DEVIL PHYSICS
THE BADDEST CLASS ON CAMPUS

AP PHYSICS

LSN 8-7: ROTATIONAL KINETIC ENERGY

Questions From Reading Activity?

Big Idea(s):

- The interactions of an object with other objects can be described by forces.
- Interactions between systems can result in changes in those systems.
- Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding(s):

- A force exerted on an object can cause a torque on that object.
- A net torque exerted on a system by other objects or systems will change the angular momentum of the system.

Enduring Understanding(s):

- Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.
- The angular momentum of a system is conserved.

Essential Knowledge(s):

- A torque exerted on an object can change the angular momentum of an object.
  - Angular momentum is a vector quantity, with its direction determined by a right-hand rule.
  - The magnitude of angular momentum of a point object about an axis can be calculated by multiplying the perpendicular distance from the axis of rotation to the line of motion by the magnitude of linear momentum.
  - The magnitude of angular momentum of an extended object can also be found by multiplying the rotational inertia by the angular velocity.
  - The change in angular momentum of an object is given by the product of the average torque and the time the torque is exerted.

Essential Knowledge(s):

- Torque, angular velocity, angular acceleration, and angular momentum are vectors and can be characterized as positive or negative depending upon whether they give rise to or correspond to counterclockwise or clockwise rotation with respect to an axis.
- The change in angular momentum is given by the product of the average torque and the time interval during which the torque is exerted.

Essential Knowledge(s):

- The angular momentum of a system may change due to interactions with other objects or systems.
  - The angular momentum of a system with respect to an axis of rotation is the sum of the angular momenta, with respect to that axis, of the objects that make up the system.
  - The angular momentum of an object about a fixed axis can be found by multiplying the momentum of the particle by the perpendicular distance from the axis to the line of motion of the object.
  - Alternatively, the angular momentum of a system can be found from the product of the system's rotational inertia and its angular velocity.

Essential Knowledge(s):

- For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.
- If the net external torque exerted on the system is zero, the angular momentum of the system does not change.

Essential Knowledge(s):

- The angular momentum of a system is determined by the locations and velocities of the objects that make up the system.
- The rotational inertia of an object or system depends upon the distribution of mass within the object or system.

Essential Knowledge(s):

- Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.
- Examples should include elliptical orbits in an Earth-satellite system. Mathematical expressions for the moments of inertia will be provided where needed. Students will not be expected to know the parallel axis theorem.

Learning Objective(s):

- The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum.
- In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object.

Learning Objective(s):

- The student is able to plan data collection and analysis strategies designed to test the relationship between torques exerted on an object and the change in angular momentum of that object.
- The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system.

Learning Objective(s):

- The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data.
- The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems.

Learning Objective(s):

- The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems.
- The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum.

Learning Objective(s):

- The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted.
- The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

Learning Objective(s):

- The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque.
- The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero.

Learning Objective(s):

- The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

Rotational Kinetic Energy

- Linear Kinetic Energy is $$KE = 1/2mv^2$$
- To convert tangential (linear) velocity to angular velocity $$v = r\omega$$
- Rotational kinetic energy is then $$KE = 1/2mr^2\omega^2$$
- For the entire mass it is $$KE = 1/2\Sigma mr^2\omega^2$$

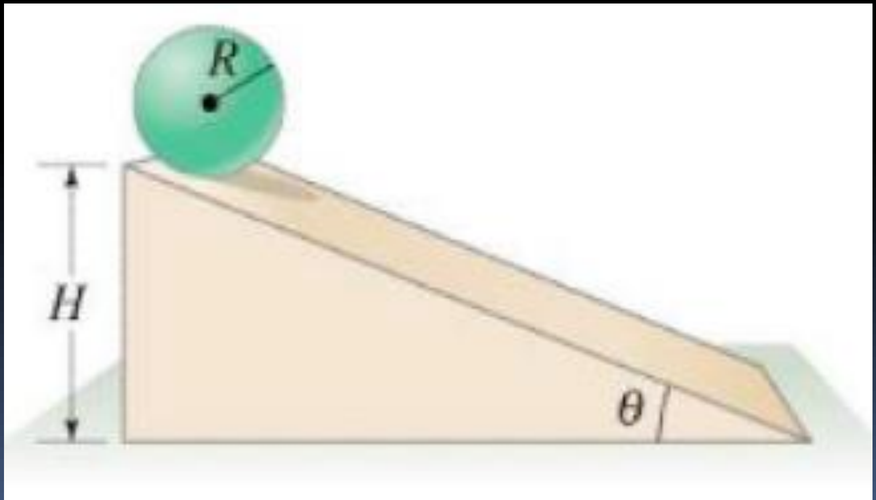
Rotational Kinetic Energy

- Kinetic energy for the entire mass it is $$KE = 1/2 \Sigma m r^2 \omega^2$$
- But, the moment of inertia is $$I = \Sigma m r^2$$
- So, kinetic energy in terms of the moment of inertia is $$KE = 1/2 I \omega^2$$

Total Kinetic Energy

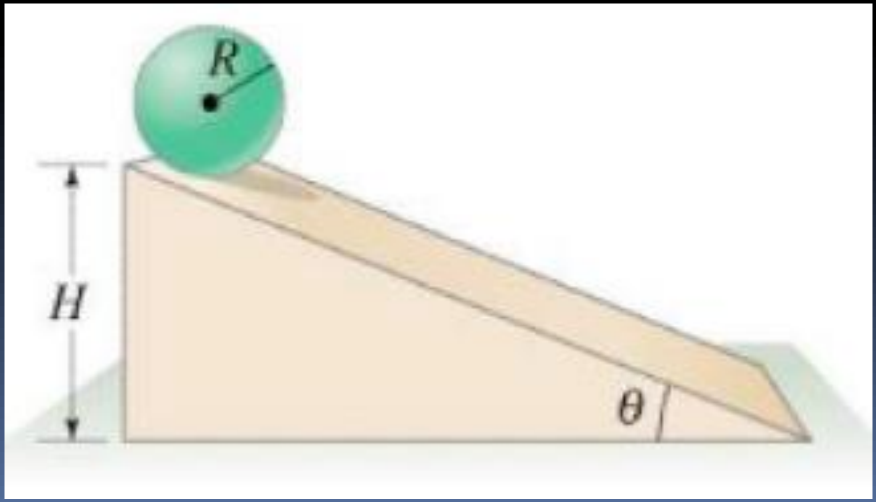
- But what happens if something has both rotational and translational motion
- This is when depression sets in
- The total kinetic energy is the sum of the translational kinetic energy and the rotational kinetic energy

$$KE = \frac{1}{2} m v_{cm} + \frac{1}{2} I_{cm} \omega^2$$



The Situation

The height of the ramp is 3m and the radius of the ball is 0.3m. What is the velocity of the ball at the bottom of the ramp? How would you solve this with energy?



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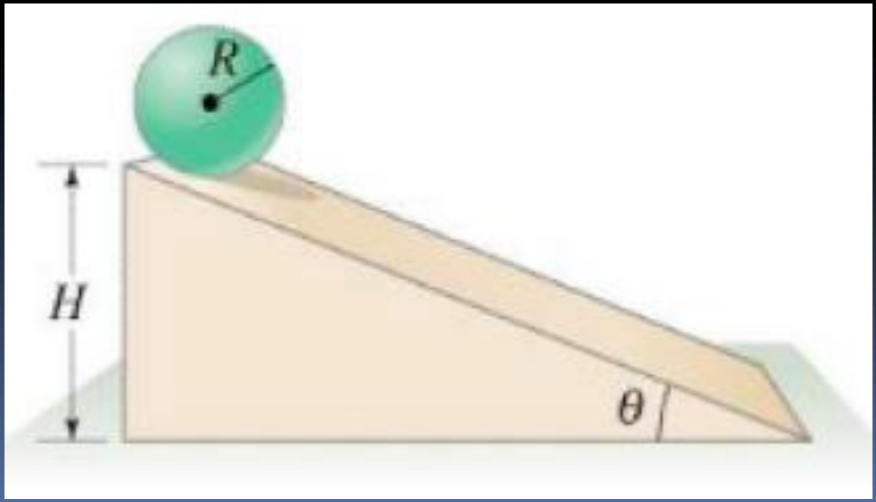
$$PE = KE$$



The Situation

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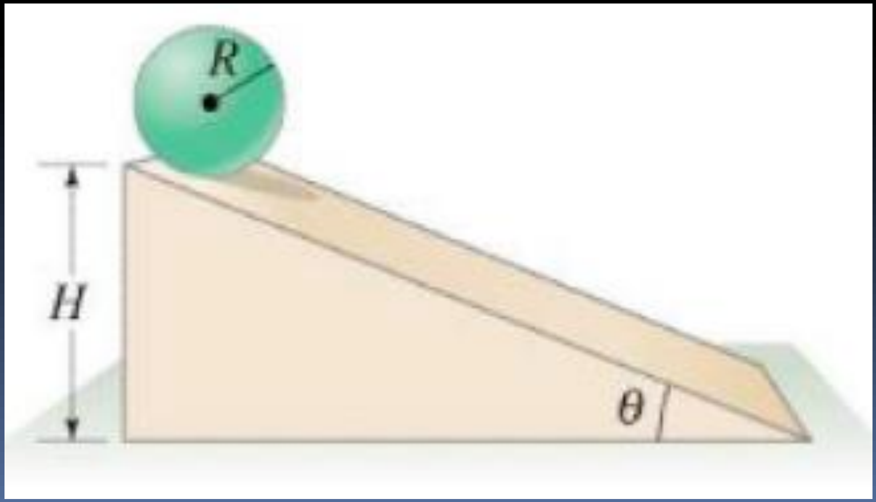
$$PE = KE$$

What type of KE?

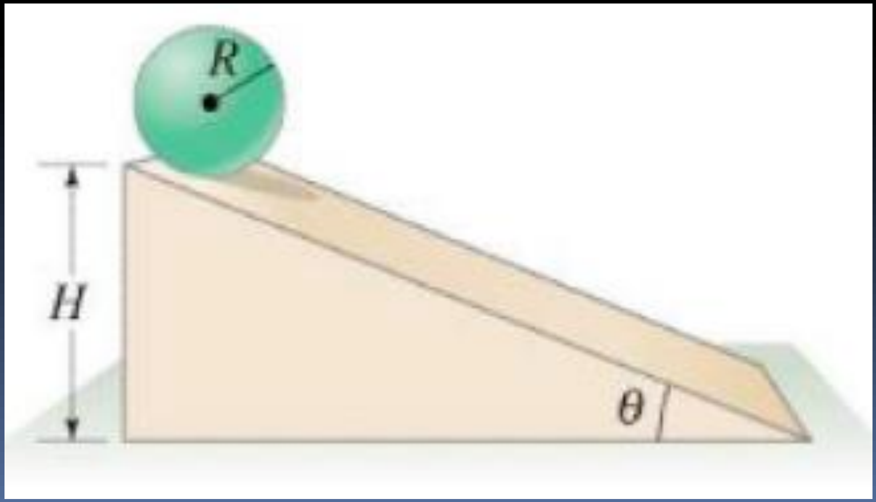


The Situation

The height of the ramp is 3m and the radius of the ball is 0.3m. What is the velocity of the ball at the bottom of the ramp? How would you solve this with energy?

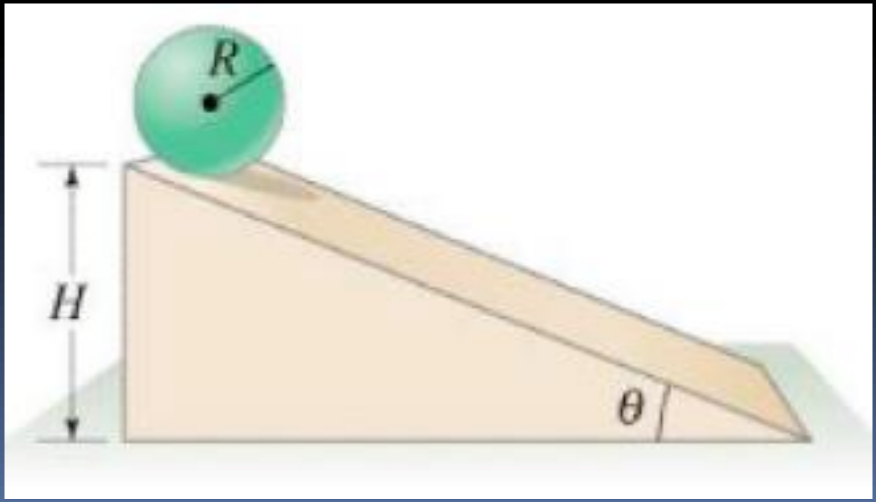
$$PE = KE_R + KE_T$$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$



Question 1?

How would the situation change if I told you that there was a high coefficient of friction between the ball and the ramp?



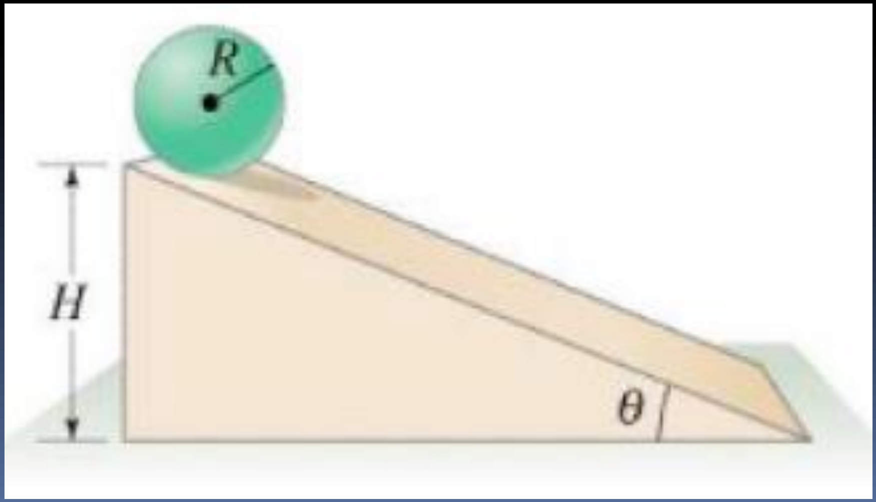
Question 1?

How would the situation change if I told you that there was a high coefficient of friction between the ball and the ramp?

Not at all. In fact the situation assumes a high coefficient of friction to make the ball roll instead of slide.

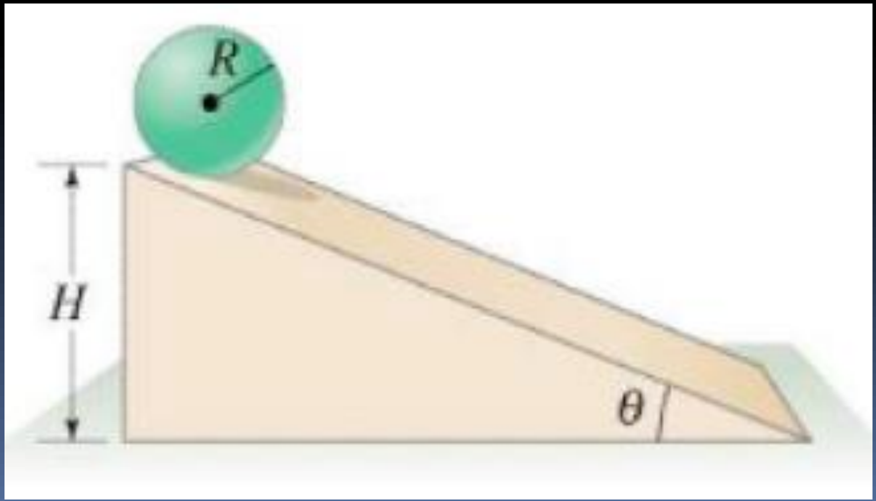
$$PE = KE_R + KE_T$$

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$



Question 2?

How would the situation change if I told you that the ramp was frictionless?



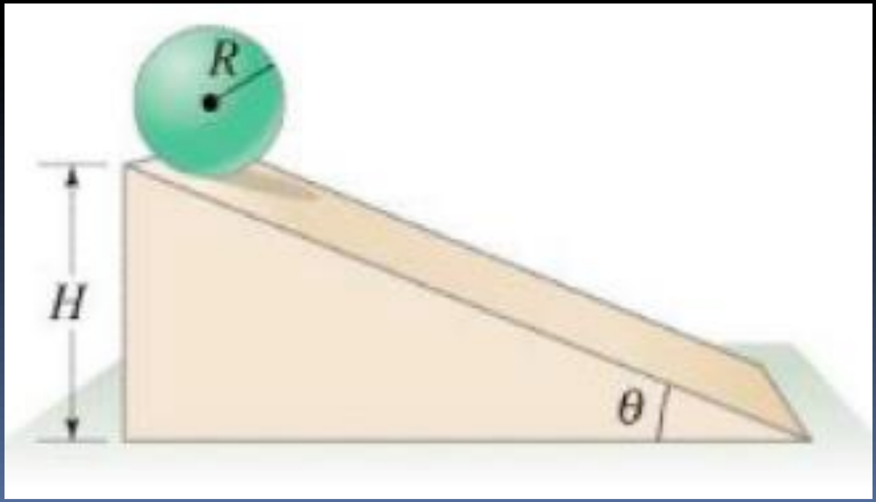
Question 2?

How would the situation change if I told you that the ramp was frictionless?

The ball would slide instead of rolling. There would be no rotational kinetic energy, just translational – the same as a box sliding down a ramp.

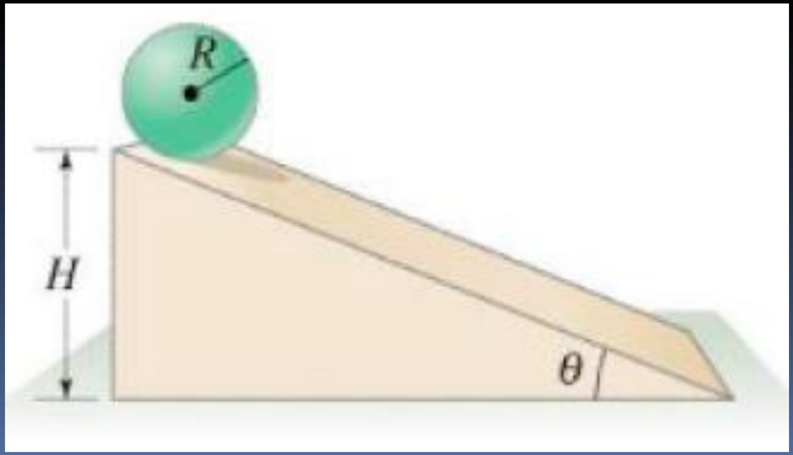
$$PE = KE_T$$

$$mgh = \frac{1}{2}mv^2$$



Question 3?

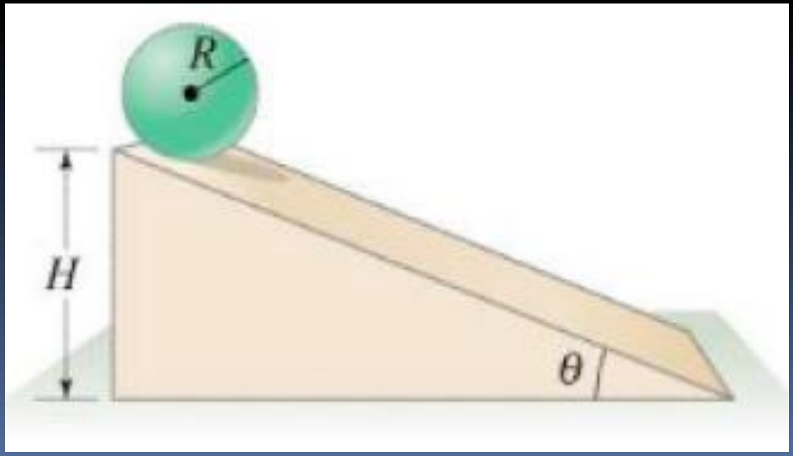
Which would be going 'faster' at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?



Question 3?

Which would be going 'faster' at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?

What does 'faster' mean?

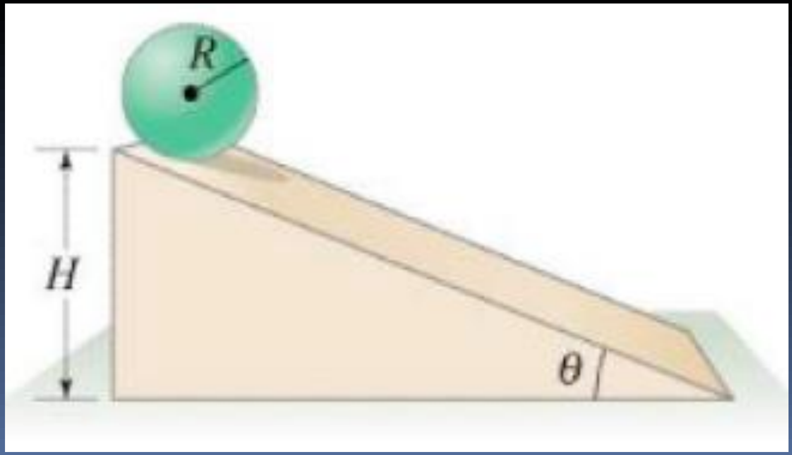


Question 3?

Which would be going 'faster' at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?

What does 'faster' mean?

- *It could mean rotating faster*
- *Most people would be talking about translational velocity*



Question 3?

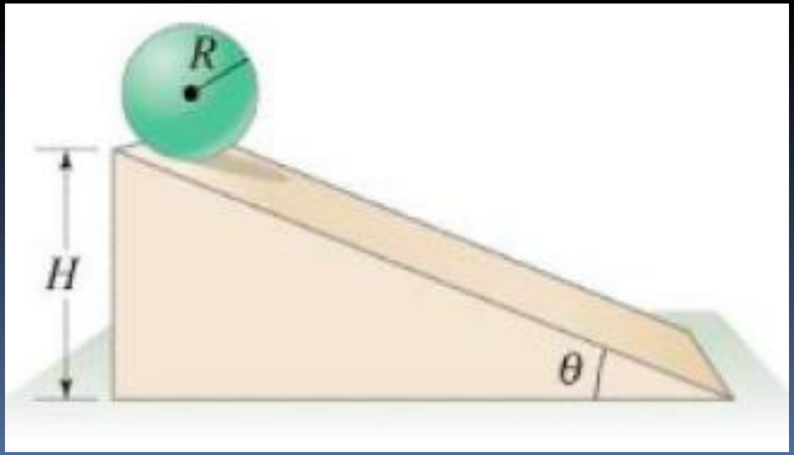
Which would be going faster at the bottom: the ball on a high friction ramp or the ball on a frictionless ramp?

The ball on the frictionless ramp. In this case, all of the potential energy is converted into translational kinetic energy. On a frictionless ramp, some of the PE must be translated into rotational KE.

$$PE = KE$$

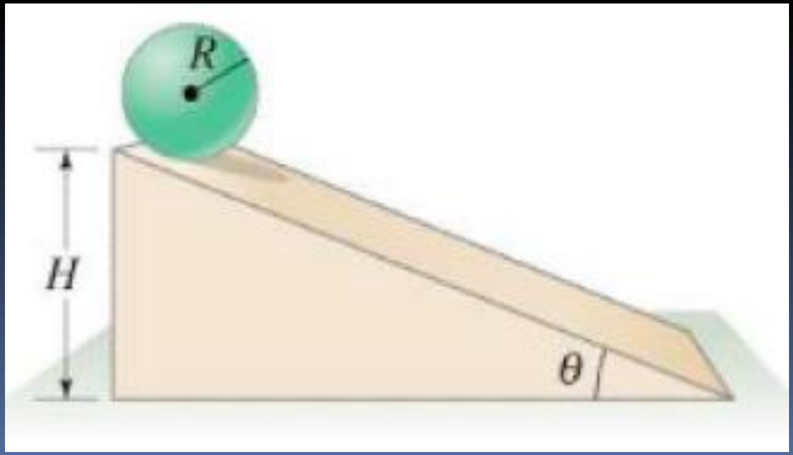
$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2$$



Work Done By Friction

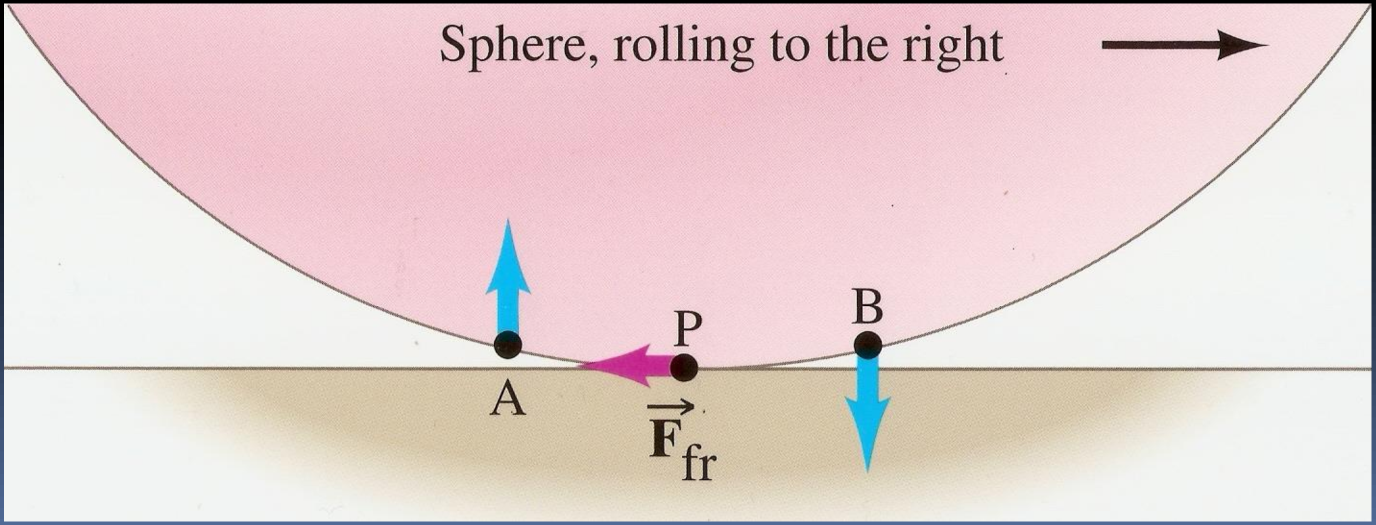
Speaking of friction, how do you calculate the work done by friction on a ball sliding down a ramp?



Work Done By Friction

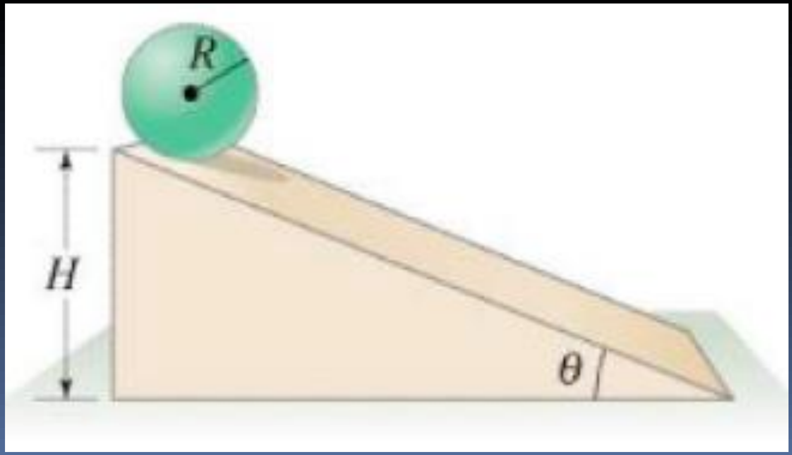
Speaking of friction, how do you calculate the work done by friction on a ball rolling down a ramp?

Trick question – you don't – there is no work done by friction. At the point of contact, the surface of the ball moves up, perpendicular to the force of friction. Force must be in direction of motion to do work.



Work Done By Torque

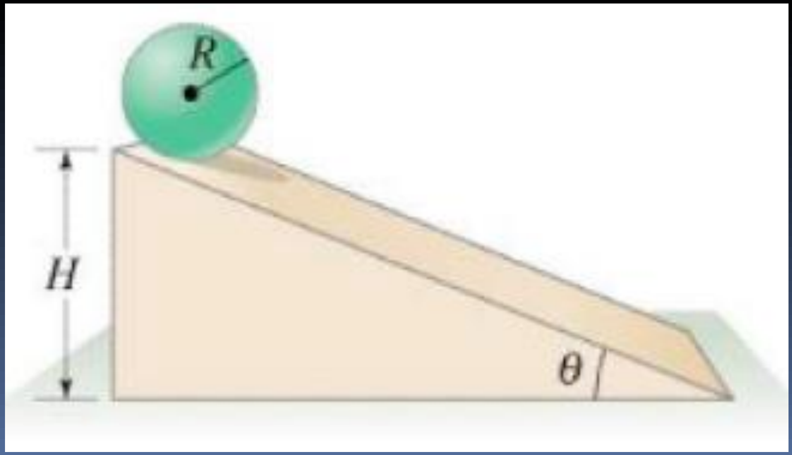
Does torque do work?



Work Done By Torque

Does torque do work?

Yes

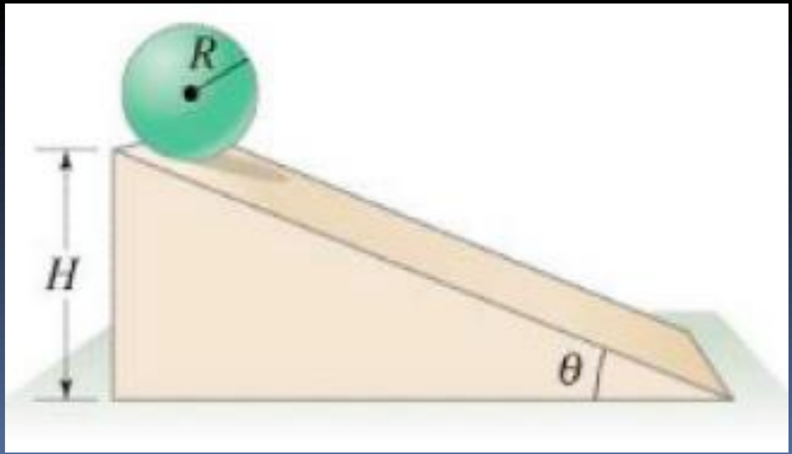


Work Done By Torque

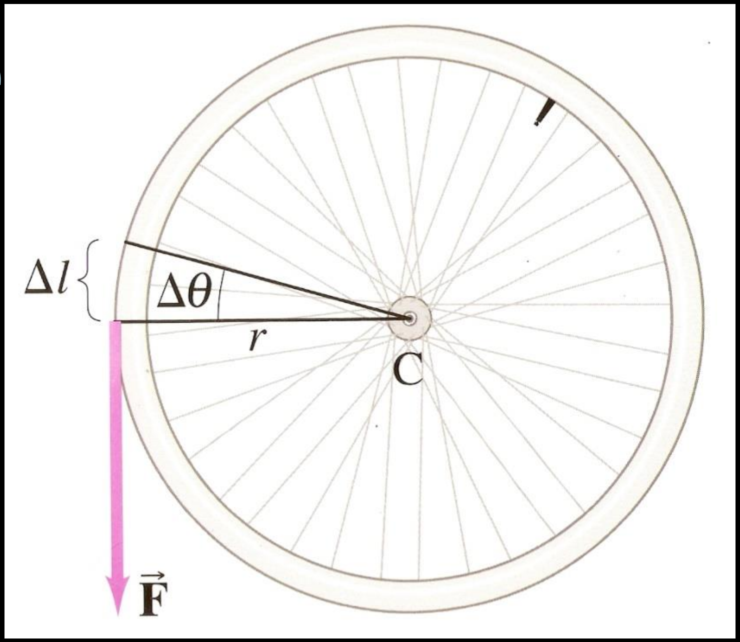
Does torque do work?

Yes

Oh, you want an explanation?



Work Done By Torque



- Torque is force times a moment arm

$$\tau = Fr$$

- Work is force times distance

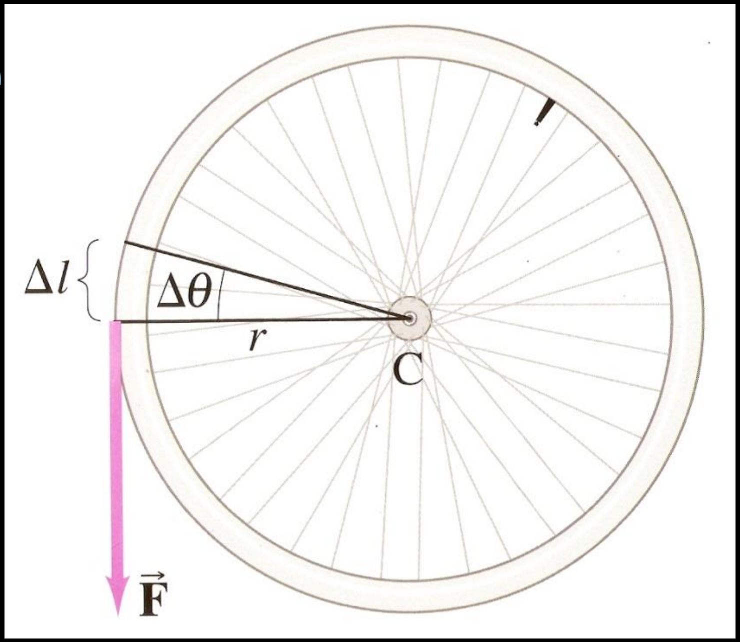
$$W = Fd$$

- A small amount of rotation (small Δθ) will produce a Δl and we know

$$\Delta\theta = \frac{\Delta l}{r}$$

$$r\Delta\theta = \Delta l$$

Work Done By Torque



- Torque is force times a moment arm

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- For small changes in θ, the force can be considered in the direction of motion so torque does work

$$\Delta\theta = \frac{\Delta l}{r}$$

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Work Done By Torque

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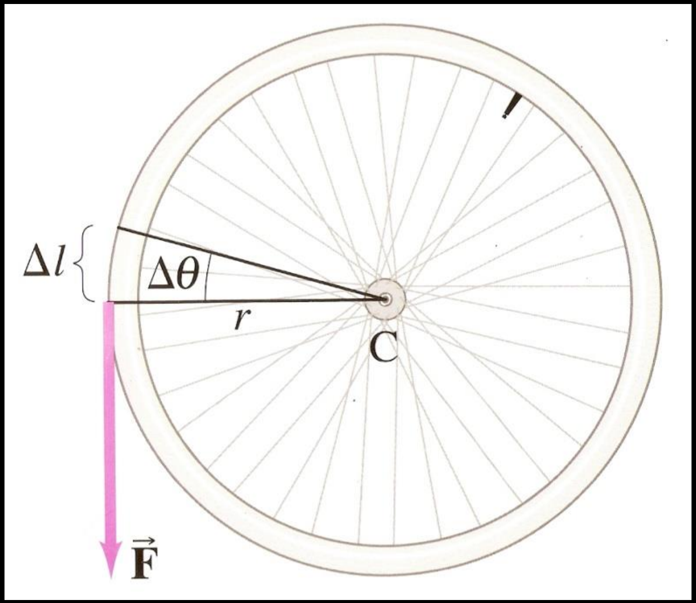


- Work is force times distance

$$W = Fd$$

- Using the marvels of Algebra,

$$r\Delta\theta = \Delta l$$

$$W = Fr\Delta\theta$$

$$\tau = Fr$$

$$W = \tau\Delta\theta$$

Power Generated By Torque

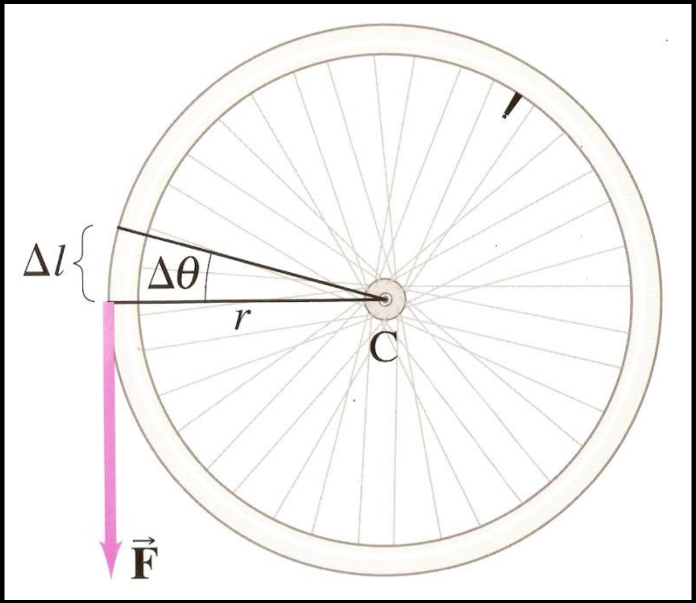
- Power is work divided by time so,

$$W = \tau\Delta\theta$$

$$P = \frac{W}{t} = \frac{\tau\Delta\theta}{t}$$

$$\omega = \frac{\Delta\theta}{t}$$

$$P = \tau\omega$$



Equations

In The Data Guide

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(2\pi f t)$$

$$\vec{\alpha} = \frac{\Sigma\vec{\tau}}{I} = \frac{\tau_{net}}{I}$$

$$\tau = r_\perp F = rF\sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\Delta t$$

$$K = \frac{1}{2}I\omega^2$$

NOT In The Data Guide

$$\theta = \frac{l}{r}$$

$$360° = 2\pi$$

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$$

$$v = r\omega$$

$$a_{\tan} = r\omega$$

$$a_R = r\omega^2$$

$$\omega = 2\pi f$$

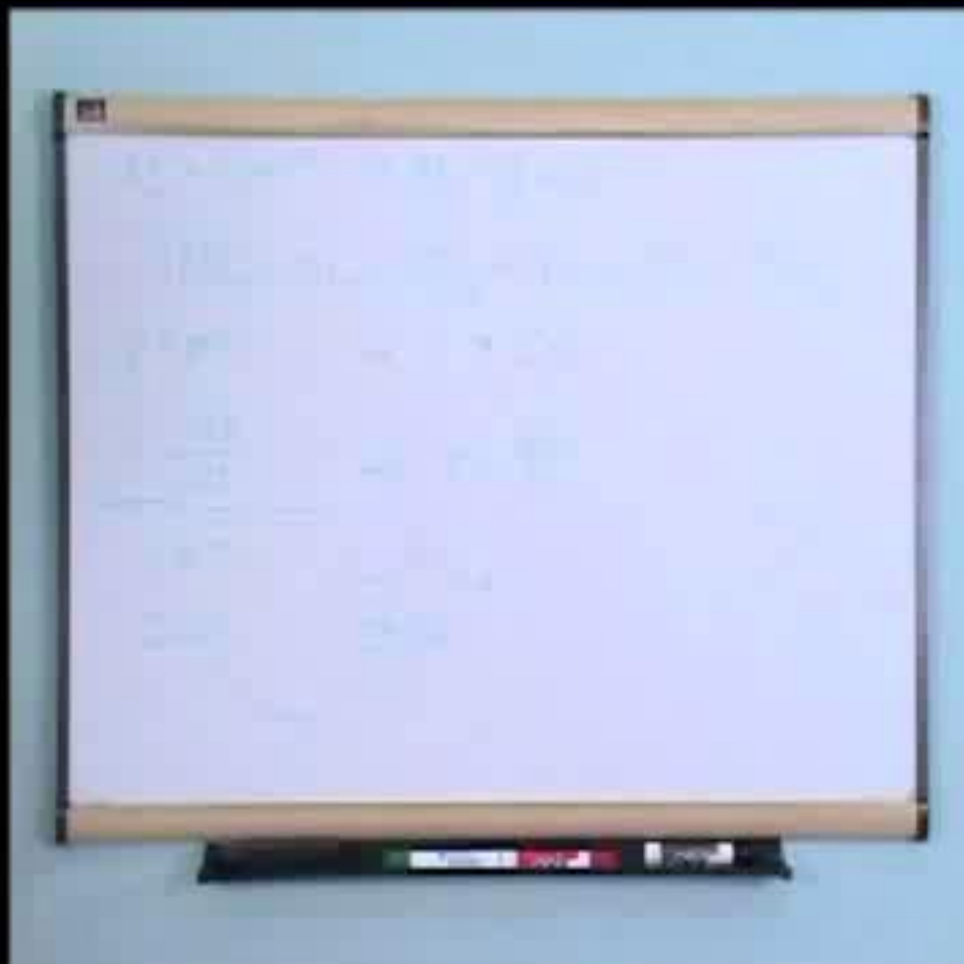
$$T = \frac{1}{f}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\Sigma\tau = I\alpha$$

$$I = \Sigma mr^2$$

Rotational Kinetic Energy - Example Problem



PROBLEMS ANYONE?

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Big Idea(s):

- The interactions of an object with other objects can be described by forces.
- Interactions between systems can result in changes in those systems.
- Changes that occur as a result of interactions are constrained by conservation laws.

QUESTIONS?

Homework

#43-48