DEVIL PHYSICS
THE BADDEST CLASS ON CAMPUS

AP PHYSICS

LSN 8-5: ROTATIONAL DYNAMICS; TORQUE AND ROTATIONAL INERTIA

LSN 8-6: SOLVING PROBLEMS IN ROTATIONAL DYNAMICS

Questions From Reading Activity?

Big Idea(s):

- Interactions between systems can result in changes in those systems.
- Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding(s):

- A net torque exerted on a system by other objects or systems will change the angular momentum of the system.
- The angular momentum of a system is conserved.

Essential Knowledge(s):

- The angular momentum of a system may change due to interactions with other objects or systems.
  - Alternatively, the angular momentum of a system can be found from the product of the system's rotational inertia and its angular velocity.

Essential Knowledge(s):

- The angular momentum of a system is determined by the locations and velocities of the objects that make up the system.
- The rotational inertia of an object or system depends upon the distribution of mass within the object or system.

Essential Knowledge(s):

- Changes in the radius of a system or in the distribution of mass within the system result in changes in the system’s rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum.

Essential Knowledge(s):

- Examples should include elliptical orbits in an Earth-satellite system. Mathematical expressions for the moments of inertia will be provided where needed. Students will not be expected to know the parallel axis theorem.

Learning Objective(s):

- The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems.

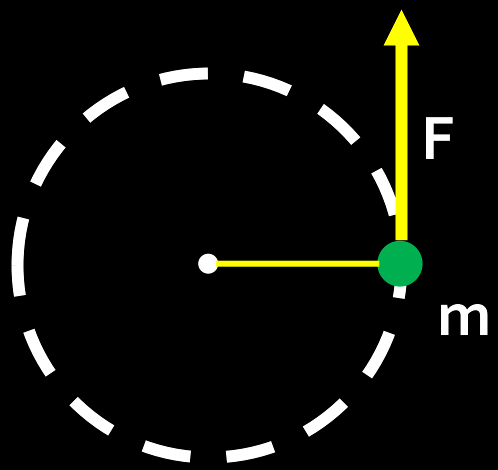
Learning Objective(s):

- The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

Introductory Video

Moment of Inertia

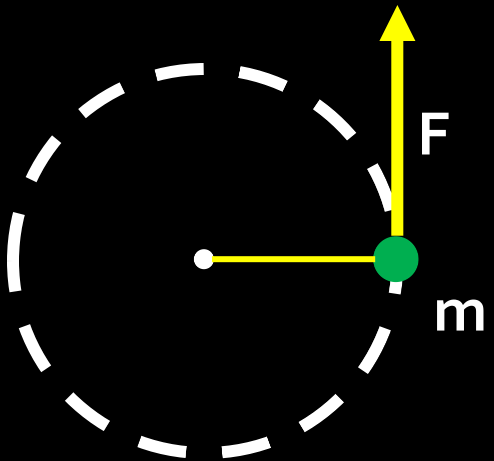
- Angular acceleration is proportional to the net torque applied to it
  - Force along a moment arm is torque
  - More force means more acceleration



$$\boxed{\alpha \propto \sum \tau}$$

Moment of Inertia

- Everything in life relates to Newton's Second Law



$$\Sigma F = ma$$

$$a_{\tan} = r\alpha$$

$$F = mr\alpha$$

$$\tau = rF$$

$$\frac{\tau}{r} = F$$

$$\frac{\tau}{r} = mr\alpha$$

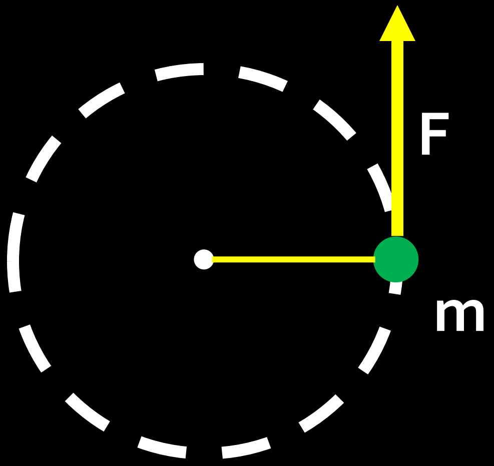
$$\tau = mr^2\alpha$$

$$\tau = \left(\Sigma mr^2\right)\alpha$$

Moment of Inertia

- Moment of Inertia defined:

$$\boxed{I = \Sigma m r^2}$$

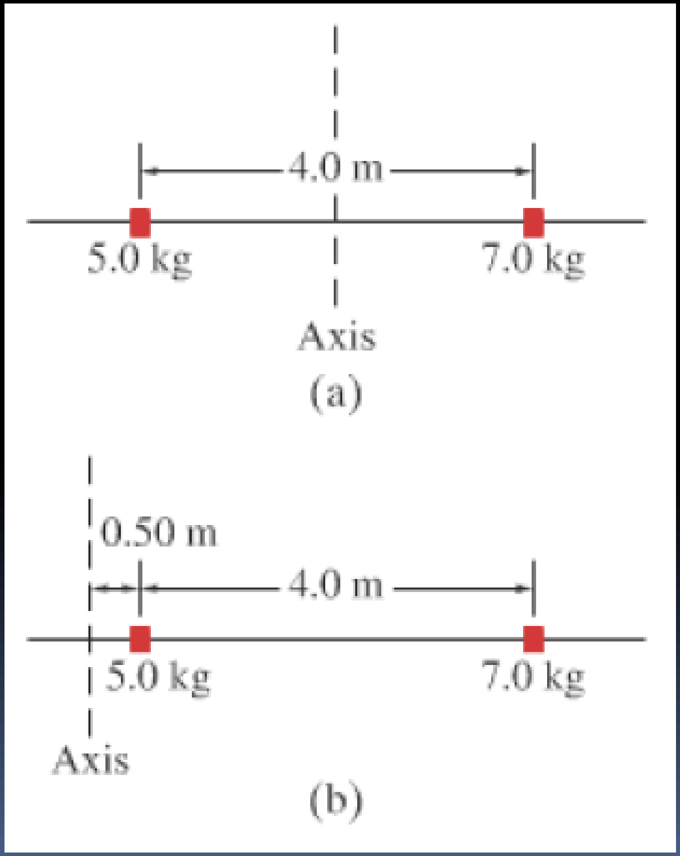


Note that the moment of inertia is much more affected by distance than by mass.

Determination of the axis of rotation (reference point) is critical!

Moment of Inertia

- ***Example 8-10: Find the moment of inertia for (a) and (b)***



Moment of Inertia

- ***Example 8-10: Find the moment of inertia for (a) and (b)***

$$(a.)\, I = \Sigma m r^2$$

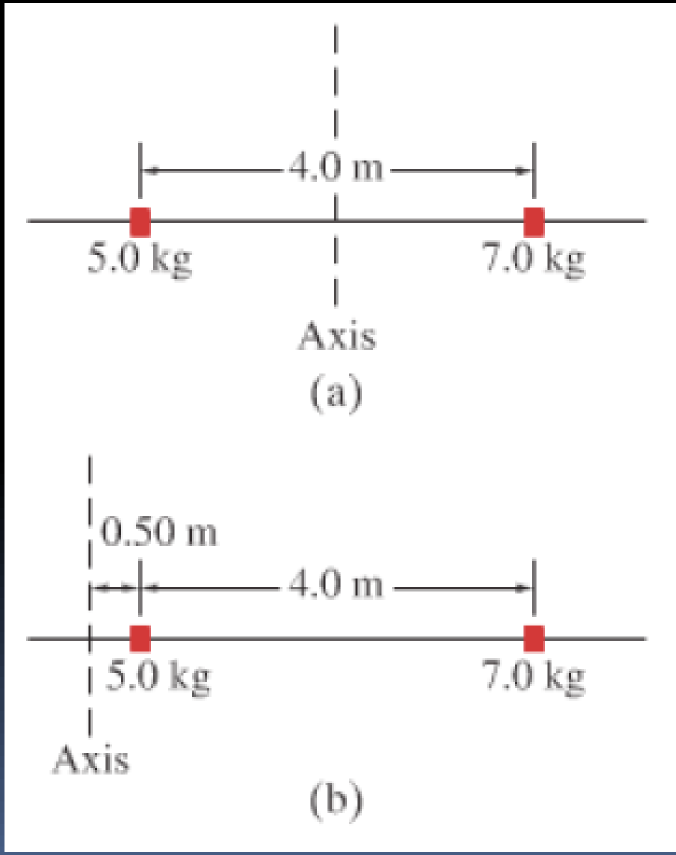
$$I = (5kg)(2.0)^2 + (7kg)(2.0)^2$$

$$I = 48 kg \cdot m^2$$

$$(b.)\, I = \Sigma m r^2$$

$$I = (5kg)(0.5)^2 + (7kg)(4.5)^2$$

$$I = 1.3 + 142 = 143 kg \cdot m^2$$



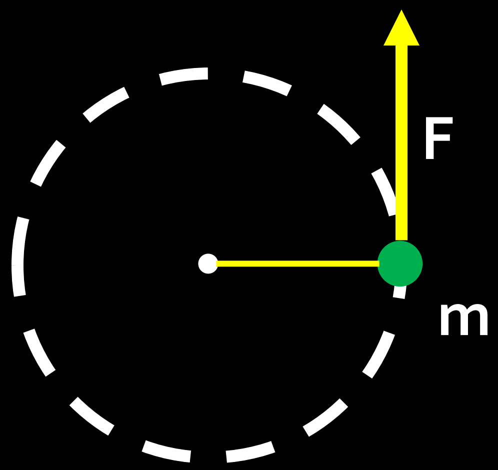
Moment of Inertia

- Moment of Inertia defined:

$$\tau = \left(\Sigma m r^2\right)\alpha$$

$$I = \Sigma m r^2$$

$$\tau = I\alpha$$



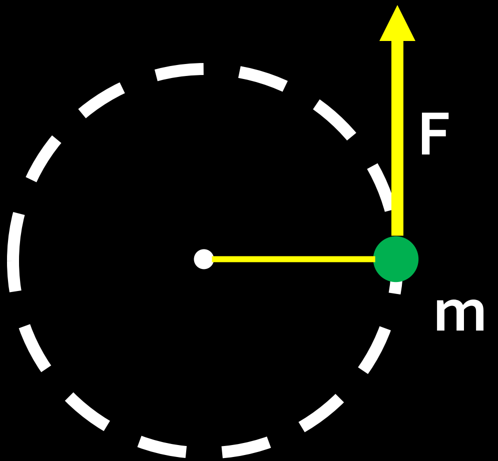
- *This is Newton's Second Law for rotation!*

Moment of Inertia

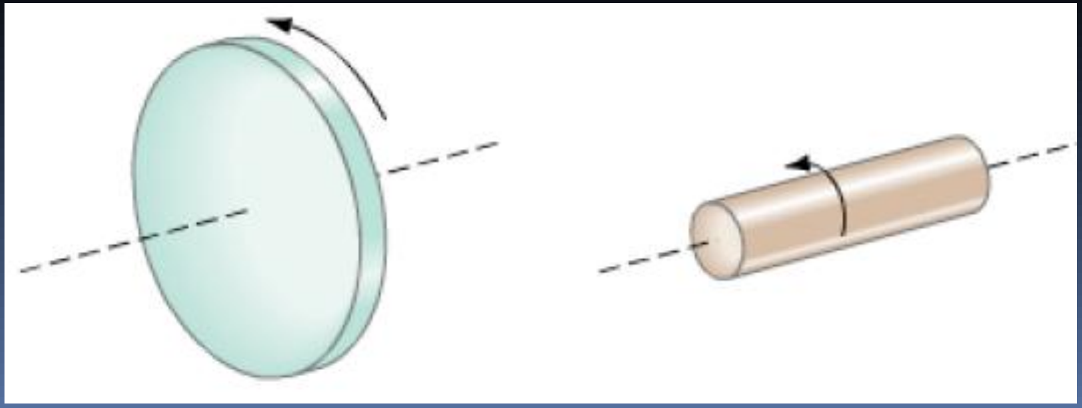
- Moment of Inertia defined:

$$I = \Sigma m r^2$$

$$\Sigma \tau = I\alpha$$



- ***So how do you find the moment of inertia and the torque on these objects?***



Moment of Inertia

- Without calculus, finding the moment of inertia can be difficult as seen by this chart
- It can be determined experimentally by finding the acceleration from a known torque

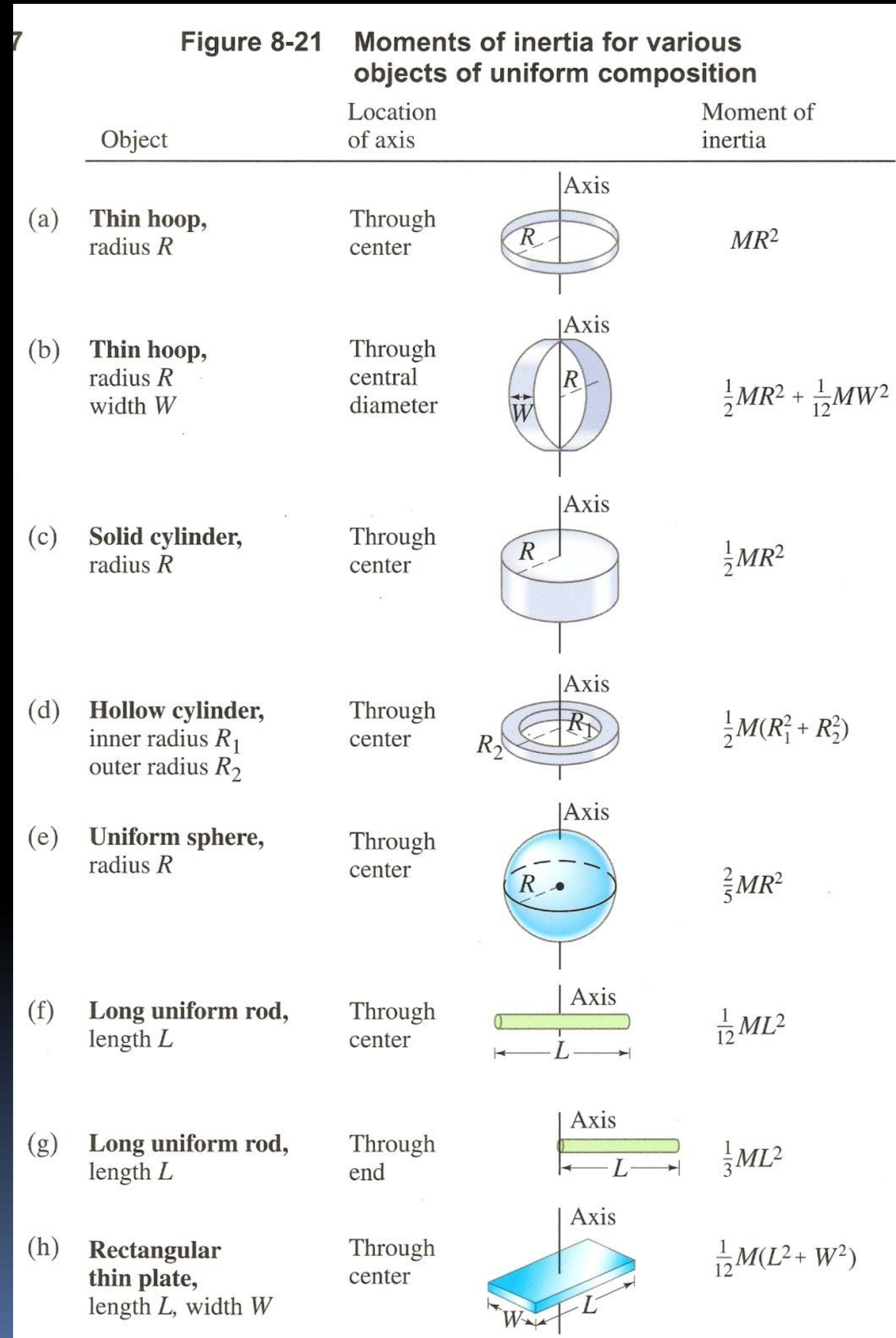


Figure 8-21 Moments of inertia for various objects of uniform composition

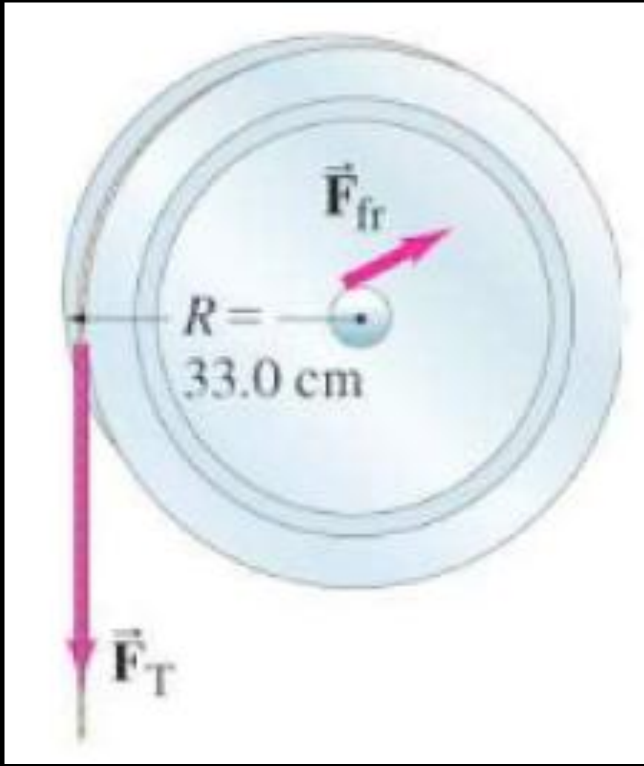
	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R	Through center	MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter	$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center	$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center	$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center	$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end	$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center	$\frac{1}{12}M(L^2 + W^2)$

Problem Solving – Rotational Motion

1. Draw a diagram of the system.
2. Draw a free-body diagram for the object of interest.
3. Identify the axis of rotation (reference point) and determine the torques (remember directions!)
4. Apply Newton's Second Law for rotation ($\Sigma\tau = I\alpha$).
5. Apply Newton's Second Law for translation as needed.
6. Solve and check for reasonability.

Example 8-11

- Determine the moment of inertia of the pulley



$$F_T = 15.0N$$

$$m = 4.00kg$$

$$\Delta\omega = 30.0rad / s$$

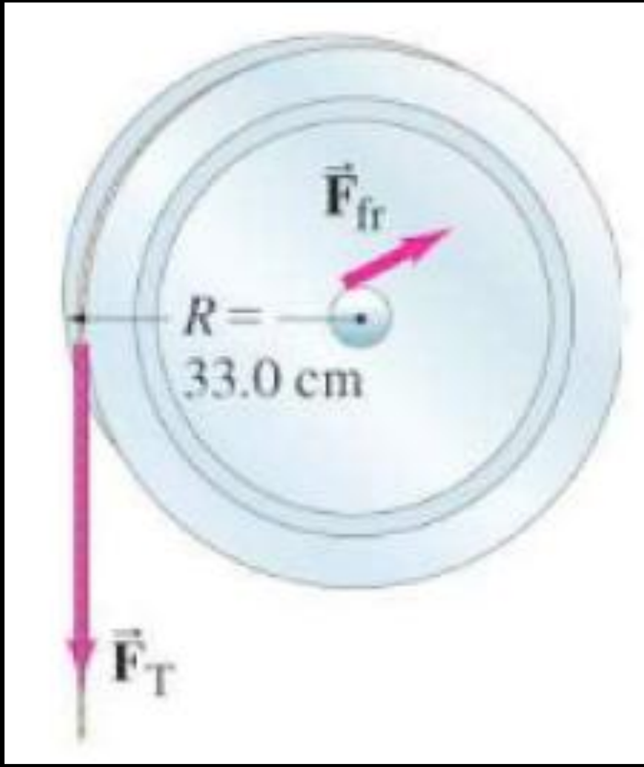
$$\Delta t = 3.00s$$

Example 8-11

- Determine the moment of inertia of the pulley

$$\Sigma\tau = I\alpha$$

$$\frac{\Sigma\tau}{\alpha} = I$$



$$F_T = 15.0N$$

$$m = 4.00kg$$

$$\Delta\omega = 30.0 rad/s$$

$$\Delta t = 3.00s$$

$$\tau_{fr} = 1.10 m\cdot N$$

Example 8-11

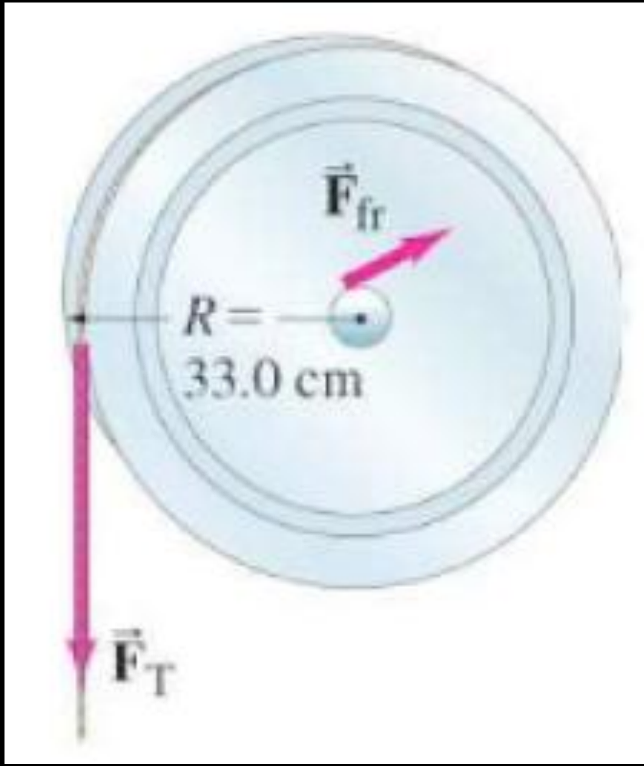
- Determine the moment of inertia of the pulley

$$I = \frac{\Sigma\tau}{\alpha}$$

$$\Sigma\tau = F_T R - \tau_{fr}$$

$$\Sigma\tau = (15)(0.33) - 1.10$$

$$\Sigma\tau = 3.85 m \cdot n$$



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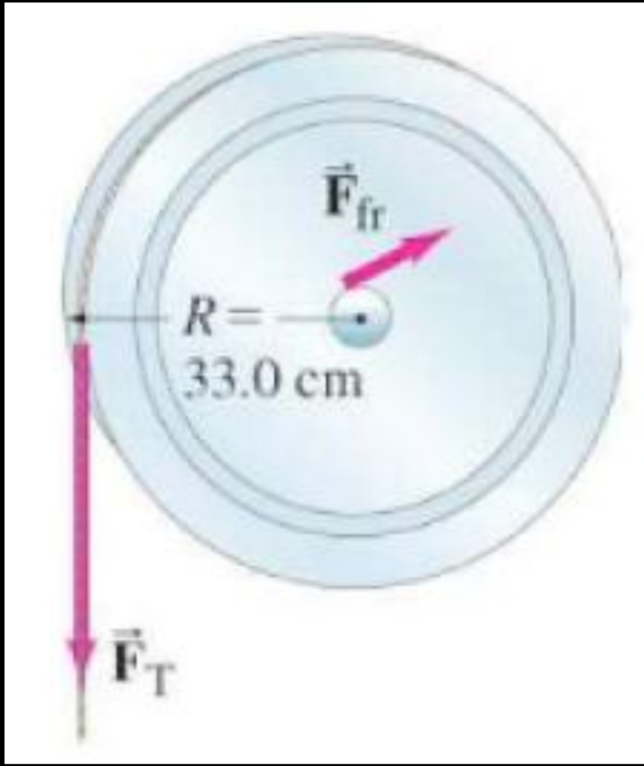

$$I = \frac{\Sigma\tau}{\alpha}$$

$$\Sigma\tau = 3.85 m \cdot n$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30.0}{3.00}$$

$$\alpha = 10.0\, rad/s^2$$

$$F_T = 15.0 N$$

$$m = 4.00 kg$$

$$\Delta\omega = 30.0 rad / s$$

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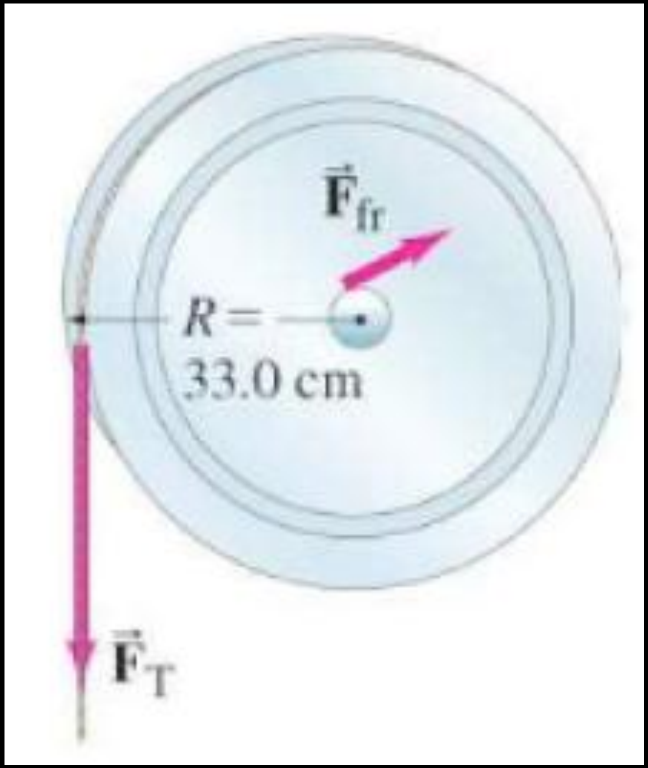
$$\tau_{fr} = 1.10 m \cdot N$$

Example 8-11

- Determine the moment of inertia of the pulley



$$I = \frac{\Sigma\tau}{\alpha}$$

$$\Sigma\tau = 3.85 m \cdot n$$

$$\alpha = 10.0\, rad/s^2$$

$$I = \frac{\Sigma\tau}{\alpha} = \frac{3.85}{10} = 0.385 kg \cdot m^2$$

$$F_T = 15.0 N$$

$$m = 4.00 kg$$

$$\Delta\omega = 30.0 rad/s$$

$$\Delta t = 3.00 s$$

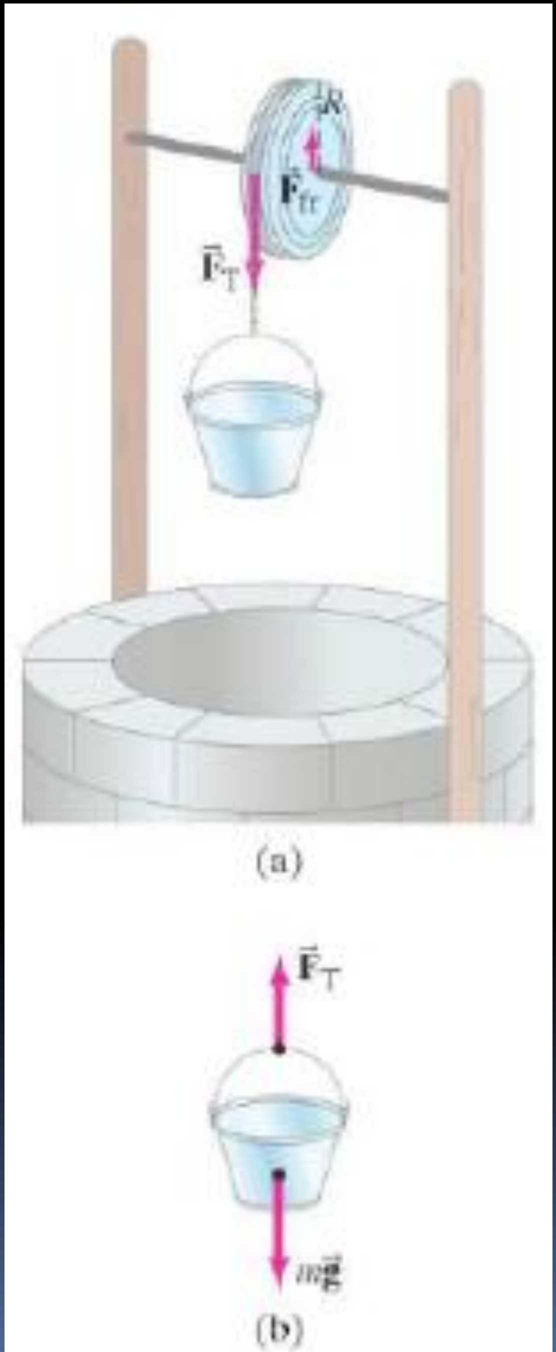
$$\tau_{fr} = 1.10 m \cdot N$$

Example 8-12

- Calculate the angular and linear acceleration of the pulley.

$$w = 15.0N$$
$$m = 1.53kg$$
$$I = 0.385kg \cdot m^2$$
$$\tau_{fr} = 1.10m \cdot N$$

$$I\alpha = \Sigma\tau = F_T R - \tau_{fr}$$
$$mg - F_T = ma$$



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$$mg - F_T = ma$$
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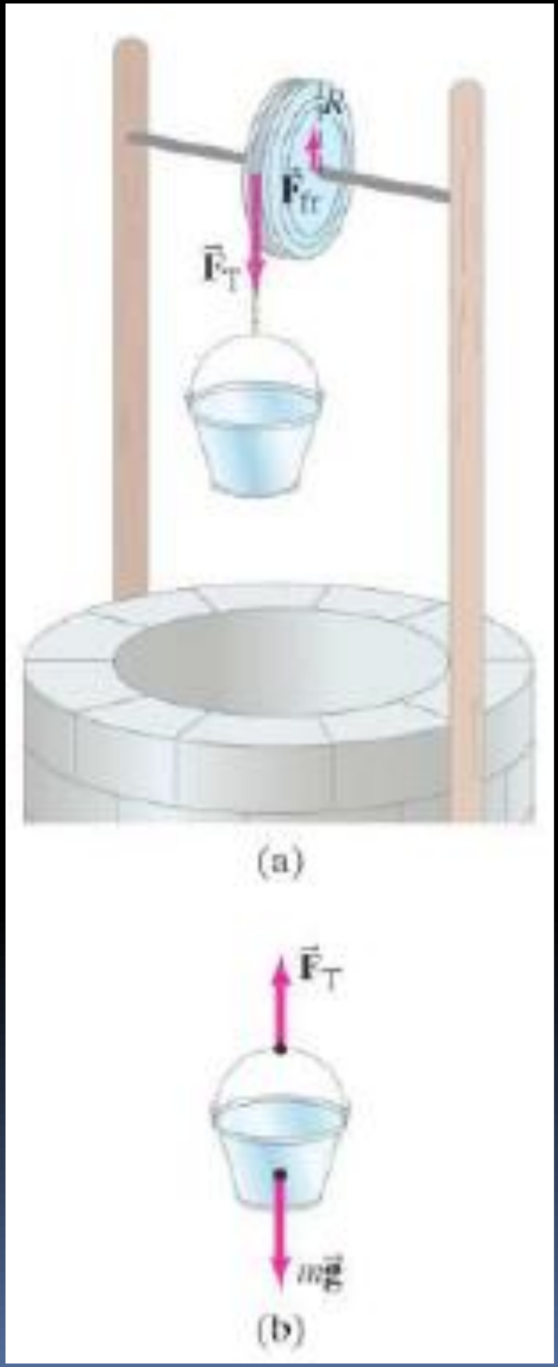

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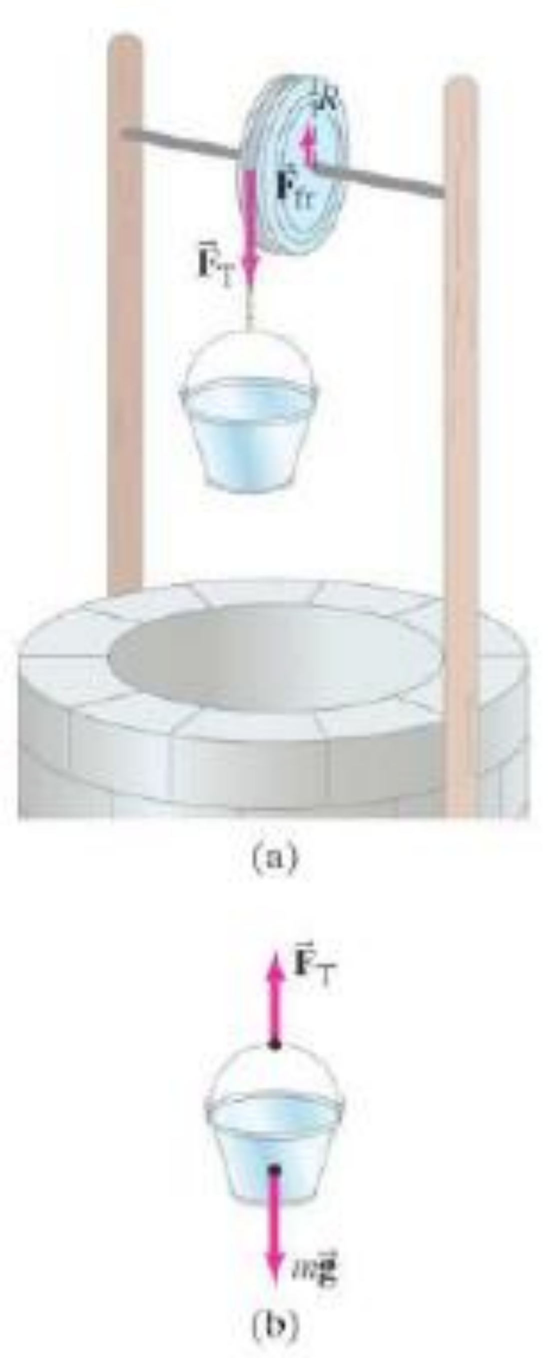

$$I\alpha = F_T R - \tau_{fr}$$
$$mg - ma = F_T$$
$$I\alpha = (mg - ma)R - \tau_{fr}$$
$$I\alpha = mgR - maR - \tau_{fr}$$

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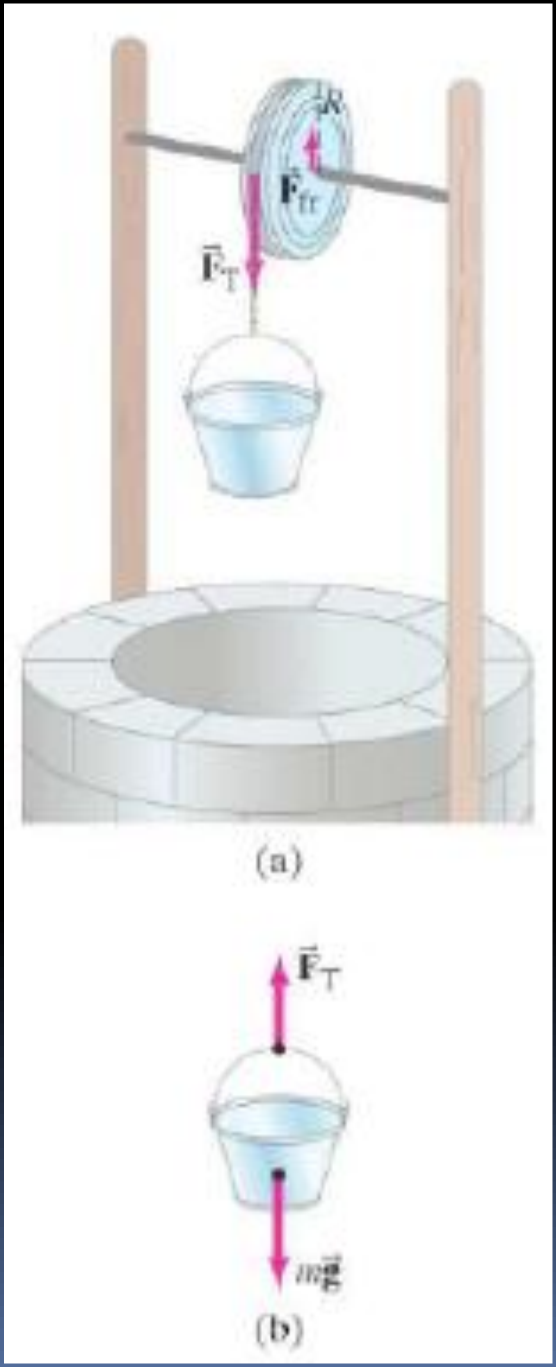

$$I\alpha = mgR - maR - \tau_{fr}$$
$$a = a_{\tan} = R\alpha$$
$$I\alpha = mgR - m(R\alpha)R - \tau_{fr}$$
$$I\alpha = mgR - mR^2\alpha - \tau_{fr}$$

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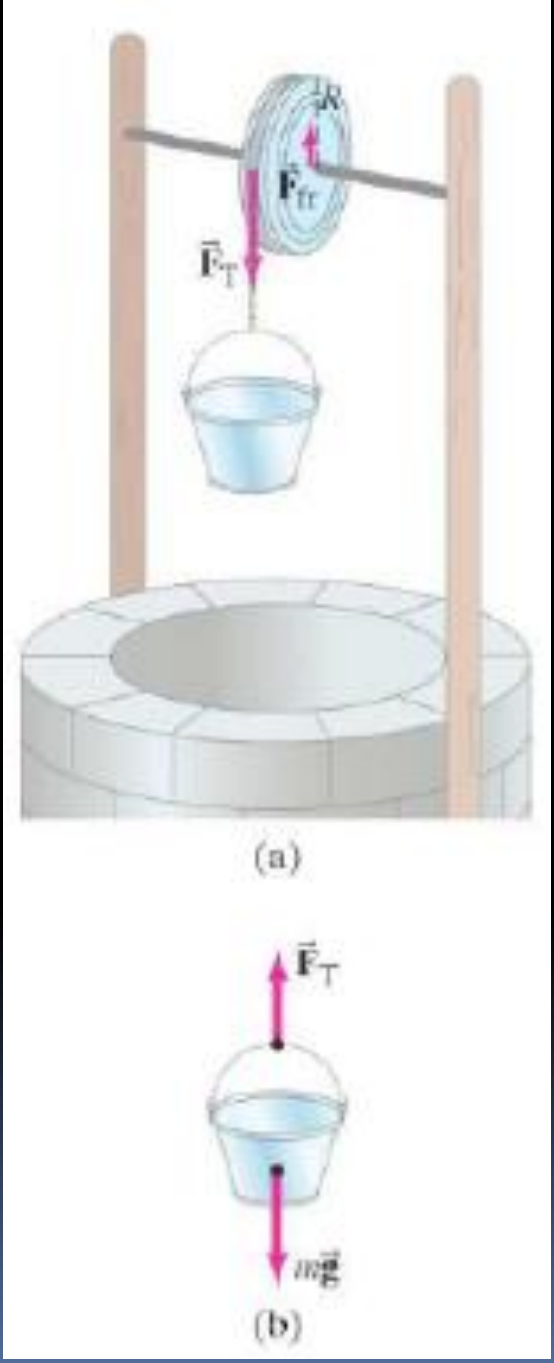
$$I = 0.385kg \cdot m^2$$
$$\tau_{fr} = 1.10m \cdot N$$

$$I\alpha = mgR - mR^2\alpha - \tau_{fr}$$
$$I\alpha + mR^2\alpha = mgR - \tau_{fr}$$
$$\left(I + mR^2\right)\alpha = mgR - \tau_{fr}$$
$$\alpha = \frac{mgR - \tau_{fr}}{\left(I + mR^2\right)}$$



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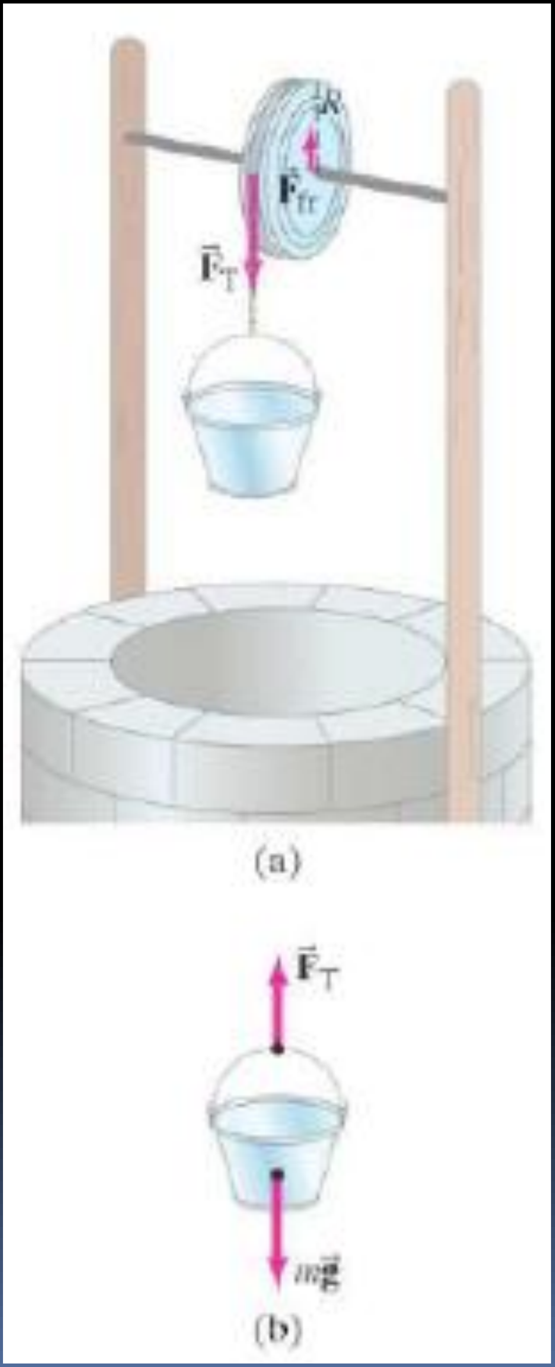
$$\alpha = \frac{mgR - \tau_{fr}}{\left(I + mR^2\right)}$$

$$\alpha = \frac{(15)(0.33) - (1.10)}{\left([0.385] + (1.53)(0.33)^2\right)} = 6.98rad/s^2$$



Review Video

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QUESTIONS?

Homework

#27-40