
1. (Two days prior to the experiment to allow time for drying) Spray paint the five aluminum bars with the black spray paint made for metals. This is to allow the IR thermometer, which assumes an emissivity of ε=.95, to take accurate measurements of the temperature of the metal. Since the aluminum naturally has a high albedo and low emissivity, it must be spray painted.
2. Thoroughly clean the oven so no oil or residue will catch fire.
3. Measure the temperature of the room with a basic indoor thermometer and continue monitoring this temperature throughout the experiment. The room temperature is an essential part of the Newton's Law of Cooling model.
4. Position the bricks on the base in groups of two, with the pair parallel to each other and far enough that when the metal is placed on them, it will only be supported by 0.5 to 1.0 cm on each side. The bricks should be a couple feet apart so that the heat from one piece of metal does not affect another piece.
5. Position the wood pieces on the inner edges of the brick pairs. The heated metal will be placed on the wood because the thermal conductivity of wood ranges from 0.12 to 0.16 W m^{-1} K^{-1} which is significantly lower than brick (which ranges from 0.6 to 1.6 W m^{-1} K^{-1})[7]

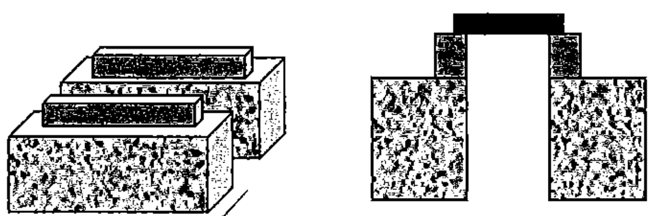
Setup Diagrams

Metal:

Wood:

Brick:





[7] "Thermal Conductivity of Some Common Materials and Gases," *The Engineering Toolbox*, N.p., n.d. Web, 21 Feb, 2013, <http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html>.

Experiment

6. Heat the oven to about 300°C.
7. Place the metal bars into the oven for five to ten minutes to heat up.
8. Remove one metal block from the oven using tongs and place it onto its respective position on the wood.
9. Set the IR thermometer to the "max" setting. This function will make the thermometer record the maximum temperature of the entire reading. Since the temperature should be decreasing the entire time, this reading will allow you to capture an accurate value for the temperature of the piece of metal at the time of measurement.
10. Press and hold the trigger on the IR thermometer, keep it a couple inches above a piece of metal and move it around over the surface of the metal. At the same time, start a timer for the metal.
11. Record the temperature with the time of each recording for each metal.
12. Run several trials for each metal in the same way.

minimal time

sounds very haphazard

3.6 Safety Considerations

- Goggles must be worn at all times.
- Must have a fire extinguisher nearby because the experiment involves work with an oven and hot metals.
- Care must be taken when using tongs to move the metal so it does not slip.
- Closed toed shoes must be worn in case a block of metal were to fall.
- Objects that may come in contact with the hot metal if an accident were to occur should be resistant to heat.

4 Data Collection and Processing

4.1 Explanation and Examples of Calculations

The following is a brief explanation of the process used to find means/averages, compute the coefficient of determination, and calculate uncertainty.

Finding the Mean [8]

Mean value: $\bar{x} = \frac{\sum_{i=1}^{n} x}{n}$

x = A given data point

n = The total number of data points for that group

The mean is the sum of all the data points divided by the total number of those points. Here is an example of finding the mean convective heat transfer coefficient, h, from the experiments:

$$\bar{h} = \frac{(13.9 + 15.2 + 14.7 + 14.4 + 15.4)}{5} = 14.7$$

data needed before the calculations

Computing the Coefficient of Determination [9]

Coefficient of determination (ranges from 0 to 1, where 1 is the best approximation): $R^2 = 1 - \frac{SS_{err}}{SS_{tot}}$

Residual sum of squares: $SS_{err} = \sum_i (y_i - f_i)^2$

Total sum of squares: $SS_{tot} = \sum_i (y_i - \bar{y})^2$

$\bar{y}$ = Mean value (of the temperatures of a given trial)

y_i = Observed value (empirical data of temperature)

f_i = Modeled value (temperature estimated by the model)

---

[8] Roberts, Bill, and Sandy MacKenzie, *Mathematics Higher Level for the IB Diploma*, Oxford: Oxford UP, 2007, Print, Page 529.

[9] "Coefficient of Determination," *Wikipedia*, Wikimedia Foundation, 21 Feb, 2013, Web, 26 Feb, 2013, <http://en.wikipedia.org/wiki/Coefficient_of_determination>.

To show an example of such a calculation, reference will be made to the values in Table 2 on page 11.

$$SS_{err} = \sum_i (y_i - f_i)^2 = 717.4847$$

$$SS_{tot} = \sum_i (y_i - \bar{y})^2 = 76147.46$$

$$R^2 = 1 - \frac{SS_{err}}{SS_{tot}} = 0.990578$$

Calculating Uncertainty[10]

When adding like values, absolute uncertainties can simply be added to each other. When multiplying or dividing values, the fractional uncertainty (or percent uncertainties) of each value must be determined and then added to each other. This is an example (using aluminum block 1) of determining the uncertainty, ΔQ, of a surface area value which involves both types of uncertainties:

$$\Delta Q = 2\left(\left(\frac{0.00025}{0.202} + \frac{0.00025}{0.078}\right)(0.202 \times 0.078) + \left(\frac{0.00025}{0.202} + \frac{0.00025}{0.001}\right)(0.202 \times 0.001) + \left(\frac{0.00025}{0.001} + \frac{0.00025}{0.078}\right)(0.001 \times 0.078)\right) = \pm 0.0003$$

3.0 x 10^-4

Above, three fractional uncertainties are found (one for the area of each side of the metal block) and are then converted to absolute uncertainties by multiplying by the area of that side. These three absolute uncertainties are then added to find the absolute uncertainty of the three sides combined. This is multiplied by two to get the total absolute uncertainty of the surface area.

unnecessary complexity

[10] Tsokos, K. A. *Physics for the IB Diploma*, Cambridge: Cambridge UP, 2008, Print, Page 34.

4.2 Properties of Aluminum Bars

Table 1: Measured and Calculated Properties of Aluminum Bars

Metal Block	1	2	3	4	5
Length (m) ±0.00025	0.20200	0.14350	0.10850	0.08850	0.07925
Width (m) ±0.00025	0.07800	0.03825	0.06450	0.08825	0.05075
Height (m) ±0.00025	0.00100	0.00650	0.01050	0.01925	0.03800
Volume (m^3) (Calculated by LWH)	0.000016 ± 0.000004	0.000036 ± 0.000002	0.000073 ± 0.000002	0.000150 ± 0.000003	0.000153 ± 0.000002
Volume (m^3) (Calculated by Density)	0.0000156 ± 0.0000004	0.0000356 ± 0.0000004	0.0000737 ± 0.0000004	0.0001500 ± 0.0000004	0.0001526± 0.0000004
Surface Area (m^2)	0.0321 ± 0.0003	0.0133 ± 0.0002	0.0176 ± 0.0002	0.0224 ± 0.0002	0.0179 ± 0.0003
Mass (kg) ± 0.001	0.042	0.096	0.199	0.405	0.412
Capacitance (J/K) ±1	38	87	180	366	372

scientific notation could have been used.

The highest uncertainty value for the calculated properties above is the Volume (Calculated by LxWxH) of Metal Block 1 at 25% uncertainty. This is because the smallest measurement of the block was its height of just one millimeter with an uncertainty of 25%.To verify the volume values, and to get values with less uncertainty, the volume was calculated by using the density and mass of the metal blocks. Since these values match those calculated by LxWxH, and because they have much lower uncertainty, they will be used in further calculations.

4.3 Using R^2 to Measure the Validity of the Model and find "k-values"

The formula for the Newton's Law of Cooling model is:[11]

$$T(t) = T_E + (T_O - T_E)e^{-kt}$$

Where T(t) is the temperature of the metal as a function of time, T_E is the temperature of the environment, T_O is the initial temperature of the metal, k is a constant based on the convective heat transfer coefficient, the surface area of the metal, and the heat capacity of the metal.

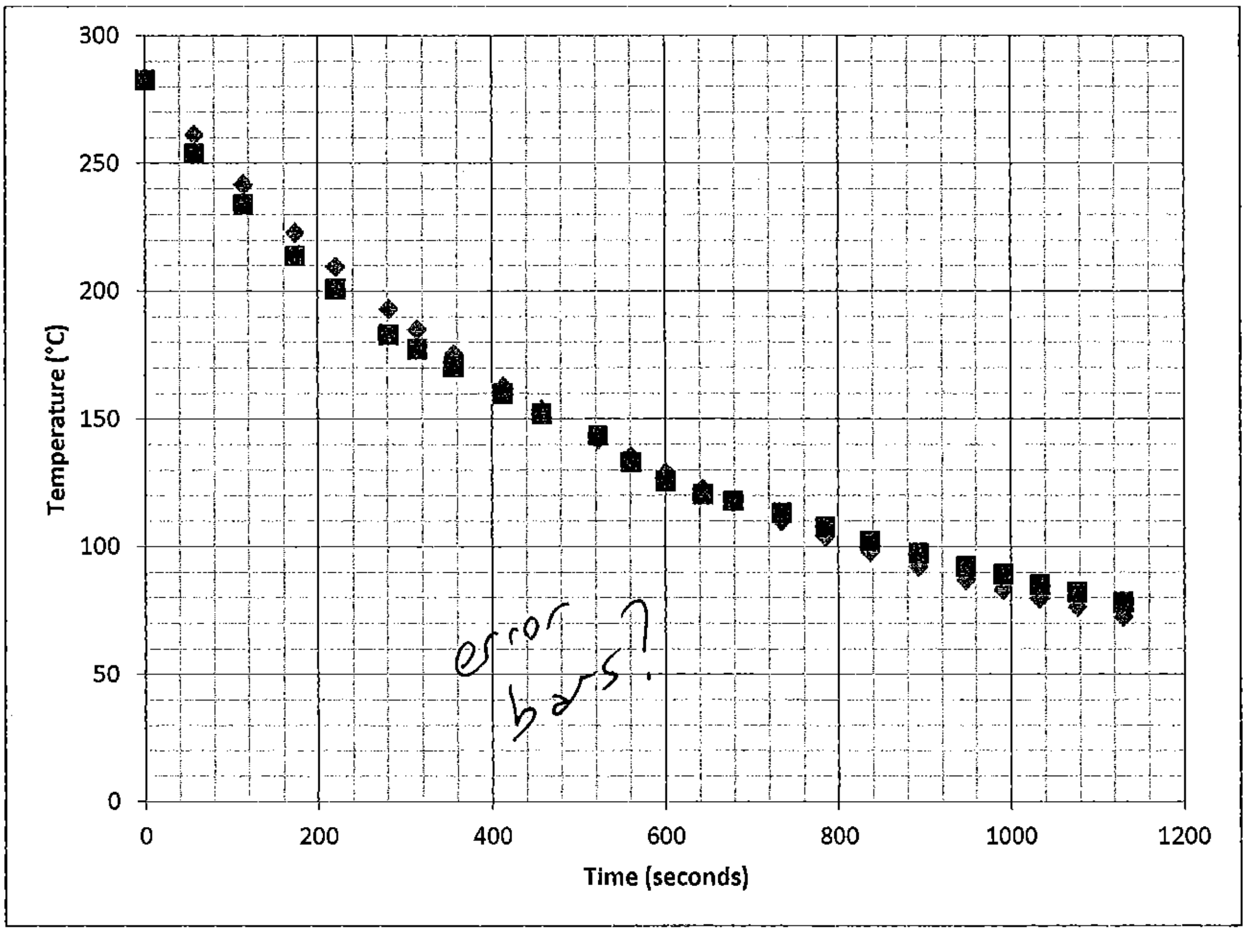
[11] "BioMath: Newton's Law of Cooling," *The Biology Project,* The University of Arizona, Dec. 2005, Web, 25 Feb, 2013, <http://www.biology.arizona.edu/biomath/tutorials/applications/cooling.html>.

The times and temperatures were graphed and a formula was created based on Newton's law of cooling. The formula was used to graph a second set of points for the same times and then to find the best coefficient of determination. In this way, it was possible to empirically determine the "k-value" which provided the best model. Table 2 and Graph 1 below are examples of how the R^2 value was found and the level of determination that the model exhibited for trial two for aluminum bar three.

Table 2: Model Temperatures, Experimental Temperatures, and R^2 Calculations

Time	Fi (Model Temperature in °C)	Yi (Experimental Temperature in °C)	SStot	SSreg	SSerr
K-value:	0.00154				
0	282.8	282.8	18548.17	18548.17	0
56.93	261.4124	253.9	11511.5	13179.97	56.43572
113.53	241.9287	234	7637.303	9085.97	62.86409
173.37	223.0955	214	4541.637	5850.285	82.72791
219.46	209.7279	200.9	2947.585	3984.08	77.93185
280.99	193.2987	183.1	1331.642	2179.993	104.0141
313.75	185.1662	177.5	954.2951	1486.71	58.77081
355.87	175.2953	170.5	570.8117	822.9396	22.99449
412.9	162.9107	160	179.3367	265.7665	8.472053
457.35	153.9846	152.1	30.1584	54.40977	3.551834
522.16	142.0176	143.6	9.050069	21.07515	2.504098
560.51	135.4788	133.3	177.1117	123.8667	4.747125
600.45	129.0672	125.9	428.8351	307.6904	10.03134
642.95	122.6642	120.6	676.4334	573.3214	4.260934
678.02	117.6872	118	818.4367	836.4316	0.09784
734.15	110.2602	113.3	1109.445	1321.184	9.240119
784.56	104.1159	107.8	1506.087	1805.611	13.57296
836.27	98.28955	102.3	1963.228	2334.705	16.08372
892.76	92.43317	97.5	2411.628	2934.949	25.67279
947.67	87.20862	92.3	2949.395	3528.326	25.92216
990.42	83.43616	89.4	3272.793	3990.723	35.56736
1033.26	79.89687	85.2	3770.983	4450.419	28.12316
1076.42	76.55961	82.3	4135.562	4906.824	32.95207
1129.66	72.73707	78.3	4666.028	5456.963	30.94616
	Mean:	146.6083			
		Sums:	76147.46	88050.38	717.4847
				R^2	0.990578

Graph 1: Model and Experimental Temperatures VS Time



Note: The blue diamonds represent the model and the red squares represent the empirical data.

At an R^2=.99058, this trial had the lowest determination to the data out of all the trials completed on all the metals. Still, this indicates that the model matches the real life system fairly accurately considering that the least accurate instance is 99% correlated to the actual data by R^2.

4.4 Confirming the Coefficient of Convective Heat Transfer

The problem with only looking at the coefficient of determination, however, is that the method of determining the equation of the model was based on empirically finding which value of "k" in the formula would generate the highest possible R^2 value. This means that "k" could be any of a relatively wide range of numbers with no regard as to what it actually stands for. This constant is actually:

$$k = \frac{hA}{C}$$

but not defined

Where "h" is the convective heat transfer coefficient (as discussed in section 2.2), A is the surface area of the body, and C is the absolute heat capacity of the body (the specific heat capacity of the material multiplied by the mass of the body). Since "A" and "C" will be constant for each block of aluminum, only "h" is left as a variable.

Now, the average of the best "k-values" for each block of aluminum can be used to calculate a theoretical convective heat transfer coefficient. Since the experiment was done all in the same room, under the same conditions, the actual heat transfer coefficient for each block should have been the same. Therefore, if the average values of the theoretical heat transfer coefficient are close to each other, then the Newton's Law of Cooling model should be fairly accurate in the case of aluminum metal of different surface area to volume ratios.

Table 3: Average "h-value" for Aluminum Blocks

Metal Block	Average "h-value" ($W\ m^{-2}\ K^{-1}$)
1	13.9 ± 0.5
2	15.2 ± 0.4
3	14.7 ± 0.2
4	14.4 ± 0.2
5	15.4 ± 0.2
Mean	14.7 ± 0.3

no data to support these values

From table 3, it is clear that the coefficient of convective heat transfer was not correlated to the dimensions of the block and remained fairly constant for the trials with a mean of about 14.7 $W\ m^{-2}\ K^{-1}$. The literature value of the range of "h-values" for stationary air is 5 to 25 $W\ m^{-2}\ K^{-1}$ so this is a reasonable value.[12]

[12] "Convective Heat Transfer," *The Engineering Toolbox*, N.p., n.d. Web, 21 Feb, 2013, <http://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html>.

4.5 Finding a Linear Relationship

Although this is evidence that the model is accurate, the strongest measure of validity would be to find a linear relationship between some measurable property of the aluminum bars and the "k-value" because the "k-value" is the only part of the Newton's Law of Cooling model which has to otherwise be determined experimentally. Finding this linear relationship would allow for the prediction of the correct "k-value" for a particular aluminum bar.

Logically, the best place to look is the formula:

$$k = \frac{hA}{C}$$

which table ?

Under constant conditions, the "h-value" should be the same. As the table shows, it seemed to remain fairly steady throughout the experiment. Capacitance, C, is not a constant value, but that is only because it accounts for mass. Assuming the above, the equation becomes:

$$k \propto \frac{A}{m}$$

The surface area to mass ratio was not considered in this investigation, but a similar property was calculated. The surface area to volume ratio was considered because it was hypothesized that this would be important in determining how accurate the Newton's Law of Cooling is. Table 4 displays the surface area to volume ratios and the mean "k-values" for each aluminum block:

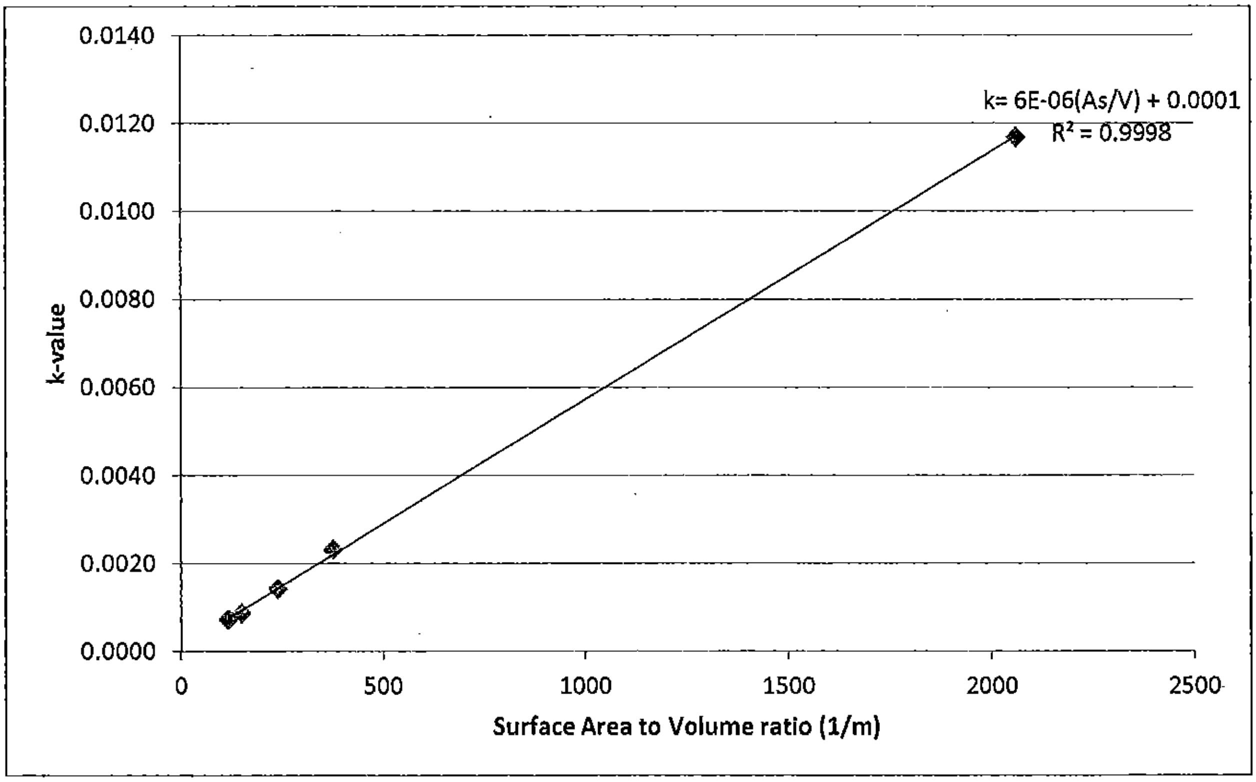
data ?

Table 4: Mean Surface Area to Volume Ratios and k-values

Metal Block	As/V ratio	Mean k-value
1	2062 ± 67	0.0117
2	375 ± 9	0.00233
3	239 ± 4	0.00144
4	150 ± 2	0.000885
5	117 ± 1	0.000743

Graph 2 illustrates this:

Graph 2: Mean k-values VS Surface Area to Volume Ratio



There is a direct relationship between the surface area to volume ratio and the k-value in the Newton's Law of Cooling model. The proportionality constant is the slope of the graph: 6×10^{-6}. This is the best indicator that the model is accurate for aluminum meter, at least at these sizes. Given this relationship it is possible to predict the cooling curve of a piece of Aluminum metal at the same environmental conditions. I.e. the model is valid because it can be used to make accurate predictions.

To the graph; slope should have a unit.

5 Evaluation of Methods: Possible Sources of Error

While the analysis of data obtained in the experiment may indicate that the Newton's Law of Cooling is/is not accurate, it is essential to evaluate the methods by which the data was collected to look for potential sources of systematic and random error. One obvious issue that must be addressed is the shape of the actual temperature curves compared to the theoretical model. The actual temperature seems to drop faster than the model in the beginning, but then slow down enough to intersect the model. Afterwards, the modeled temperature continues to drop faster than the empirical temperature. It is important to investigate the potential causes of this phenomenon.

5.1 Radiation

As mentioned earlier, the temperature readings for this experiment were taken by an infrared thermometer which measured the infrared electromagnetic radiation being emitted by the metal blocks. This also means that the metal blocks lost energy through radiation. The amount of energy they lost can be determined by the formula[13]:

$$P = e\sigma AT^4$$

The emissivity, e, can be assumed to be 0.95 because the metal was spray painted black. The Stefan-Boltzmann constant, σ, is 5.670 x 10^{-8} W m^{-2} K^{-4}, the surface area, A, depends on the selected block, and the absolute temperature, T, depends on the trial and decreases throughout each trial.

Lets calculate the theoretical power lost to radiation by a piece of metal at 500K or about 227 °C since there were trials with starting temperatures both above and below this. Using the above power equation for metal block 1 which has the highest surface area:

$$P = (0.95)(5.670\ \times\ 10^{-8}\,Wm^{-2}K^{-4})(0.0321\,m^2)(500^4) \cong 108\,W$$

[13] Tsokos, K. A. *Physics for the IB Diploma*, Cambridge: Cambridge UP, 2008, Print, Page 435.

For the first metal piece, this is quite significant. It amounts to nearly a temperature drop of three degrees per second. The metal blocks with lower surface area to volume ratios, however, would lose less energy to radiation.

It is clear that the metal may be losing a significant amount of energy to radiation while it is still hot. This could explain part of the curve seen in some of the graphs of various trials, especially when the metal started at very hot temperatures. At the same time, this would affect the optimal "k-value" which will result in a model curve that matches neither the first half of the real curve, nor the second half because it is skewed by the temperature decrease that is caused by radiation.

This could be remedied by performing the experiment in colder environmental conditions and not raising the temperature of the aluminum as high as it was raised in this experiment. Since the power emitted through radiation is based on the fourth power of absolute temperature, decreasing the maximum temperature that the metal reaches would have an extreme effect on the amount of energy lost through radiation.

5.2 Wood Pieces

The procedure used wood pieces to hold up the metal blocks because the wood was the best available insulator that could withstand the heat of the metal. The problem is that, initially, the wood is of a very different temperature than the metal, but is in contact with it. Despite being an insulator, the wood is still a better conductor than air and absorbs a significant amount of heat early in the trial. The wood warms during the trial because it cannot conduct the heat away very quickly and therefore begins slowing down the heat loss in the metal. These two things, like radiation, would have the combined effect of causing the temperature of the aluminum to decrease quickly in the beginning (faster than the predictive model) and then decrease slower than the model once the wood warmed up.

A possible solution to this error would be to use a more sophisticated setup with heat resistant insulators. Styrofoam would not work (it was tried and it melted due to the high temperatures of the metal), but silica aerogel, with a conductivity of 0.02 W m^{-1} K^{-1}, would be great for this application.[14] It is expensive, however, so it may not be practical. Still, if only a small amount was used to support the metal at each corner, it could significantly reduce the error due to conduction of heat into the wood.

5.3 Random Convection Currents

While any convection caused by the large difference in temperature between the aluminum bars and the surrounding air is meant to be part of the Newton's Law of Cooling model, the movement of air caused by the measurement of the temperatures themselves is another limitation. The researcher had to move around, move the thermometer, and breathe while measuring the temperatures. This could have caused air to move more and cool the aluminum blocks faster.

Having a more sophisticated setup might solve this issue. If the setup were improved as mentioned in 5.2, additional improvements could be made. The experiment could be run in a more tightly controlled environment. Furthermore, an IR thermometer that doesn't require manual activation could be used. If the thermometer sensor was placed a specific distance above the metal, remained stationary, and could be controlled and read remotely, it would reduce this type of error.

[14] "Thermal Conductivity of Some Common Materials and Gases," *The Engineering Toolbox*, N.p., n.d. Web, 21 Feb, 2013, <http://www.engineeringtoolbox.com/thermal-conductivity-d_429.html>.

6 Conclusion

The data gathered demonstrates that the Newton's Law of Cooling model is valid to a large extent for aluminum bars in stationary air. Using the formula, it was possible to find a "k-value" that provides a fairly accurate theoretical curve of the temperature of an aluminum bar. It was found that a linear relationship exists between the "k-value" and the surface area to volume ratio which would allow for estimation of the "k-value" for any piece of aluminum provided that its surface area to volume (or mass) ratio can be established. Using the averages of every trial, this model worked for every metal block from the one with the highest surface area to volume ratio to the one with the lowest ratio.

But none of this is in the text

These results, however, do raise new questions. This experiment was done on aluminum bars which all had biot numbers much lower than 0.1 and therefore the Newton's Law of Cooling model should have been very accurate. Further investigation could be done into the applicability of this model as the biot number increases. This could be accomplished through the use of different materials or by increasing the convective heat transfer coefficient. Different materials would be ones that conduct heat more slowly within themselves. The coefficient could be increased by using moving air or changing the fluid surrounding the object. Another opportunity for investigation is the applicability of this model at more extreme surface area to volume ratios. The model might fail when the volume is much greater than the surface area because heat transfer within the object might not be rapid enough to maintain a fairly even temperature distribution. ✓

Flawed through lack of any raw data in the text; mathematical treatment, resulting in lack of physics; EE is not understandable without the Appendix ✓

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Appendix A: Raw Data, R-Squared, and Graphs for All Trials

Note: In all graphs, blue diamonds represent the model and red squares represent the empirical data.

Aluminum block 1:

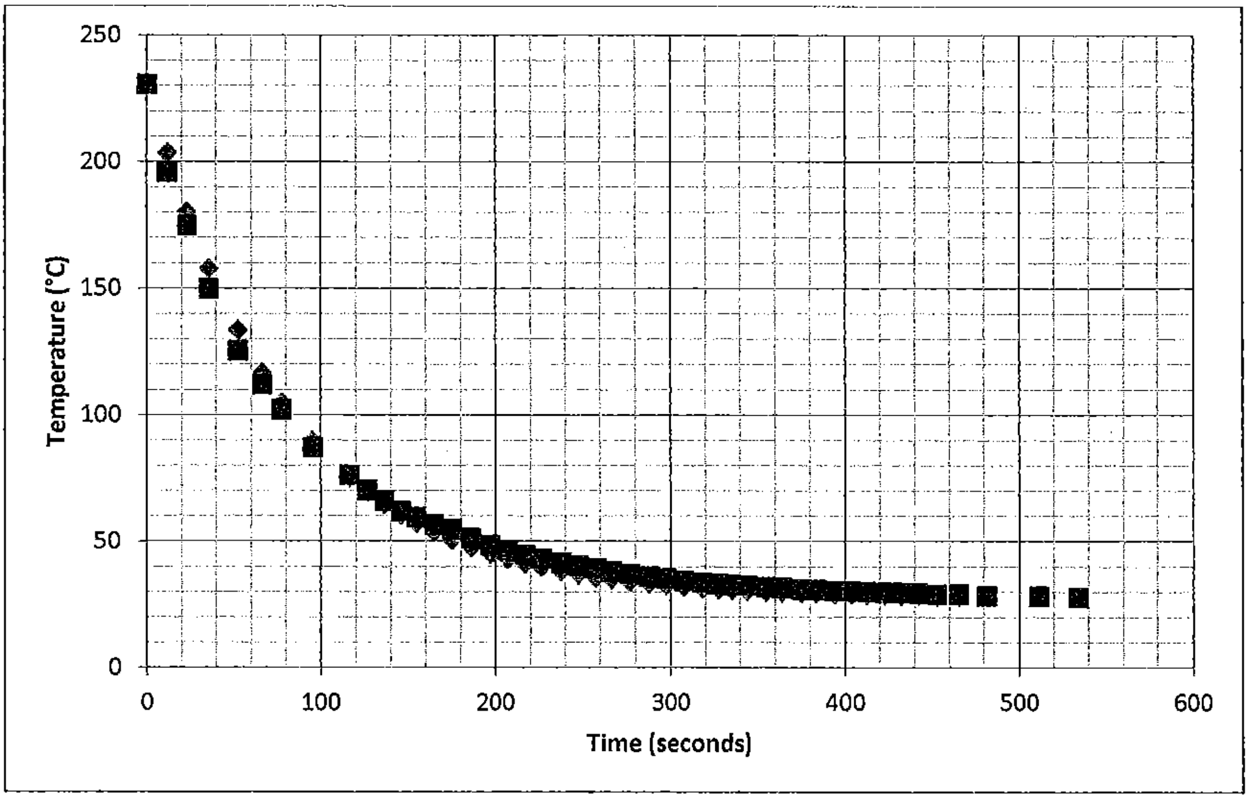
Data Table for Trial 1:

Time	Fi	Yi #1 T1	SStot	SSreg	SSerr
k-value:	0.0125				
0	230.7	230.7	29699.02	29699.02	0
11.43	203.713	196	18943.13	21125.77	59.49056
22.78	180.4714	174.9	13580.18	14909.74	31.04053
35.56	157.96	149.8	8360.184	9918.982	66.58633
52.1	133.6865	125.4	4493.563	5673.182	68.66589
66.13	116.686	112	2876.611	3401.227	21.95861
77.45	104.9843	102.1	1912.666	2173.274	8.31943
95.17	89.68878	87.2	831.402	981.1192	6.194022
116.03	75.52957	76.4	325.2267	294.5896	0.757648
126.17	69.87136	70.4	144.8182	132.3743	0.279461
136.13	64.96983	66.1	59.81541	43.61107	1.277294
145.64	60.8262	62	13.20627	6.052806	1.377801
154.4	57.42154	59.6	1.522861	0.891925	4.745689
164.69	53.87047	56.5	3.481797	20.20943	6.914443
174.91	50.76791	54.8	12.71605	57.73034	16.25776
185.87	47.85294	51.3	49.92775	110.5236	11.88223
197.15	45.24207	48.5	97.33712	172.2365	10.61414
207.07	43.23129	46.5	140.8009	229.0581	10.68445
217.14	41.42981	44.6	189.5016	286.833	10.05009
226.12	40.00385	43.3	226.9831	337.1672	10.86463
237.51	38.41089	41.6	281.0973	398.2048	10.17044
247.66	37.17036	40.5	319.1924	449.2532	11.08647
257.45	36.11409	39.5	355.9244	495.1456	11.46439
266.42	35.25345	38.2	406.6658	534.1881	8.682164
276.75	34.3748	37.2	447.9978	575.5758	7.981779
287.6	33.56628	36.1	495.7729	615.024	6.419735
297.56	32.91468	35.3	532.0384	647.7674	5.689742
307.76	32.32636	34.5	569.5839	678.0605	4.724701
318.24	31.79516	33.8	603.4863	706.0072	4.019376
327.5	31.38034	33.2	633.3254	728.2234	3.311153
335.1	31.07399	32.8	653.6182	744.8514	2.979104

NOT ASSESSED AS PART OF THE ESSAY

344.09	30.74725	32.7	658.7414	762.7928	3.813215
354.95	30.39852	32	695.1637	782.1775	2.564737
363.56	30.15379	31.7	711.0733	795.9266	2.390779
374.86	29.87007	31.2	737.9892	812.0155	1.768708
383.31	29.68262	30.9	754.3788	822.7339	1.482014
393.9	29.474	30.6	770.9484	834.7454	1.267885
403.29	29.31075	30.4	782.0948	844.2049	1.186458
412.32	29.17085	30.2	793.3212	852.3545	1.059157
422.81	29.02696	29.9	810.3107	860.7768	0.762201
432.31	28.91197	29.8	816.0139	867.5374	0.788597
441.78	28.81016	29.7	821.7371	873.5451	0.791814
452.71	28.7067	29.1	856.4963	879.6715	0.154684
465.45	28.60266	29.2	850.6531	885.8537	0.356812
481.5	28.49311	28.6	886.0122	892.387	0.011425
511.18	28.34027	28.4	897.9586	901.5421	0.003568
534.11	28.25547	28	922.0914	906.6414	0.065265
	Mean:	58.36596			
		Sums:	100025.8	109720.8	442.9574
				R^2	0.995572

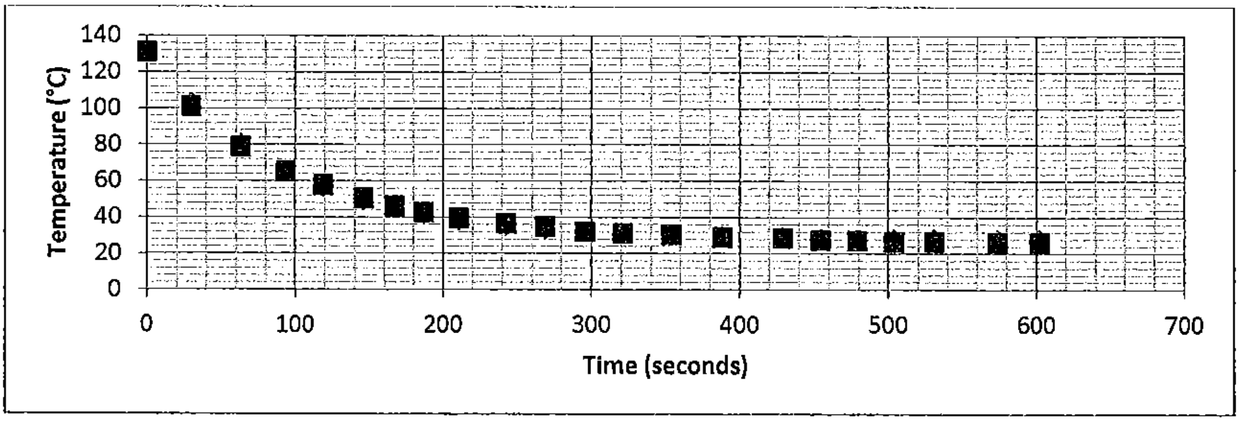
Graph for trial 1:



Data Table for Trial 2:

Time	Fi	Yi #1 T2	SStot	SSreg	SSerr
k-value	0.0107				
0	131.3	131.3	7369.442	7369.442	0
29.36	103.4511	101.1	3096.417	3363.606	5.527888
62.77	80.77293	79.1	1132.017	1247.388	2.798696
93.41	66.02145	65.5	401.8202	422.9974	0.271906
118.88	56.95148	57.7	149.9512	132.1795	0.56028
145.88	49.68718	50.8	28.57388	17.91521	1.238365
166.81	45.33574	46.3	0.714793	0.014114	0.929794
186.76	42.00347	42.9	6.525702	11.9099	0.803761
210.6	38.85057	39.9	30.85298	43.61253	1.101309
242.37	35.72358	36.7	76.64207	94.69163	0.95339
268.51	33.83912	35	109.2975	134.9181	1.347639
295.91	32.35533	32.3	173.0421	171.5894	0.003061
320.81	31.33666	31.4	197.5302	199.3147	0.004012
353.88	30.34227	30.5	223.6384	228.3808	0.024878
388.22	29.62204	29.2	264.2102	250.6683	0.178116
428.85	29.05016	28.7	280.7148	269.1039	0.122612
454.46	28.79845	27.7	315.2239	277.4256	1.206589
478.87	28.61491	27.4	325.9666	283.5732	1.476018
503.72	28.47134	26.9	344.2712	288.4291	2.469123
530.74	28.353	26.8	347.9921	292.4628	2.411816
573.95	28.22232	26.4	363.0757	296.9495	3.320858
601.67	28.16526	26.4	363.0757	298.9194	3.116145
	Mean:	45.45455			
		Sums:	15600.99	15695.49	29.86626
				R^2	0.998086

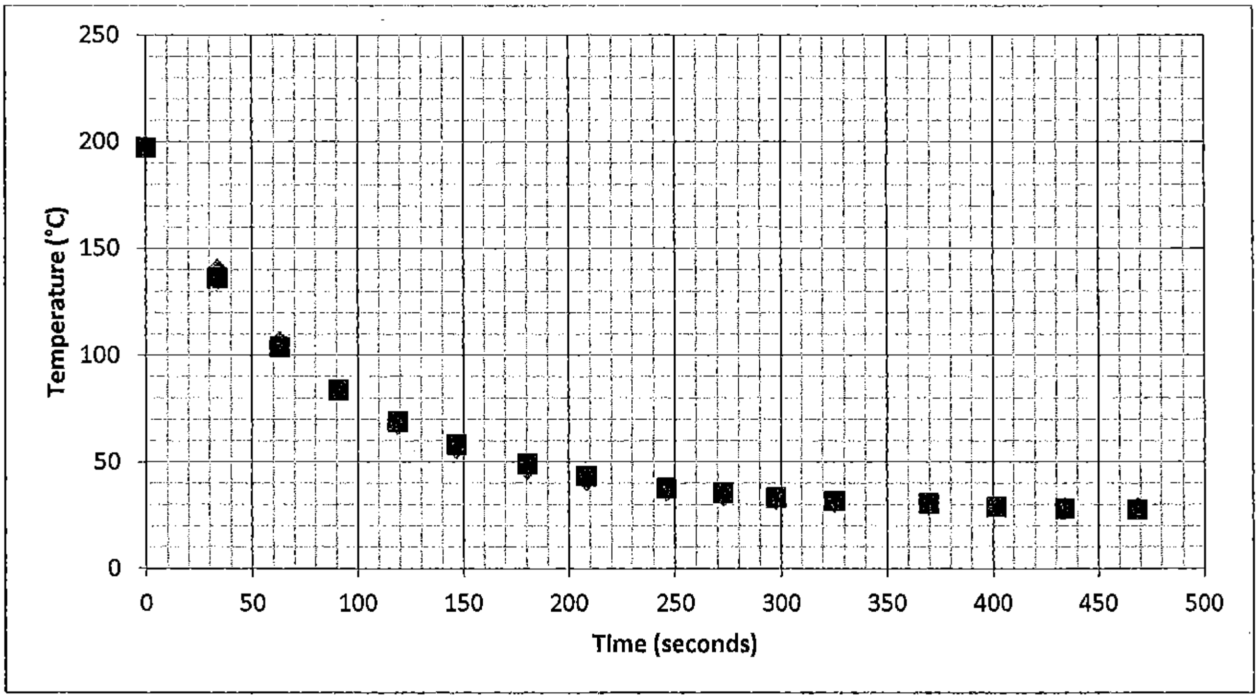
Graph for Trial 2:



Data Table for Trial 3:

Time	Fi	Yi #1 T3	SStot	SSreg	SSerr
k-value:	0.0122				
0	197.5	197.5	18295.94	18295.94	0
33.57	140.5389	136.2	5470.451	6131.109	18.82604
62.91	106.677	103.9	1735.764	1974.87	7.711757
90.7	84.05386	83.8	464.9414	475.9533	0.064442
118.72	67.82397	69.1	47.09391	31.20862	1.628259
146.59	56.34512	58.3	15.50391	34.72013	3.821551
180.24	46.80128	49.2	169.9764	238.2769	5.75386
208.36	41.34125	43.4	354.8514	436.6532	4.238441
245.81	36.44835	37.9	592.3139	665.0804	2.107293
272.68	34.08701	35.7	704.2389	792.4501	2.601738
297.64	32.48907	33.5	825.8439	884.9693	1.021985
325.15	31.2092	31.8	926.4414	962.7552	0.34904
369.53	29.86747	30.6	1000.931	1047.819	0.536594
401.37	29.26635	29	1104.731	1087.097	0.070942
433.92	28.85132	28.1	1165.369	1114.637	0.564477
468.5	28.55831	27.8	1185.941	1134.288	0.575029
	Mean:	62.2375			
		Sums:	34060.34	35307.83	49.87145
				R^2:	0.998536

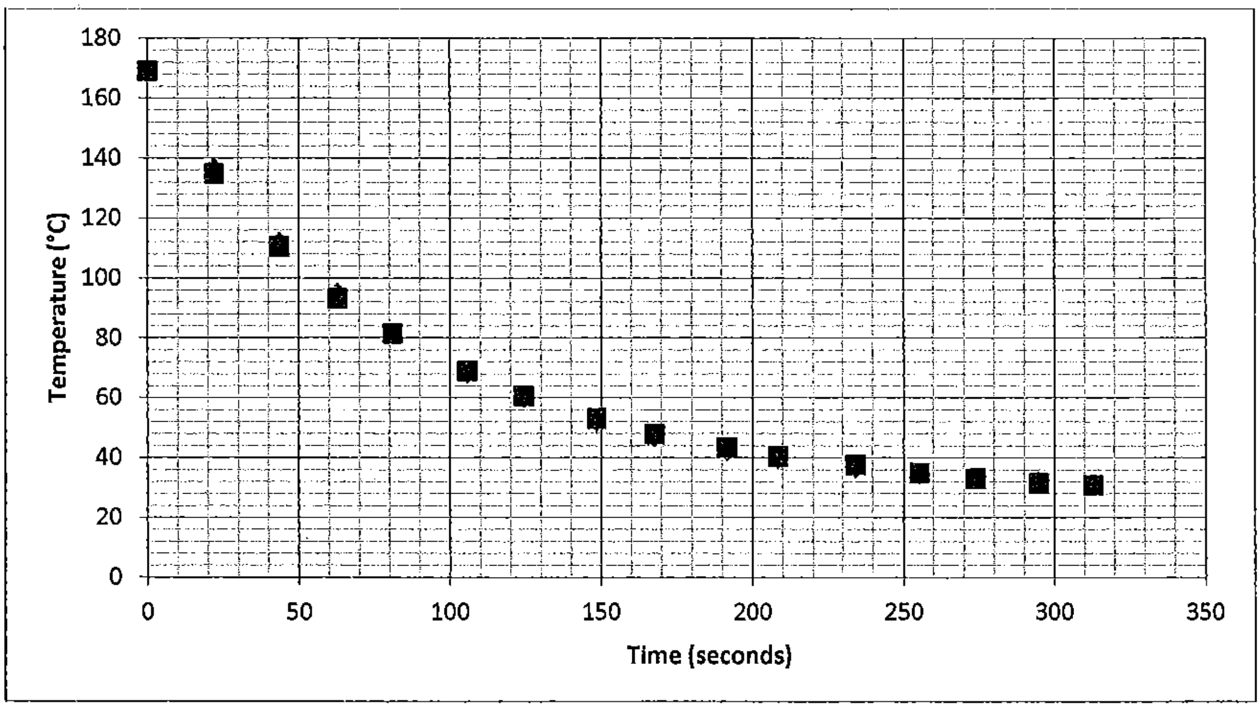
Graph for Trial 3:



Data Table for Trial 4:

Time	Fi	Yi #1 T4	SStot	SSreg	SSerr
k-value:	0.0119				
0	169.2	169.2	10435.9	10435.9	0
21.95	136.7414	134.9	4604.471	4857.758	3.390649
43.51	112.1338	110.6	1897.147	2033.112	2.352526
62.57	95.06044	93.3	689.3907	784.935	3.099154
80.94	81.8925	81.5	208.9832	220.4854	0.154056
105.61	68.18197	69.1	4.228164	1.295539	0.842784
124.28	60.17679	60.7	40.24316	47.15517	0.273751
148.39	52.15125	53.2	191.6494	221.7864	1.099867
167.73	47.18618	47.9	366.4832	394.323	0.509536
191.68	42.42821	43.6	549.6094	605.9248	1.373092
208.42	39.8222	40.4	709.8894	741.0127	0.33385
234.16	36.70305	37.8	855.1969	920.5582	1.203303
255.24	34.77217	35	1026.802	1041.455	0.051907
273.8	33.4301	33.1	1152.178	1129.878	0.108965
294.72	32.23342	31.5	1263.358	1211.759	0.5379
312.85	31.41188	30.9	1306.371	1269.63	0.262017
	Mean:	67.04375			
		Sums:	25301.9	25916.97	15.59336
				R^2:	0.999384

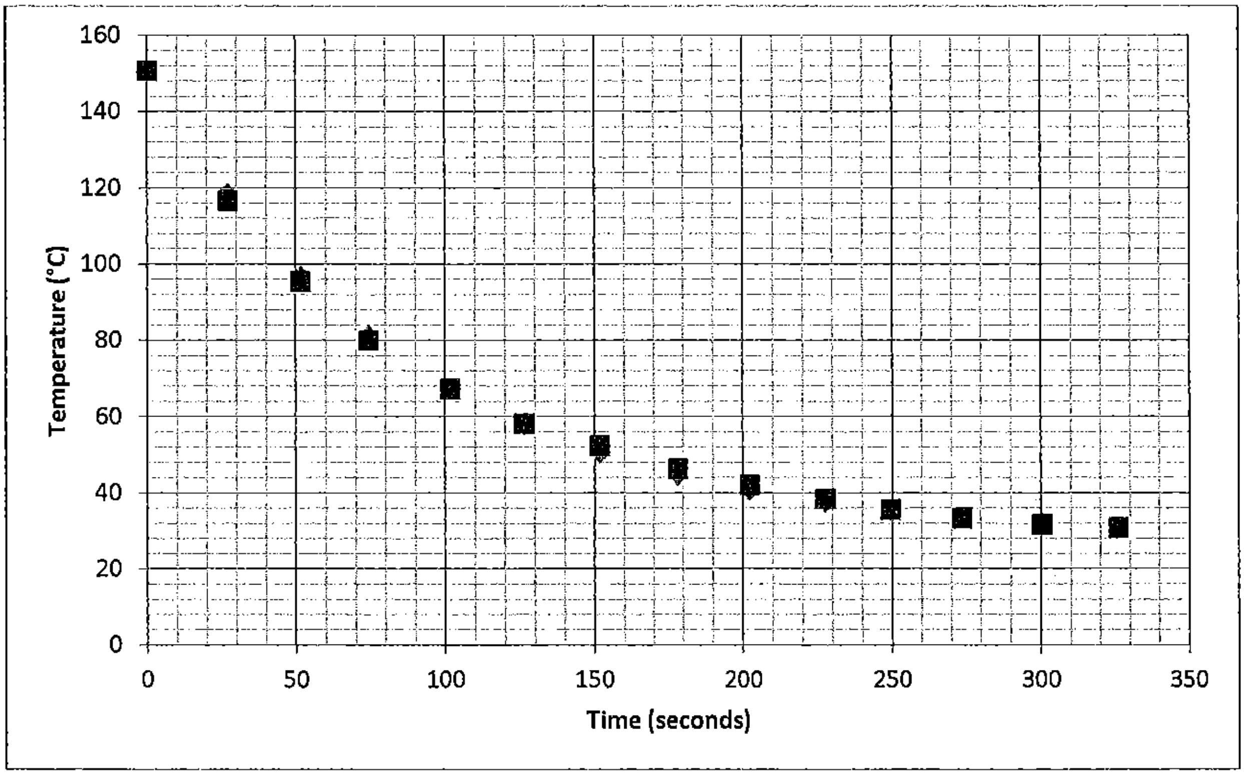
Graph for Trial 4:



Data Table for Trial 5:

Time	Fi	Yi #1 T5	SStot	SSreg	SSerr
k-value:	0.0112				
0	150.7	150.7	7726.41	7726.41	0
26.96	118.7211	116.6	2894.44	3127.173	4.499214
51.48	96.93516	95.5	1069.29	1165.209	2.059684
74.25	81.41771	80	295.84	346.6192	2.009904
101.67	67.29268	67.3	20.25	20.18421	5.35E-05
126.34	57.80673	58.2	21.16	24.9327	0.154658
151.61	50.45943	52.4	108.16	152.2898	3.765831
177.94	44.72349	46.3	272.25	326.7602	2.485382
202.18	40.74739	42.1	428.49	486.3177	1.82956
227.49	37.60088	38.5	590.49	634.9958	0.808424
249.38	35.51339	35.6	739.84	744.5593	0.007502
273.49	33.73538	33.4	864.36	844.752	0.112481
300.25	32.2501	31.7	967.21	933.2964	0.302609
326.04	31.18386	30.9	1017.61	999.5803	0.080577
	Mean:	62.8			
		Sums:	17015.8	17533.08	18.11588
				R^2:	0.998935

Graph for Trial 5:

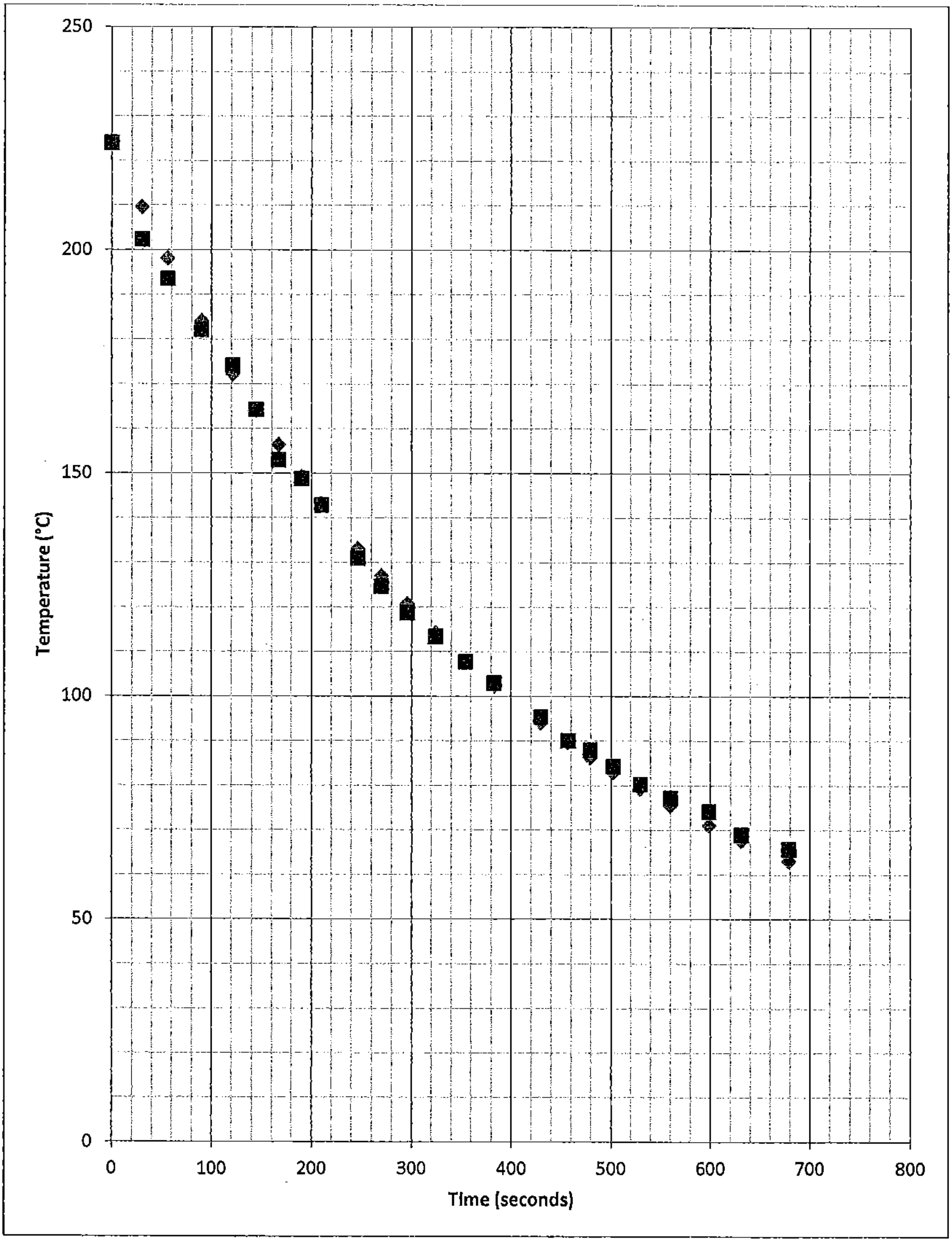


Aluminum Block 2:

Data Table for Trial 1:

Time	Fi	Yi #2 T1	SStot	SSreg	SSerr
k-value:	0.00253				
0	224	224	9732.645	9732.645	0
30	209.6741	202.4	5937.345	7111.264	52.91316
55.82	198.1857	193.6	4658.631	5305.643	21.02843
89.77	184.178	182.2	3232.396	3461.227	3.912593
121.11	172.2729	174.2	2386.73	2202.146	3.713873
143.93	164.1792	164.3	1517.427	1508.031	0.01459
166.92	156.4843	153	764.7529	969.6068	12.14065
189.93	149.2181	148.8	550.0979	569.8872	0.174844
209.87	143.2546	142.9	308.1488	320.7238	0.125738
246.29	133.1093	131	31.9696	60.27174	4.449234
269.77	127.0472	124.7	0.417101	2.894667	5.509373
295.41	120.8261	118.7	44.1671	20.4284	4.520107
324.19	114.3073	113.4	142.7029	121.8496	0.823158
354.16	108.0051	107.8	307.8563	300.7025	0.042049
382.89	102.3961	103	499.3363	526.6922	0.364746
429.46	94.12725	95.4	896.7529	974.6002	1.619902
456.47	89.75936	90.1	1242.269	1266.397	0.116035
478.99	86.33895	88	1394.711	1521.537	2.759088
501.94	83.04805	84.3	1684.76	1789.102	1.567376
529.34	79.36127	80.3	2029.127	2114.58	0.881207
559.63	75.5723	77.2	2318.021	2477.404	2.649392
598.21	71.14831	74.2	2615.896	2937.371	9.312783
630.51	67.7625	69	3174.853	3315.84	1.531396
678.52	63.21454	65.8	3545.706	3860.298	6.684605
	Mean:	125.3458			
		Sums:	49016.72	52471.14	136.8543
				R^2	0.997208

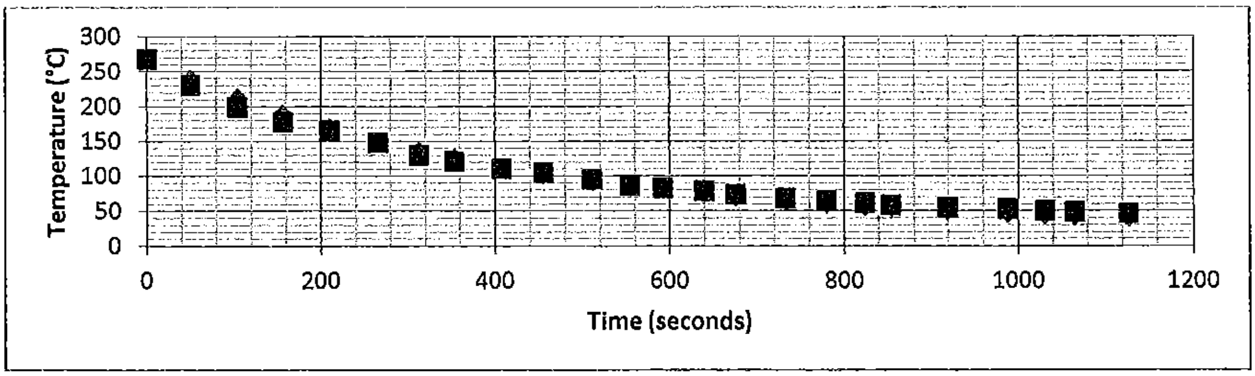
Graph for Trial 1



Data Table for Trial 2:

Time	Fi	Yi #2 T2	SStot	SSreg	SSerr
k-value:	0.00254				
0	267.9	267.9	25632.01	25632.01	0
49.91	239.3363	230.7	15104.41	17301.8	74.58592
103.93	212.2401	199.3	8372.25	10907.74	167.447
156.41	189.2476	178.3	4970.25	6633.704	119.8489
209.6	168.8701	165.9	3375.61	3729.557	8.821478
265.36	150.2671	148.9	1689.21	1803.452	1.868887
312.09	136.5829	130.6	519.84	828.453	35.79462
353.31	125.7893	121.3	182.25	323.6135	20.15347
407.12	113.2968	111.2	11.56	30.21521	4.396724
454.85	103.5582	105.5	5.29	17.9929	3.770602
510.74	93.55848	95.6	148.84	202.8209	4.167804
553.72	86.77834	87	432.64	441.9103	0.049134
592.02	81.32959	83.2	605.16	700.6825	3.498427
639.26	75.2996	79.1	823.69	1056.276	14.44306
675.23	71.16966	74.1	1135.69	1341.782	8.586883
732.15	65.35853	68.3	1560.25	1801.279	8.652267
779.44	61.13018	65	1831.84	2178.072	14.97553
823.84	57.59687	61.7	2125.21	2520.354	16.83569
853.22	55.4686	58.7	2410.81	2738.576	10.44198
918.76	51.2562	55	2787.84	3197.201	14.01604
987.54	47.52841	53.3	2970.25	3632.664	33.3112
1030.25	45.52077	50.8	3249	3878.702	27.87024
1064.57	44.05812	49.3	3422.25	4063.028	27.47735
1126.93	41.70581	46.5	3757.69	4368.442	22.98424
	Mean:	107.8			
		Sums:	87123.84	99330.33	643.9974
				R^2	0.992608

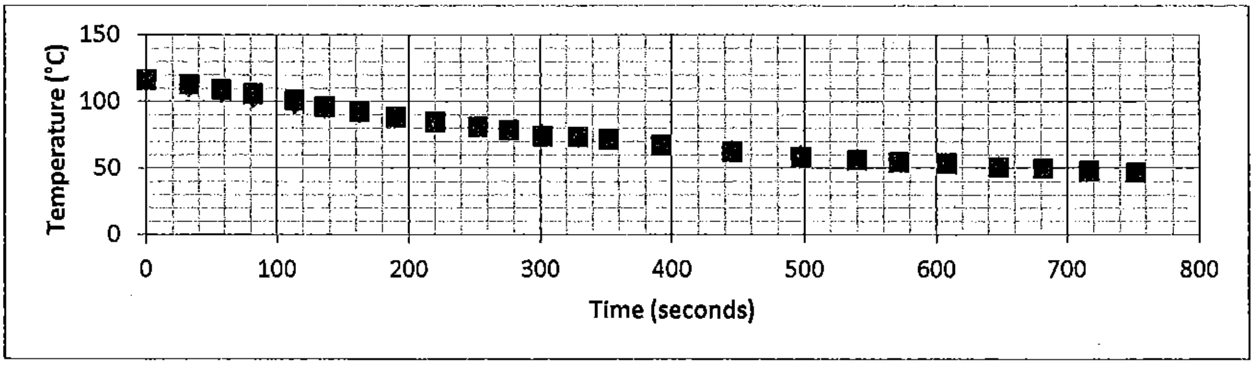
Graph for Trial 2:



Data Table for Trial 3:

Time	Fi	Yi #2 T3	SStot	SSreg	SSerr
k-value:	0.00203				
0	116.5	116.5	1597.334	1597.334	0
32.47	110.8547	113	1329.818	1177.956	4.602319
57.23	106.7931	109.2	1067.111	915.6545	5.793083
81.72	102.9717	106.1	874.1878	698.9882	9.786147
113.18	98.33343	101.2	608.4444	475.244	8.217251
135.88	95.16593	96.2	386.7778	347.1737	1.069297
162.67	91.61075	92.5	254.9344	227.3284	0.790769
189.94	88.18507	88.6	145.6044	135.763	0.172165
220.11	84.60964	85	71.68444	65.2267	0.152383
252.58	80.99858	81.2	21.77778	19.93844	0.040569
276.05	78.53272	78.8	5.137778	3.997548	0.071438
301.58	75.98052	74.1	5.921111	0.3056	3.536366
328.89	73.39291	73.6	8.604444	9.86228	0.042888
352.08	71.30552	71.8	22.40444	27.33001	0.244509
391.86	67.94593	67.5	81.60111	73.74352	0.198852
445.2	63.84652	62.5	196.9344	160.9552	1.813122
497.18	60.25674	58.7	318.0278	264.9273	2.423453
539.74	57.58686	56.3	409.3878	358.9688	1.656009
571.38	55.74626	54.7	476.6944	432.1023	1.094666
608.17	53.74956	53.6	525.9378	519.1004	0.022368
647.59	51.7693	50.7	667.3611	613.2573	1.143404
681.27	50.19849	49.9	709.3344	693.5238	0.089099
716.41	48.67014	48	814.1511	776.3574	0.449091
751.98	47.23022	47.1	866.3211	858.6723	0.016958
	Mean:	76.53333			
		Sums:	11465.49	10453.71	43.4262
				R^2	0.996212

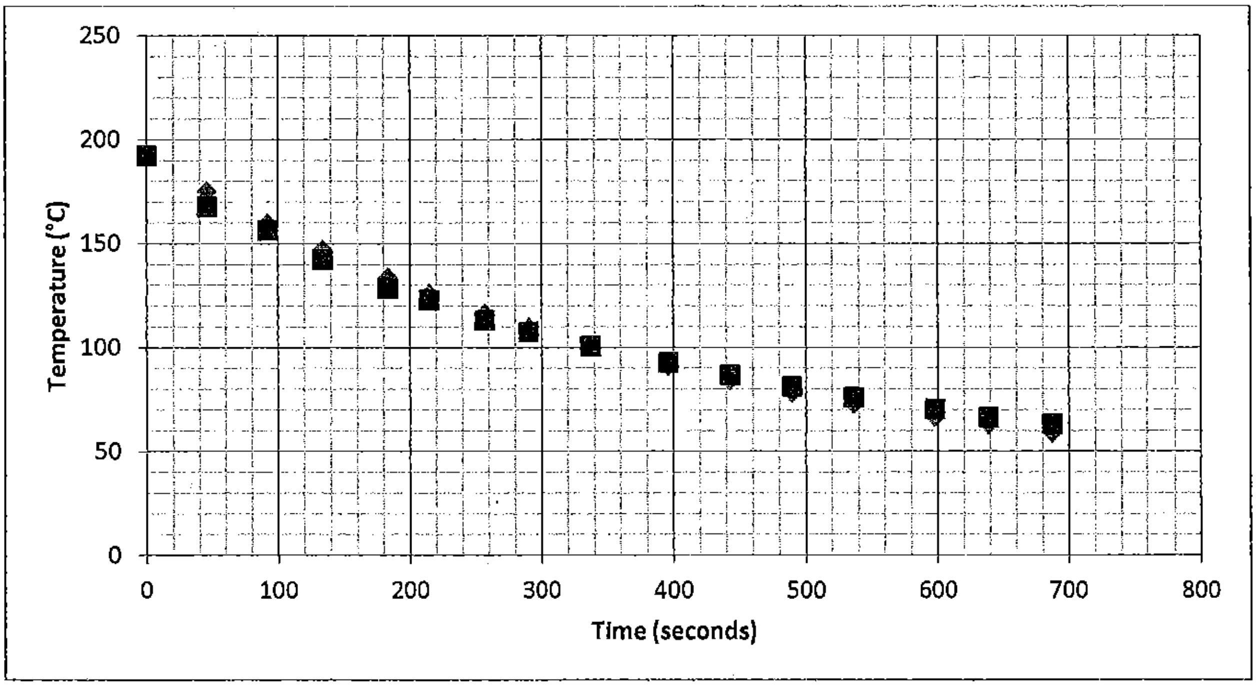
Graph for Trial 3:



Data Table for Trial 4:

Time	Fi	Yi #2 T4	SStot	SSreg	SSerr
k-value:	0.00241				
0	192.3	192.3	6685.106	6685.106	0
45.43	175.2612	167.8	3278.994	4189.154	55.66916
91.75	159.7066	156.4	2103.369	2417.598	10.93343
133.54	147.088	142.3	1008.856	1335.938	22.92484
182.98	133.7113	128.5	322.6514	537.0271	27.15812
214.65	125.9432	122.9	152.8314	237.335	9.260951
256.83	116.4762	113.2	7.088906	35.2683	10.73357
290.07	109.665	107.6	8.628906	0.761283	4.264161
337.1	100.9142	100.8	94.81891	92.60844	0.013035
396	91.26511	92.9	311.0814	371.4249	2.672859
442.49	84.5595	86.8	563.4689	674.8566	5.019855
489.12	78.54757	81.1	866.5664	1023.356	6.514911
535.84	73.16487	76	1192.839	1396.713	8.037949
597.51	66.92727	70.3	1619.056	1901.853	11.37534
638.42	63.27244	66.5	1939.301	2233.986	10.41717
686.97	59.37766	63.2	2240.839	2617.329	14.61025
	Mean:	110.5375			
		Sums:	22395.5	25750.32	199.6056
				R^2	0.991087

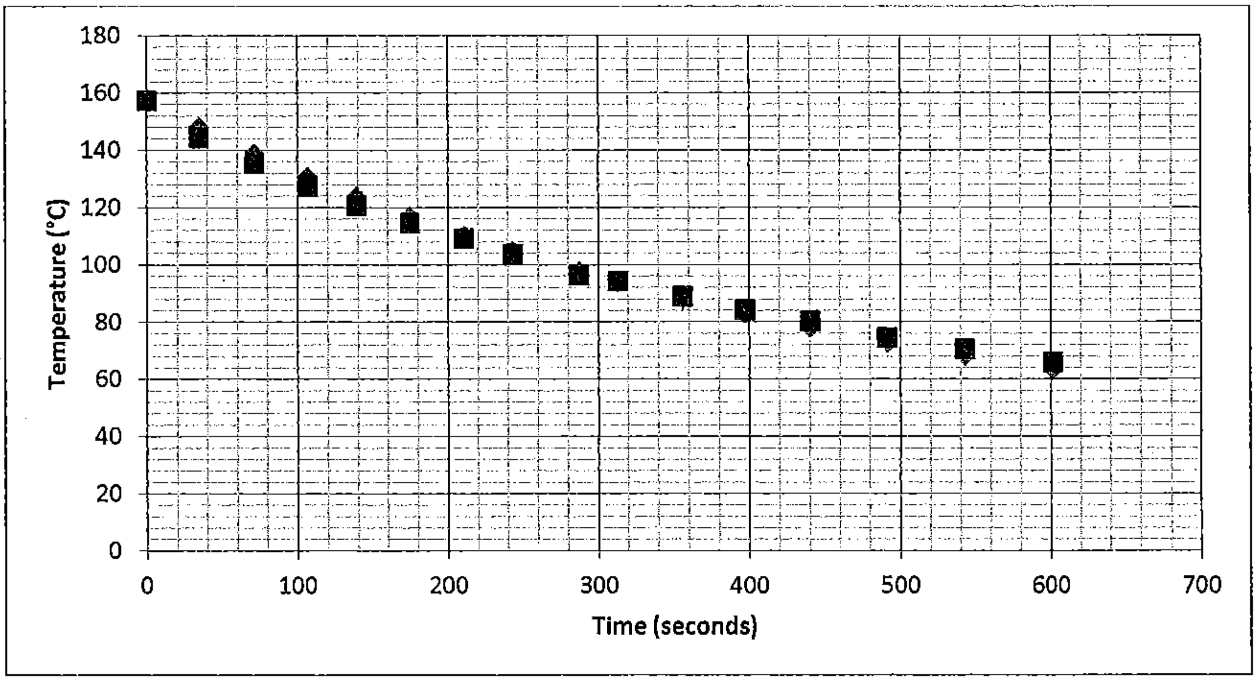
Graph for Trial 4:



Data Table for Trial 5:

Time	Fi	Yi #2 T5	SStot	SSreg	SSerr
k-value:	0.00215				
0	157.4	157.4	2824.258	2824.258	0
34.27	148.2085	144.3	1603.502	1931.802	15.27651
71.22	139.0283	135.4	969.9332	1209.097	13.16477
106.7	130.8738	127.5	540.2719	708.4961	11.3828
138.95	123.9825	120.8	273.6957	389.1249	10.12831
174.52	116.9158	114.6	106.9932	160.2654	5.363147
210.46	110.304	109.1	23.46191	36.57495	1.449551
242.96	104.7493	103.8	0.208164	0.243107	0.901187
287.01	97.81416	96.4	61.72066	41.5005	1.999854
312.84	94.04276	94.3	99.12691	104.3154	0.066173
355.93	88.19919	89	232.7532	257.8292	0.641298
397.18	83.09017	84.2	402.2532	448.0027	1.231712
440.25	78.21787	80.3	573.9019	677.9974	4.335277
491.22	73.00553	74.5	885.4344	976.6075	2.233439
542.69	68.29087	70.6	1132.743	1293.509	5.332083
601.2	63.52826	65.9	1471.202	1658.769	5.62516
	Mean:	104.2563			
		Sums:	11201.46	12718.39	79.13126
				R^2	0.992936

Graph for Trial 5:

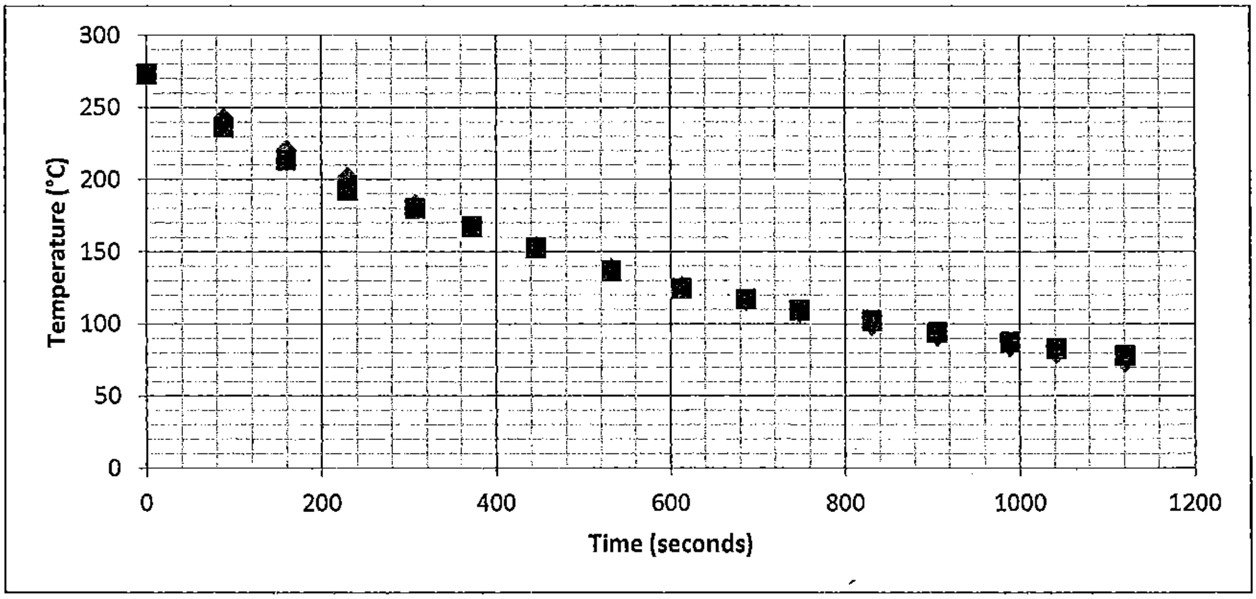


Aluminum Block 3:

Data Table for Trial 1:

Time	Fi	Yi #3 T1	SStot	SSreg	SSerr
k-value:	0.00149				
0	273.1	273.1	15898.06	15898.06	0
87.62	243.102	236.5	8008.013	9233.198	43.58685
159.44	221.2724	213.6	4433.895	5514.536	58.86601
228.88	202.2751	193.1	2124.058	3053.955	84.18248
306.34	183.2784	180.3	1108.058	1315.216	8.870918
371.75	168.8589	167.6	423.8452	477.2659	1.584868
445.59	154.1835	153	35.85016	51.42323	1.40067
531.49	139.024	137.1	98.25766	63.81649	3.701689
612.66	126.3766	124.7	497.8477	425.8407	2.810961
686.22	116.164	117.5	870.9877	951.6312	1.784949
746.92	108.5401	110	1369.925	1480.124	2.131239
830.26	99.13496	102.4	1990.275	2292.259	10.6605
905.32	91.60802	94.3	2778.608	3069.657	7.246781
988.26	84.21361	87.6	3529.845	3943.701	11.46764
1041.63	79.91655	83.2	4072.035	4501.867	10.78105
1120.3	74.17411	78.2	4735.16	5305.431	16.20778
	Mean:	147.0125			
		Sums:	51974.72	57577.98	265.2844
				R^2	0.994896

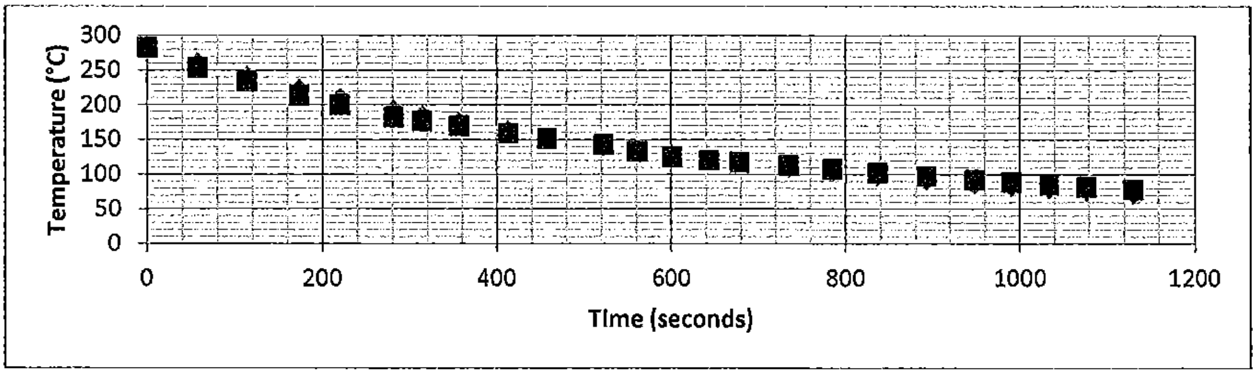
Graph for Trial 1:



Data Table for Trial 2:

Time	Fi	Yi #3 T2	SStot	SSreg	SSerr
k-value:	0.00154				
0	282.8	282.8	18548.17	18548.17	0
56.93	261.4124	253.9	11511.5	13179.97	56.43572
113.53	241.9287	234	7637.303	9085.97	62.86409
173.37	223.0955	214	4541.637	5850.285	82.72791
219.46	209.7279	200.9	2947.585	3984.08	77.93185
280.99	193.2987	183.1	1331.642	2179.993	104.0141
313.75	185.1662	177.5	954.2951	1486.71	58.77081
355.87	175.2953	170.5	570.8117	822.9396	22.99449
412.9	162.9107	160	179.3367	265.7665	8.472053
457.35	153.9846	152.1	30.1584	54.40977	3.551834
522.16	142.0176	143.6	9.050069	21.07515	2.504098
560.51	135.4788	133.3	177.1117	123.8667	4.747125
600.45	129.0672	125.9	428.8351	307.6904	10.03134
642.95	122.6642	120.6	676.4334	573.3214	4.260934
678.02	117.6872	118	818.4367	836.4316	0.09784
734.15	110.2602	113.3	1109.445	1321.184	9.240119
784.56	104.1159	107.8	1506.087	1805.611	13.57296
836.27	98.28955	102.3	1963.228	2334.705	16.08372
892.76	92.43317	97.5	2411.628	2934.949	25.67279
947.67	87.20862	92.3	2949.395	3528.326	25.92216
990.42	83.43616	89.4	3272.793	3990.723	35.56736
1033.26	79.89687	85.2	3770.983	4450.419	28.12316
1076.42	76.55961	82.3	4135.562	4906.824	32.95207
1129.66	72.73707	78.3	4666.028	5456.963	30.94616
	Mean:	146.6083			
		Sums:	76147.46	88050.38	717.4847
				R^2	0.990578

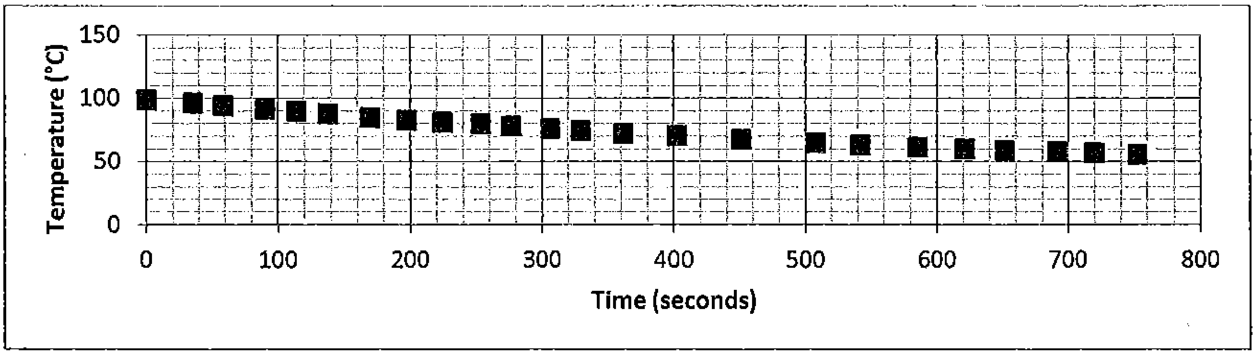
Graph for Trial 2:



Data Table for Trial 3:

Time	Fi	Yi #3 T3	SStot	SSreg	SSerr
k-value:	0.00127				
0	99	99	560.5056	560.5056	0
35.34	95.88384	96.3	439.9506	422.6661	0.173185
58.12	93.94806	94.3	360.0506	346.8184	0.123861
90.03	91.3289	91.6	264.8756	256.1248	0.073495
113.59	89.46209	90	215.3556	199.8573	0.289346
137.38	87.63289	88.1	163.2006	151.4842	0.218191
169.42	85.25509	85	93.60562	98.60661	0.065069
197.14	83.27452	82.9	57.38063	63.1949	0.140267
224.1	81.414	81.1	33.35062	37.07592	0.098596
253.23	79.47405	80	21.85563	17.21461	0.276624
276.71	77.96177	78.4	9.455625	6.952573	0.192043
306.16	76.12764	76.2	0.765625	0.644233	0.005236
329.56	74.71843	74.8	0.275625	0.36793	0.006654
361.37	72.86868	72.4	8.555625	6.033513	0.21966
402.48	70.5862	70.6	22.32563	22.45622	0.00019
450.97	68.04276	67.6	59.67563	53.03096	0.19604
507.66	65.26117	65	106.6056	101.2807	0.068208
541.62	63.68828	63.4	142.2056	135.4131	0.083108
585.13	61.76973	61.5	191.1306	183.7454	0.072753
620.94	60.26832	59.9	237.9306	226.7035	0.135663
651.36	59.04546	59	266.5056	265.0233	0.002067
691.18	57.51449	57.8	307.1256	317.2143	0.081517
719.51	56.47146	57.1	332.1506	355.456	0.395063
752.44	55.30531	55.8	381.2256	400.7881	0.244721
	Mean:	75.325			
		Sums:	4276.065	4228.658	3.161557
				R^2	0.999261

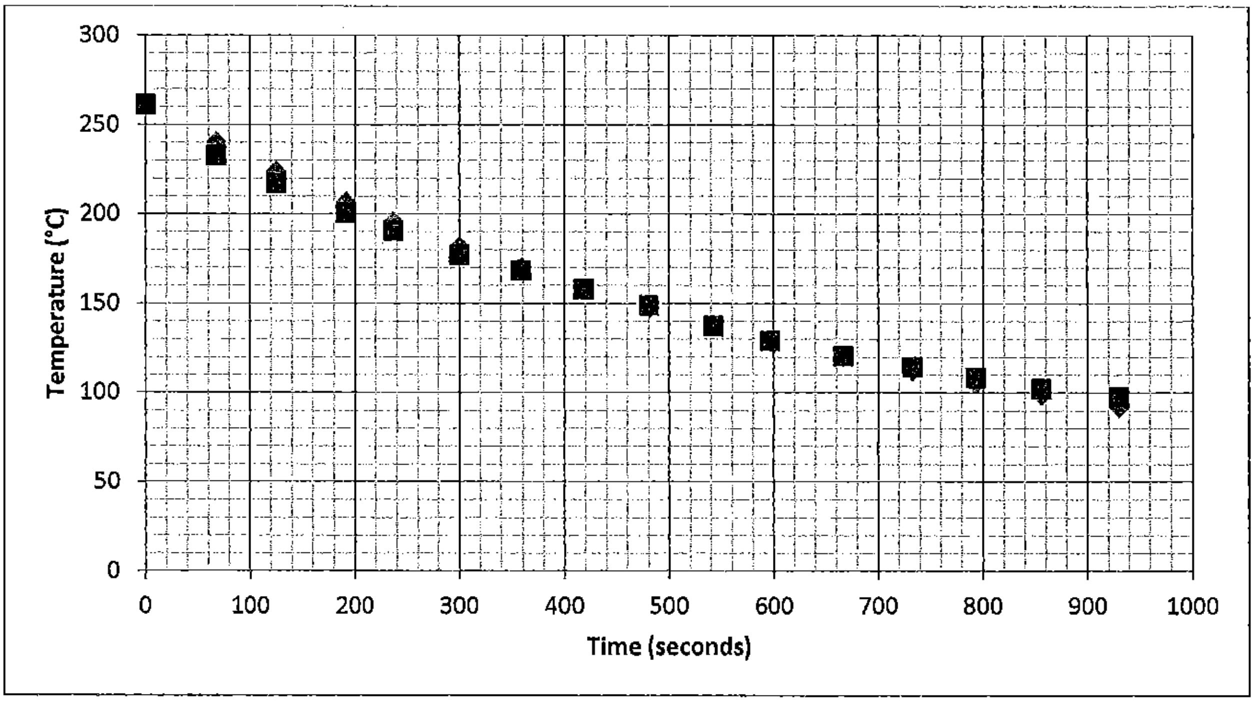
Graph for Trial 3:



Data Table for Trial 4:

Time	Fi	Yi #3 T4	SStot	SSreg	SSerr
k-value:	0.00139				
0	261.4	261.4	10194.69	10194.69	0
67.24	240.574	232.9	5251.72	6422.859	58.89014
124.84	224.218	217.1	3211.347	4068.745	50.66546
191.38	206.8836	200.8	1629.636	2157.824	37.01063
236.67	195.9695	190.7	916.1972	1262.967	27.76764
299.54	181.9139	177.4	287.9385	461.5027	20.37497
357.91	169.9193	168.5	65.10473	90.02372	2.014507
417.68	158.6051	158.2	4.978477	3.334703	0.164133
480.29	147.7194	149	130.6735	161.5913	1.639952
542	137.8783	137.5	525.8422	508.6339	0.143138
596.33	129.886	129.2	975.391	933.0093	0.470663
666.24	120.4511	120.8	1570.636	1598.41	0.121714
732.18	112.3541	114.5	2109.68	2311.412	4.604849
793.08	105.5073	108.7	2676.122	3016.637	10.19314
855.72	99.04425	102.3	3379.242	3768.364	10.59991
929.86	92.08743	97.9	3910.157	4670.877	33.78591
	Mean:	160.4313			
		Sums:	36839.35	41630.88	258.4468
				R^2	0.992984

Graph for Trial 4:



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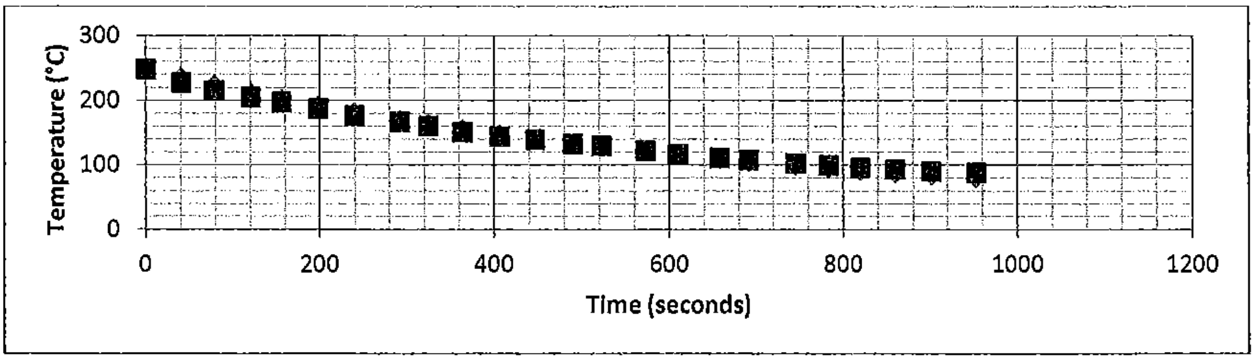
Data Table for Trial 5:

Time	Fi	Yi #3 T5	SStot	SSreg	SSerr
k-value:	0.0015				
0	248.2	248.2	10383.61	10383.61	0
39.72	235.4637	227.7	6625.96	7950.159	60.27452
78.24	223.8162	215.3	4761	6008.754	72.52486
120.39	211.8189	205.1	3457.44	4292.731	45.14409
155.7	202.3363	197.4	2611.21	3140.068	24.36711
197.81	191.6649	187.4	1689.21	2057.977	18.18962
239.25	181.8013	176.9	936.36	1260.339	24.02231
291.43	170.2223	166.7	416.16	572.2757	12.40649
324.61	163.3171	160.6	204.49	289.5831	7.382861
363.32	155.6837	151.1	23.04	88.05409	21.01044
406.15	147.7386	144.2	4.41	2.069636	12.52185
446.69	140.6743	139.5	46.24	31.64867	1.378946
489.64	133.6441	132.8	182.25	160.1709	0.712563
522.74	128.527	130	265.69	315.88	2.169766
572.8	121.2549	122.1	585.64	627.2588	0.714254
609.77	116.2242	116.9	864.36	904.5542	0.456715
657.61	110.115	111.3	1225	1309.351	1.404124
691.23	106.0767	107.8	1482.25	1617.917	2.969885
745.35	99.98885	102.7	1900.96	2144.723	7.350343
782.95	96.04105	99.8	2162.25	2525.962	14.12969
819.61	92.40049	96.1	2520.04	2905.157	13.68638
859.08	88.69833	93.5	2787.84	3317.952	23.05601
901.11	84.98974	90.2	3147.21	3758.948	27.14683
952.59	80.75461	87.9	3410.56	4296.198	51.05655
	Mean:	146.3			
		Sums:	51693.18	59961.34	444.0762
				R^2	0.991409

Graph for Trial 5:

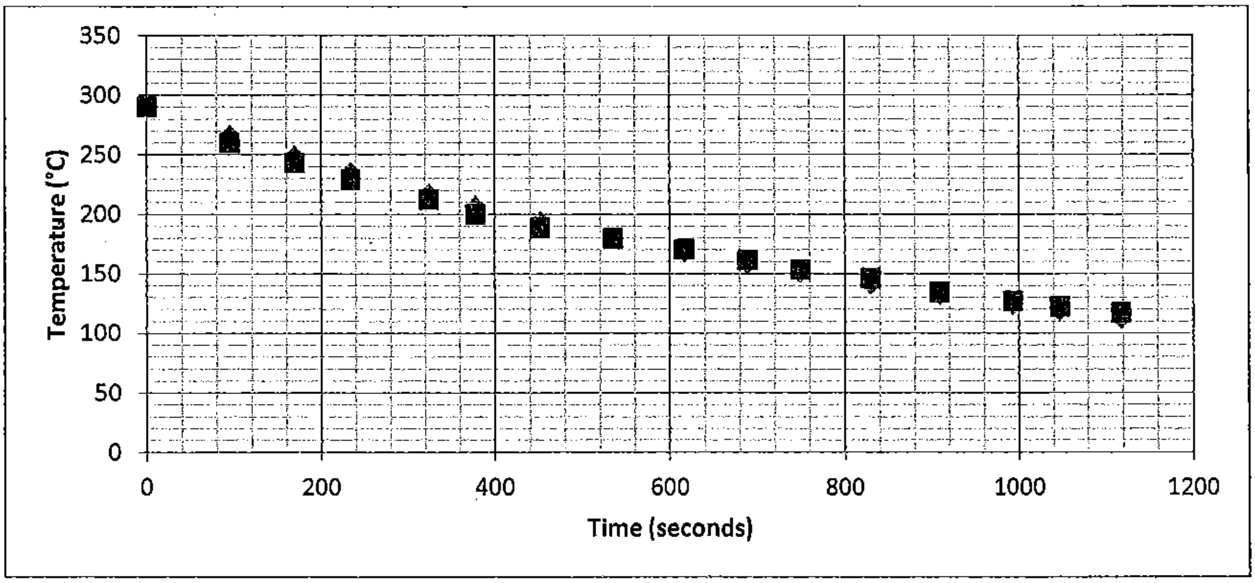


Aluminum Block 4:

Data Table for Trial 1:

Time	Fi	Yi #4 T1	SStot	SSreg	SSerr
k-value:	0.00101				
0	290.8	290.8	11431.62	11431.62	0
94.72	266.8238	260.5	5870.433	6879.468	39.99058
169.62	249.4235	243.5	3554.395	4295.784	35.08767
233.54	235.5802	229.3	2062.863	2672.784	39.44121
323.85	217.484	212.7	830.5204	1129.147	22.88702
377.52	207.4862	200.2	266.3016	557.1916	53.08809
451.27	194.6025	188.9	25.18785	114.9447	32.51822
534.76	181.1298	179.8	16.6566	7.57068	1.76827
616.27	169.0283	170.9	168.5129	220.6101	3.503257
688.41	159.1182	161.7	492.0079	613.2088	6.665705
748.77	151.3635	154.1	886.9229	1057.402	7.488226
829.32	141.7246	146.6	1389.892	1777.18	23.76914
908.41	132.9936	135	2389.377	2589.555	4.025729
992.55	124.4397	127.5	3178.845	3533.3	9.365528
1046.71	119.306	122.8	3730.919	4169.967	12.20824
1117.2	113.0315	117.8	4366.732	5019.693	22.73899
	Mean:	183.8813			
		Sums:	40661.18	46069.42	314.5459
				R^2:	0.992264

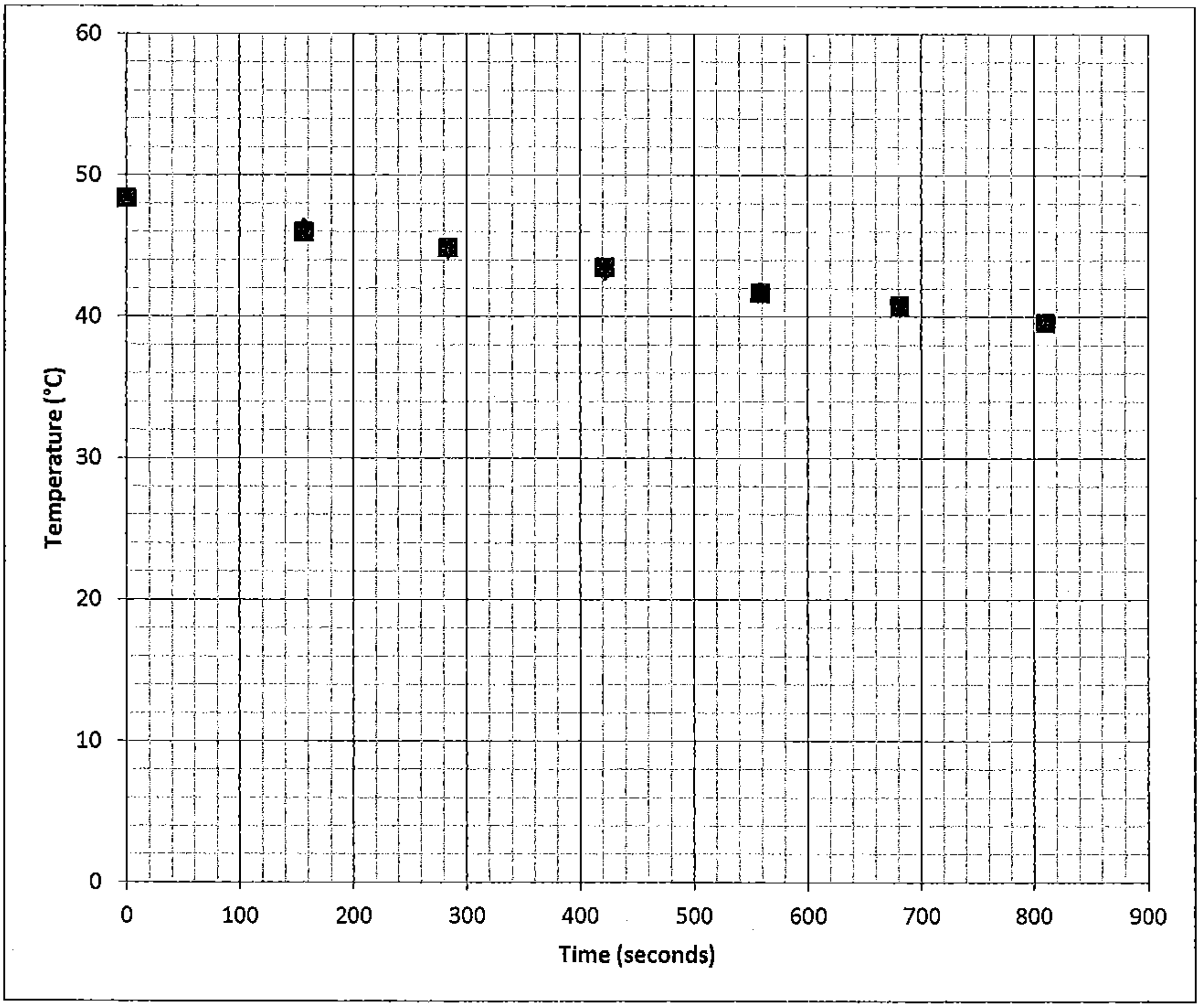
Graph for Trial 1:



Data Table for Trial 2:

Time	Fi	Yi #4 T2	SStot	SSreg	SSerr
k-value:	0.000691				
0	48.4	48.4	23.45327	23.45327	0
156.24	46.3123	46	5.967551	7.590901	0.097532
283.42	44.77168	44.9	1.803265	1.475098	0.016466
421.48	43.24562	43.5	0.003265	0.097047	0.064709
558.41	41.86924	41.7	3.44898	2.849009	0.028643
680.55	40.74673	40.8	7.601837	7.898415	0.002838
810.33	39.65339	39.6	15.65898	15.23931	0.00285
	Mean:	43.55714			
		Sums:	57.93714	58.60304	0.213039
				R^2:	0.996323

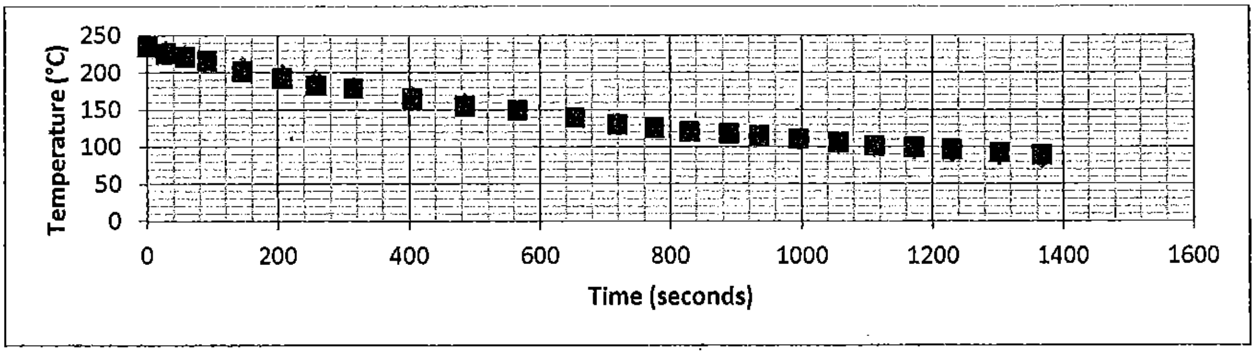
Graph for Trial 2:



Data Table for Trial 3:

Time	Fi	Yi #4 T3	SStot	SSreg	SSerr
k-value:	0.000945				
0	235.5	235.5	7474.323	7474.323	0
27.97	230.0873	225.8	5891.202	6567.716	18.38078
56.94	224.6299	221.6	5264.107	5712.944	9.180006
90.94	218.4126	216.6	4563.565	4811.742	3.285341
144.56	209.0045	202	2804.144	3595.047	49.06365
206.53	198.709	192.7	1905.686	2466.432	36.10836
257.51	190.6799	182.9	1146.105	1733.394	60.52665
314.67	182.1257	179.4	921.3754	1094.274	7.429177
404.33	169.6047	165.5	270.7396	422.6677	16.8487
484.4	159.2854	155	35.4521	104.8481	18.36438
565.17	149.6375	149.8	0.568767	0.350124	0.026391
653.54	139.8922	139.9	83.64627	83.78873	6.06E-05
719.11	133.1694	130.7	336.5696	252.0611	6.09794
775.12	127.7476	126.5	508.3146	453.615	1.556492
829.23	122.7753	120.9	792.1879	690.1401	3.516808
889.23	117.5511	118.2	951.4654	991.9206	0.42112
934.47	113.8033	115.4	1132.042	1242.038	2.549527
995.08	109.0269	111.3	1424.748	1601.518	5.167153
1056.03	104.4918	106.4	1818.667	1985.066	3.64139
1111.42	100.5909	102.1	2203.911	2347.881	2.277411
1171	96.61673	99.6	2444.89	2748.811	8.899917
1229.85	92.9049	96.9	2719.188	3151.805	15.96084
1302.28	88.61102	92.8	3163.594	3652.367	17.54755
1366.41	85.04691	89.6	3533.807	4095.862	20.73064
	Mean:	149.0458			
		Sums:	51390.3	57280.61	307.5803
				R^2	0.994015

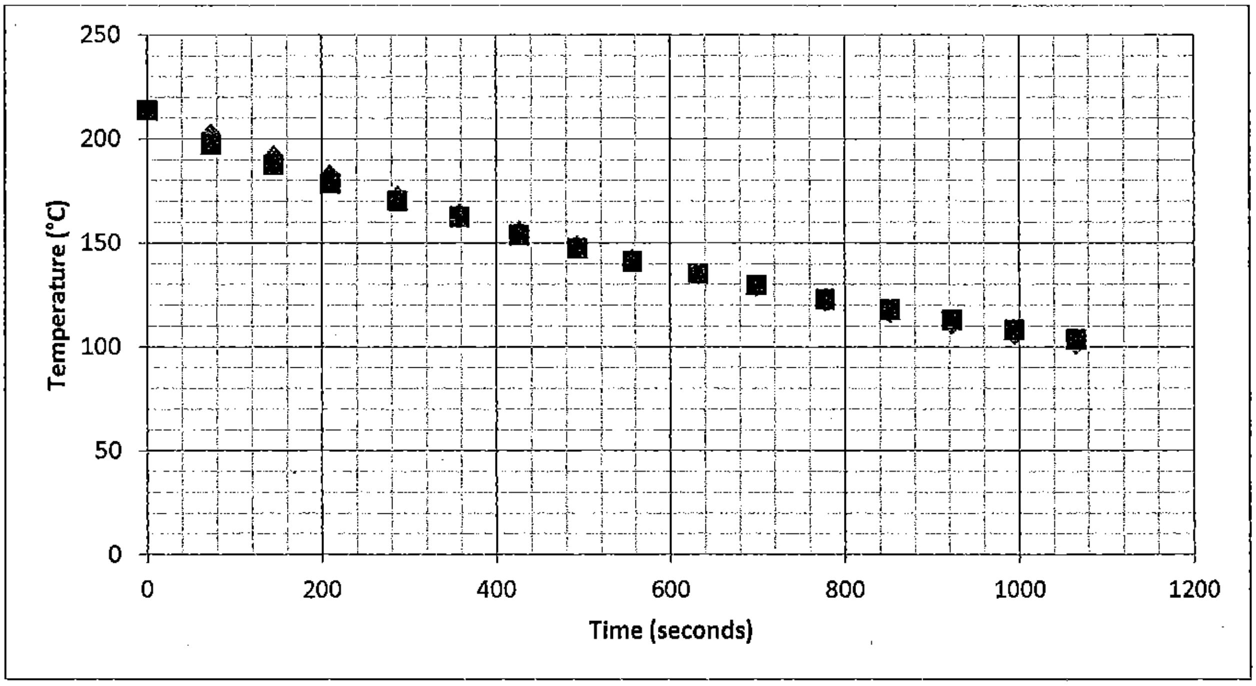
Graph for Trial 3:



Data Table for Trial 4:

Time	Fi	Yi #4 T4	SStot	SSreg	SSerr
k-value:	0.000873				
0	213.9	213.9	4207.144	4207.144	0
72.54	202.4925	197.4	2338.931	2857.432	25.93305
144.28	191.8993	187.6	1487.066	1837.137	18.48432
209.32	182.8524	178.8	885.8064	1143.449	16.42211
286.42	172.7726	170.3	452.0939	563.355	6.11375
357.19	164.0989	162.7	186.6639	226.8466	1.956996
425.7	156.1976	153.9	23.64391	51.26698	5.278949
492.46	148.9396	147.4	2.681406	0.009583	2.370393
555.81	142.4327	141.2	61.42641	43.62355	1.519518
631.75	135.0923	135.5	183.2639	194.4684	0.166214
698.23	129.0539	129.8	370.0814	399.344	0.556655
777.14	122.3268	123	677.9514	713.4599	0.453157
850.95	116.4405	118.1	957.1289	1062.567	2.754077
922.36	111.0953	113.1	1291.504	1439.608	4.018665
994.02	106.0562	108.2	1667.701	1847.39	4.59576
1065.22	101.3522	103.7	2055.489	2273.892	5.512393
	Mean:	149.0375			
		Sums:	16848.58	18860.99	96.136
				R^2:	0.994294

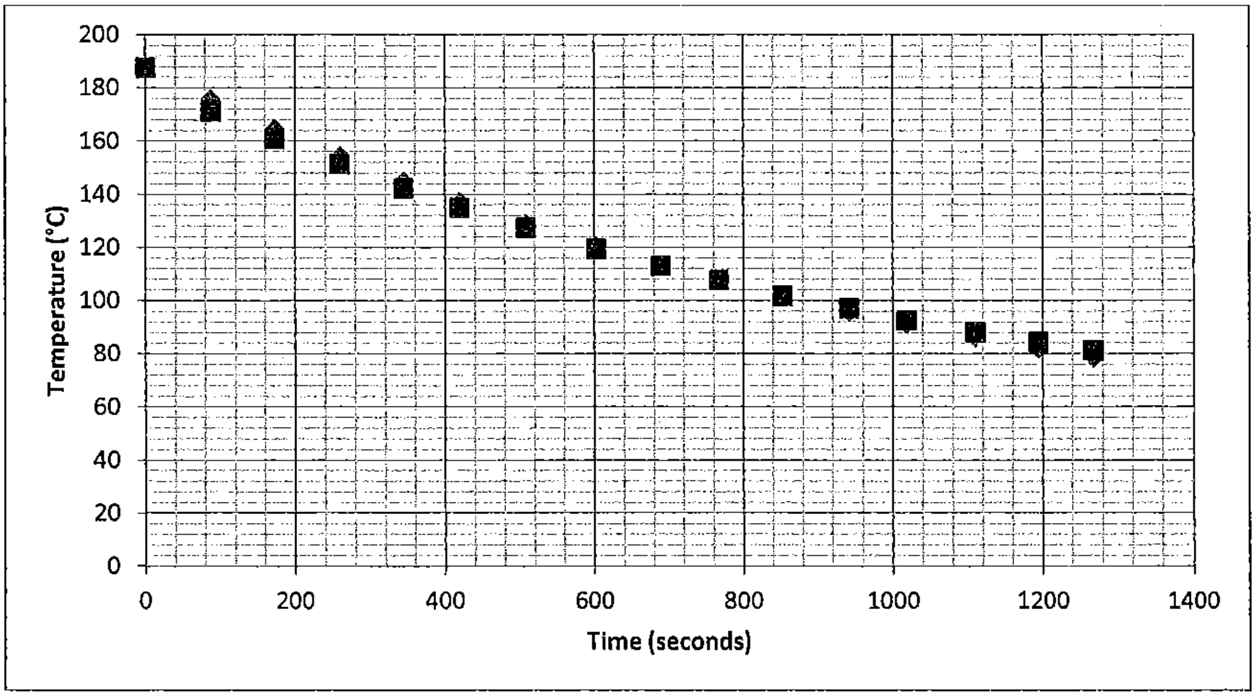
Graph for Trial 4:



Data Table for Trial 5:

Time	Fi	Yi #4 T5	SStot	SSreg	SSerr
k-value:	0.000905				
0	187.6	187.6	4221.751	4221.751	0
86.74	175.5506	171.2	2359.531	2801.116	18.92749
172.42	164.5418	160.9	1464.976	1757.016	13.2625
258.97	154.2549	151.4	828.0006	1000.45	8.150422
344.21	144.8815	142.3	387.1056	495.352	6.664163
419.26	137.2065	135	153.1406	212.6195	4.868546
508.02	128.7772	127.5	23.76563	37.85012	1.631355
602.3	120.5352	119.4	10.40062	4.367236	1.288694
688.41	113.5978	113.1	90.72562	81.49026	0.247809
767	107.7212	107.8	219.7806	222.1229	0.006208
852.39	101.7925	101.9	429.5256	433.9911	0.011546
941.58	96.07029	97.2	646.4306	705.1529	1.276254
1017.95	91.52452	92.7	895.5056	967.24	1.381759
1109.81	86.45707	88.1	1191.976	1308.119	2.69922
1194.08	82.16464	84.6	1445.901	1637.041	5.930981
1267.54	78.68079	81.3	1707.756	1931.093	6.86025
	Mean:	122.625			
		Sums:	16076.27	17816.77	73.20719
				R^2:	0.995446

Graph for Trial 5:

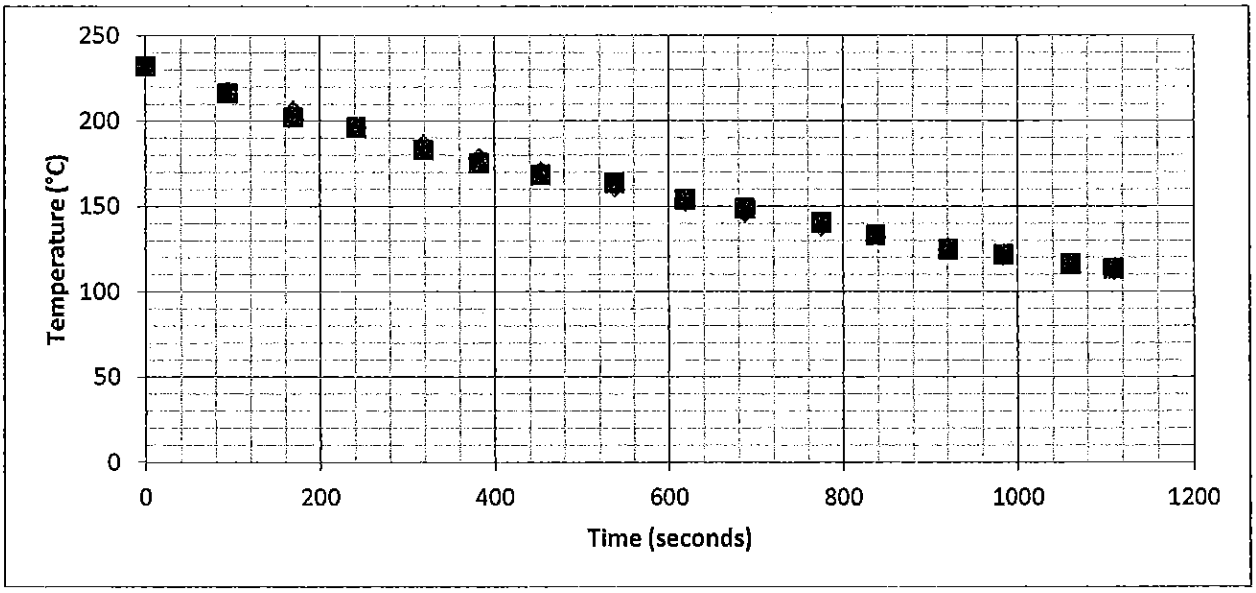


Aluminum Block 5:

Data Table for Trial 1:

Time	Fi	Yi #5 T1	SStot	SSreg	SSerr
k-value:	0.000792				
0	232.2	232.2	4911.382	4911.382	0
93.61	217.6084	216.5	2957.32	3079.099	1.228506
169.26	206.5817	202.4	1622.579	1976.951	17.4863
241.34	196.6724	196.5	1182.07	1193.958	0.029737
318.27	186.7023	183.3	448.6454	604.3505	11.57559
381.62	178.9361	175.5	179.0579	282.8248	11.80709
452.15	170.736	168.6	42.0066	74.25782	4.562701
537.21	161.437	163.9	3.172852	0.46476	6.066284
618.01	153.1654	154.3	61.13285	80.16265	1.287339
686.56	146.5511	149	172.1016	242.3506	5.99694
774.27	138.5954	140.8	454.4891	553.3502	4.860471
836.72	133.2583	133.4	824.7666	832.9235	0.020069
920.15	126.528	125	1377.802	1266.699	2.334885
982.91	121.7503	122	1609.514	1629.609	0.062335
1060.24	116.1809	116.7	2062.863	2110.291	0.269516
1109.14	112.831	113.8	2334.702	2429.282	0.938964
	Mean:	162.1188			
		Sums:	20243.6	21267.96	68.52672
				R^2:	0.996615

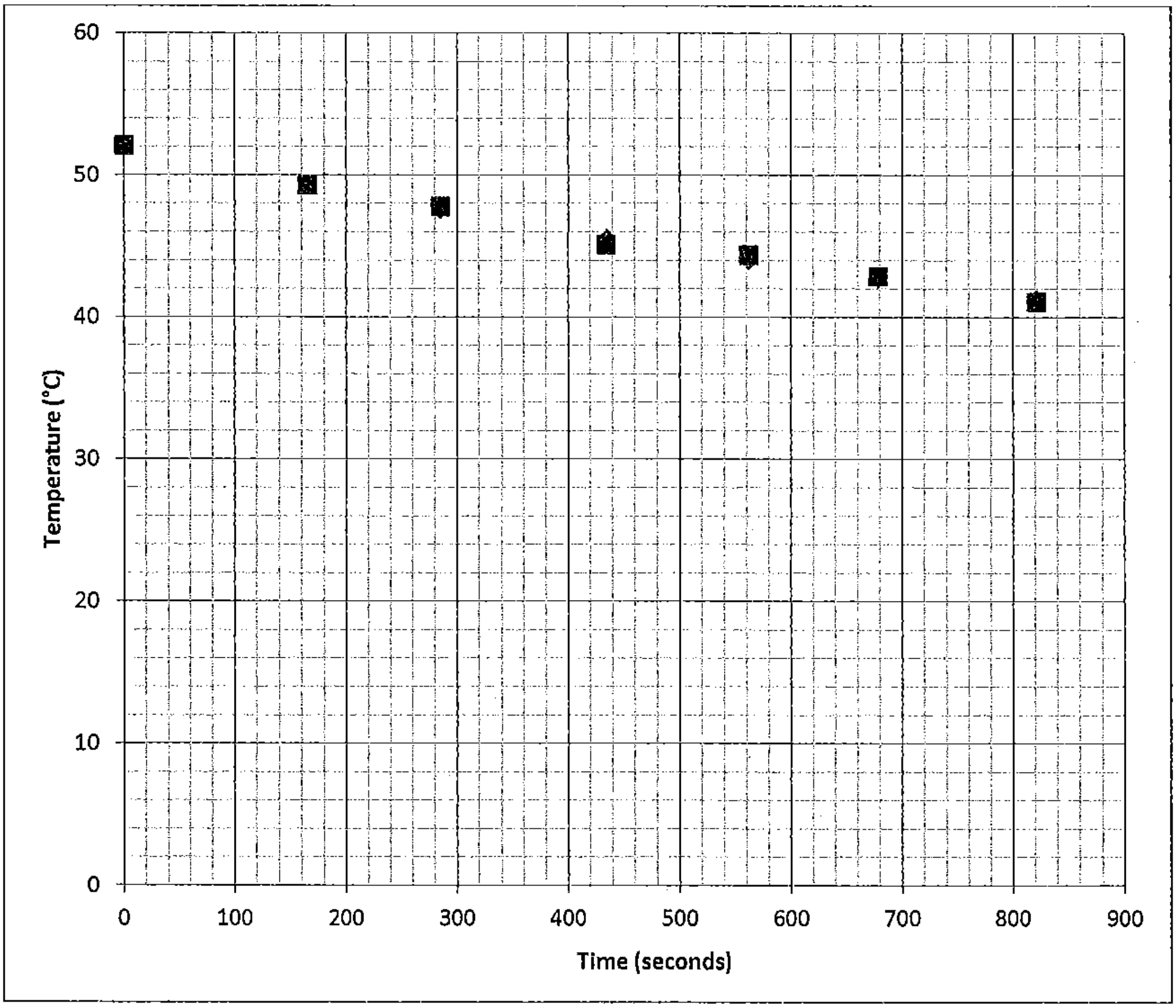
Graph for Trial 1:



Data Table for Trial 2:

Time	Fi	Yi #5 T2	SStot	SSreg	SSerr
k-value:	0.000725				
0	52.1	52.1	36	36	0
164.99	49.38295	49.3	10.24	10.77779	0.006881
284.85	47.60325	47.8	2.89	2.259774	0.038709
434.18	45.59177	45.1	1	0.258295	0.241841
561.95	44.03539	44.4	2.89	4.262619	0.132941
678.17	42.73961	42.9	10.24	11.2922	0.025724
821.05	41.28919	41.1	25	23.14394	0.035791
	Mean:	46.1			
		Sums:	88.26	87.99462	0.481887
				R^2:	0.99454

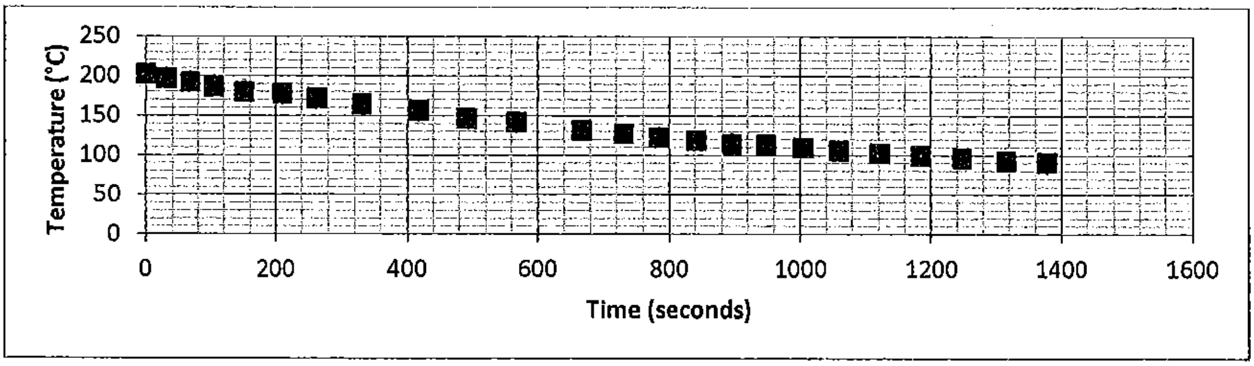
Graph for Trial 2:



Data Table for Trial 3:

Time	Fi	Yi #5 T3	SStot	SSreg	SSerr
k-value:	0.000761				
0	202.9	202.9	4005.835	4005.835	0
29.67	198.9952	197.2	3316.8	3526.8	3.222759
66.7	194.2438	192.5	2797.528	2985.04	3.04101
104.31	189.5532	186.9	2236.502	2494.489	7.039446
150.73	183.9459	180.1	1639.575	1965.816	14.79059
209.79	177.0921	178.6	1520.35	1405.032	2.273782
262.34	171.2475	172.4	1075.293	1001.034	1.328343
330.92	163.9632	164.8	634.6201	593.1595	0.700234
417.52	155.2917	157	302.4701	245.9694	2.918133
491.24	148.3472	146.5	47.49507	76.36746	3.412078
567.08	141.5981	142.3	7.245069	3.959045	0.492708
666.54	133.3173	132.1	56.37507	39.57744	1.481754
730.5	128.3139	127.4	149.0434	127.5651	0.835142
783.87	124.3213	122.8	282.5201	233.6939	2.314305
841.46	120.1911	119.2	416.5001	377.0299	0.982229
894.93	116.5151	114.5	630.4284	533.2995	4.060447
947.24	113.0607	113.7	671.2417	704.7785	0.408746
1004.47	109.4356	109.8	888.5367	910.3929	0.132776
1058.79	106.1379	106.3	1109.445	1120.27	0.026277
1121.08	102.5204	102.7	1362.225	1375.517	0.032267
1183.86	99.04382	99.9	1576.752	1645.479	0.733038
1247.02	95.70988	96.8	1832.553	1927.074	1.188354
1315.16	92.2883	93	2172.337	2239.186	0.50652
1377.32	89.31803	91.2	2343.367	2529.115	3.541826
	Mean:	139.6083			
		Sums:	31075.04	32066.48	55.46276
				R^2	0.998215

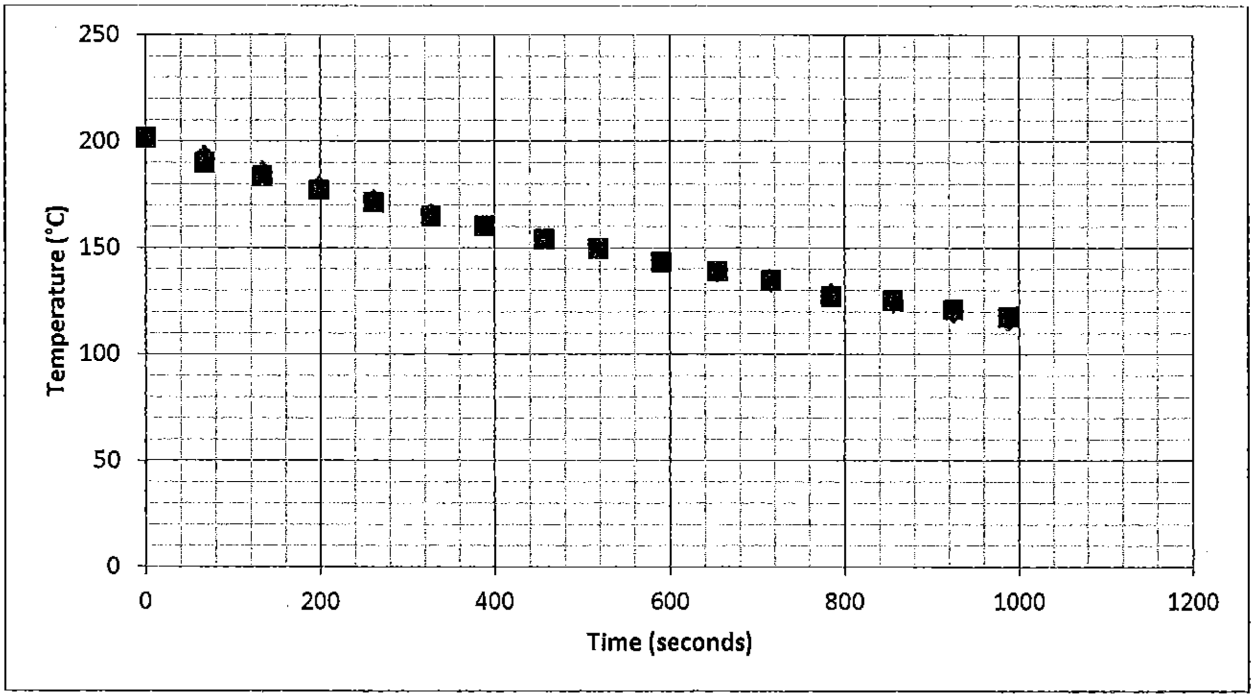
Graph for Trial 3:



Data Table for Trial 4:

Time	Fi	Yi #5 T4	SStot	SSreg	SSerr
k-value:	0.000691				
0	201.7	201.7	2280.659	2280.659	0
66.72	193.8736	190.1	1307.274	1594.394	14.24018
132.84	186.4656	183.9	897.3769	1057.668	6.582058
198.12	179.4762	177.4	550.1957	651.9078	4.310755
261.41	172.9944	171.6	311.7432	362.9286	1.944449
326.85	166.584	165.2	126.7032	159.7748	1.915335
387.49	160.897	160.3	40.40191	48.34711	0.35636
456.03	154.7495	154	0.003164	0.649283	0.561797
517.64	149.4667	149.7	18.00941	20.04367	0.054413
589.67	143.569	143.5	109.0719	107.6356	0.00476
653.29	138.5985	139.1	220.3369	235.4775	0.251529
715.2	133.9669	134.9	362.6644	399.0758	0.870737
783.99	129.0477	127.3	709.8894	619.8138	3.05442
854.72	124.2278	125.4	814.7457	883.0385	1.374085
924.18	119.7182	121.2	1072.153	1171.385	2.195606
987.37	115.7996	117.8	1306.371	1454.976	4.00158
	Mean:	153.9438			
		Sums:	10127.6	11047.78	41.71807
				R^2:	0.995881

Graph for Trial 4:



Data Table for Trial 5:

Time	Fi	Yi #5 T5	SStot	SSreg	SSerr
k-value:	0.000745				
0	185.1	185.1	2829.575	2829.575	0
79.67	176.0468	174.8	1839.874	1948.388	1.554494
157.94	167.6609	166.9	1224.563	1278.394	0.578948
241.58	159.224	157.2	639.7738	746.257	4.096395
322.1	151.5836	150.7	353.205	387.1991	0.780792
406.92	144.0159	143.5	134.415	146.6433	0.26614
484.18	137.5267	137.8	34.73629	31.58957	0.074687
563.68	131.2281	130.2	2.911289	0.459909	1.056956
640.74	125.4687	125.8	37.28629	41.44212	0.109763
723.93	119.6113	119	166.5713	151.1656	0.373695
797.59	114.7194	114.8	292.6238	295.3867	0.006491
876.31	109.7799	110.2	471.1613	489.575	0.176477
962.38	104.7006	105.1	718.575	740.1477	0.159526
1052.84	99.70186	99.7	1037.243	1037.122	3.47E-06
1111.95	96.61285	97.4	1190.681	1245.624	0.619609
1224.19	91.10885	92.3	1568.655	1664.428	1.418841
	Mean:	131.9063			
		Sums:	12541.85	13033.4	11.27282
				R^2:	0.999101

Graph for Trial 5: