



## MODELING RADIOACTIVE DECAY WITH FLUID DYNAMICS

**Note:** *Due to material and space constraints, you will work in teams of three to collect data. Each individual will be responsible for recording their own data and making their own qualitative observations. Data analysis, data presentation, conclusions, evaluation of limitations, and suggestions for improvement will all be individual effort.*

### Research Question

**Can you create a model of radioactive decay and half life using water flowing out of a graduated cylinder?** We have learned that an unstable atom will decay into more stable products and will release ions with significant kinetic energy in the process. We have also learned this decay is an exponential function and that the number of undecayed atoms can be determined using the equation  $N = N_0 e^{-\lambda t}$  where  $N$  is the number of undecayed atoms at any given time,  $N_0$  is the original number of undecayed atoms,  $\lambda$  is the decay constant and  $t$  is the time period being observed. Furthermore, we have learned that the half life of a radioactive material is the time it takes for half of a given amount of material to decay given the material's decay constant. Using the above equation,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda t}$$

$$\frac{1}{2} = \frac{1}{e^{\lambda t}}$$

$$2 = e^{\lambda t}$$

$$\ln(2) = \ln(e^{\lambda t})$$

$$0.693 = \lambda t$$

Our goal then, is to create a model of exponential decay,  $e^x$ , where  $x$  is equal to 0.693.

From thermodynamics we have learned that pressure in a gas results from the collision of molecules with the sides of a container. In fluids, pressure on the sides of a container is a result of the accumulated weight of the fluid above the point of interest. Thus pressure will increase with depth and will be a maximum at the bottom of the container. If we have a graduated cylinder full of water, the pressure will be greatest at the bottom of the cylinder. If we decrease the amount of water, the weight of the water decreases and the pressure at the bottom decreases.

Pressure can be defined as force per unit area or,  $= \frac{F}{A}$ . If we were to cut a hole into the side of the graduated cylinder near its bottom, the force acting on the cross-sectional area of the hole would be,  $F = PA$ . When

the cylinder is full and the pressure is high, the force of water will be high so the acceleration of water through the hole will be large. But, as the water level decreases the pressure decreases and the flow of water decreases. It is fairly easy to see then that the rate of change of the volume of water will exhibit exponential decay.

Your mission then, is to determine what diameter hole in a given graduated cylinder will produce an exponential decay of  $\lambda t = 0.693$ .

### **Variables**

**Independent variable: hole diameter/area.** You will be given a graduated cylinder with six holes of varying diameter drilled into the sides of the cylinder near the bottom.

**Dependent variable: change in volume of water in the cylinder.** You will measure the volume of water at specified time intervals in order to determine the exponential decrease in water volume.

### **Controlling the variables:**

- Use the same container for all of your trials.
- Fill your container to the same level each time.
- Ensure the person marking the volume on the cylinder is at eye-level with the cylinder to reduce parallax.
- To reduce random error, conduct at least three trials for each hole size.
- To create a reasonable graph, you will need 5-10 data points per hole size. Because of the different flow rates, you may have to reduce the timing intervals for the larger hole sizes. It is okay to change the timing interval for different hole sizes, but use the same timing interval for all trials with a single hole size.

### **Relevant variables:**

- Water density: changes in water density could affect the flow rate, however we will assume that the tap water we use will be of uniform density.
- Water temperature: temperature could affect density which could affect flow rate. We will assume the tap water we use will be at a constant temperature.

### **Process**

Materials:

- Graduated cylinder with six pre-cut holes
- Calipers to measure hole size
- Water source
- Sink for water to drain into
- Duct tape to cover holes not being used
- Masking tape and pen to mark volume levels
- Stopwatch
- Data collection table

Procedure:

1. Make qualitative observations throughout the experiment. Deviations from the given design should be noted on this sheet during the experiment.
2. Use the calipers to measure the diameter of the hole. Calculate the area of the hole using  $A = \pi r^2$  and record this value.
3. Use two long pieces of duct tape to cover the holes that aren't being used. Fold the outside end of the duct tape over on itself to use as a tab for easy removal and replacement.
4. Place a strip of masking tape vertically on the cylinder next to the volume markings.
5. Take a picture of the lab set-up for inclusion in your report.
6. Assign duties to the people in your team. To minimize random error, the same person should do each duty for each trial:
  - a. One person will fill the cylinder, cover the hole with his/her finger, release their finger when told, and will ensure the draining water goes into the sink and not onto the countertop or floor.
  - b. One person will act as a timer.
  - c. One person will mark the volume on the masking tape when prompted by the timer.
7. Run a few practice trials to determine the timing interval you will use for the first hole size. The goal is to be able to get 5-10 data points per trial. The more points the better, but don't take points so fast that you increase the random error of the measurements. This practice will also allow you to coordinate the actions of the team.
8. While holding a finger over the hole, fill the cylinder to its highest volume mark. It is not crucial to be exactly on the mark, but make sure to mark and record the actual starting point.
9. Simultaneously release the water and start the stopwatch.
10. At the designated intervals, the timer will call "TIME" or "MARK" and the recorder will mark the volume level of the water on the masking tape. Continue this process until the water level is just above the top of the hole.
11. Record the elapsed time and water volume remaining for each point marked on the masking tape.
12. Repeat steps 2 and 7-10 for at least three trials at each hole size, and test at least five different hole sizes.
13. Put all of your materials away when data collection is complete. Make sure all spilled water is wiped up.

### Data Collection

Hole Size: _____					
Trial #1		Trial #2		Trial #3	
Time	Volume	Time	Volume	Time	Volume

Hole Size: \_\_\_\_\_

<b>Trial #1</b>		<b>Trial #2</b>		<b>Trial #3</b>	
<b>Time</b>	<b>Volume</b>	<b>Time</b>	<b>Volume</b>	<b>Time</b>	<b>Volume</b>

Hole Size: \_\_\_\_\_

Trial #1		Trial #2		Trial #3	
Time	Volume	Time	Volume	Time	Volume

Hole Size: \_\_\_\_\_

Trial #1		Trial #2		Trial #3	
Time	Volume	Time	Volume	Time	Volume

Hole Size: \_\_\_\_\_

<b>Trial #1</b>		<b>Trial #2</b>		<b>Trial #3</b>	
<b>Time</b>	<b>Volume</b>	<b>Time</b>	<b>Volume</b>	<b>Time</b>	<b>Volume</b>



## Data Analysis

1. Create an Excel scatterplot graph for each trial with volume on the y-axis and time on the x-axis. Each graph should contain an exponential trendline, an equation for that trendline, and an  $R^2$  correlation value. *Attach these graphs to the end of this report.*
2. Compile the decay constants for each trial into the table below and obtain an average value for each hole size.

Hole Size	Trial 1 Constant	Trial 2 Constant	Trial 3 Constant	Average

3. Create an Excel scatterplot with average decay constant on the y-axis and hole size on the x-axis. Each graph should contain an exponential trendline, an equation for that trendline, and an  $R^2$  correlation value. Use the equation on this graph to determine the ideal hole size to yield an exponential decay constant that best models radioactive decay, 0.693. *Attach this graph to the end of this report.*
4. The optimum hole size for a 300 mL graduated cylinder to model radioactive decay is \_\_\_\_\_

## Evaluation

1. List five potential sources of error or uncertainty.

- a. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- b. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- c. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- d. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- e. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

2. For each of the five potential sources of error or uncertainty listed above, state whether the data would tend to be skewed higher than actual, lower than actual, or randomly higher or lower.

- a. \_\_\_\_\_.
- b. \_\_\_\_\_.
- c. \_\_\_\_\_.
- d. \_\_\_\_\_.
- e. \_\_\_\_\_.

3. For each of the five potential sources of error or uncertainty listed above, state how these could be minimized in future experiments within a high school lab setting.

- a. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.
- b. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.
- c. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.
- d. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.
- e. \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_.

RECOMMENDATIONS FOR IMPROVEMENT OF THIS LAB ASSIGNMENT:

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\_\_\_\_\_  
\_\_\_\_\_.