IB PHYSICS

Name: ___________________________________

Period: ________ Date: ___________________

DEVIL PHYSICS

BADDEST CLASS ON CAMPUS

TSOKOS HOMEWORK SOLUTIONS LSN 9-1

Answers to test yourself questions

Topic 9

9.1 Simple harmonic motion

1 They are not simple harmonic because as shown in the textbook the restoring force whereas opposite to, is not proportional to the displacement away from the equilibrium position. If however the amplitude of oscillations is small the force does become approximately proportional to the displacement and the oscillations are then approximately simple harmonic.

2 **a** We notice that $x_0 \cos\left(\omega t - \frac{\pi}{2}\right) = x_0 \sin \omega t$ and so the phase is $-\frac{\pi}{2}$.

b At $t = 0$ the equation says that $x = x_0 \cos\phi$. The next time x assumes this value is at a time given by $x_0 \cos(\omega T + \phi) = x_0 \cos\phi$. Thus we must solve the equation $\cos(\omega T + \phi) = \cos\phi$. This means that the angles $\omega T + \phi$ and ϕ differ by 2π and so solutions are $\omega T + \phi = \phi + 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

3 **a** At $t = 0$ we have $y = 5.0\cos(0) = 5.0$ mm.

b At $t = 1.2$ s we use the calculator (in **radian** mode) to find $y = 5.0\cos(2 \times 1.2) = -3.7$ mm.

c $-2.0 = 5.0\cos(2t) \Rightarrow 2t = \cos^{-1}\left(-\frac{2}{5}\right) = 1.98 \Rightarrow t = 0.99$ s.

d Use $v = \pm\omega\sqrt{x_0^2 - x^2}$. We know that $\omega = 2.0\ \text{s}^{-1}$. Therefore,

$6.00 = \pm 2.0\sqrt{25 - x^2} \Rightarrow 25 - x^2 = 9.0 \Rightarrow x = \pm 4.00$ mm.

4 **a** The equation is simply $y = 8.0\cos(2\pi \times 14t) = 8.0\cos(28\pi t)$.

b The velocity is therefore $v = -8.0 \times 28\pi \sin(28\pi t)$ and the acceleration is $a = -8.0 \times (28\pi)^2 \cos(28\pi t)$.
At $t = 0.025$ s we evaluate (in radian mode) $y = 8.0\cos(28\pi \times 0.025) = -4.7$ cm,
$v = -8.0 \times 28\pi \sin(28\pi \times 0.025) = -5.7\ \text{m s}^{-1}$ and $a = -8.0 \times (28\pi)^2 \cos(28\pi \times 0.025) = 3.6 \times 10^2\ \text{m s}^{-2}$.

5 The angular frequency is $\omega = 2\pi f = 2\pi \times 460 = 920\pi$. The maximum velocity is $\omega A = 920\pi \times 5.0 \times 10^{-3} = 14\ \text{m s}^{-1}$ and the maximum acceleration is $\omega^2 A = (920\pi)^2 \times 5.0 \times 10^{-3} = 4.2 \times 10^4\ \text{m s}^{-2}$.

6 **a** The equation of the string may be rewritten as $y = \left(6.0\sin(\pi x)\right)\cos(2\pi \times 520t)$ from which we deduce that the frequency of all points is 520 Hz and that the phase of all points is zero.

b From **a** the amplitude is $A = 6.0\sin(\pi x)$ and so is different for different points on the string.

c The maximum amplitude is obtained when $\sin(\pi x) = 1$, i.e. the maximum amplitude is 6.0 mm.

d The displacement is always zero at the ends of the string, in particular at the right end where $x = L$, the length of the string. The displacement is zero *all the time* when $6.0\sin(\pi x) = 0$ i.e. when $\pi x = \pi \Rightarrow x = 1.0$ m.

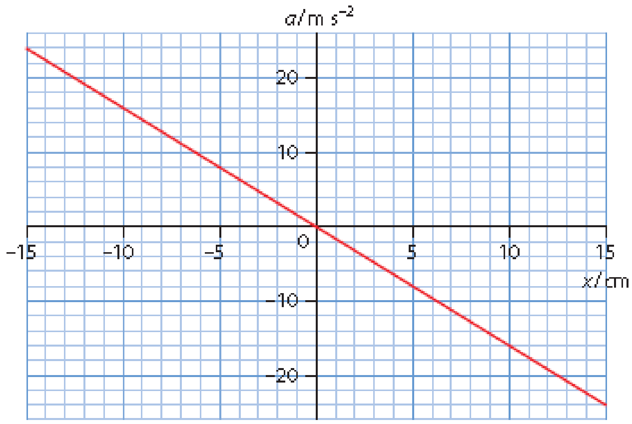
e When $x = \frac{3L}{4} = 0.75$ m the amplitude is $6.0\sin(\pi x) = 6.0\sin(0.75\pi) = 4.2$ mm.

7 **a** The area is approximately 0.50 cm (the exact value is 0.51 cm).

b This is the displacement from when the velocity is zero to when it is zero again i.e. from one extreme position until the other i.e. twice the amplitude.

c The period is 0.4 s and so the equation for displacement is $x = -0.25\sin\left(\frac{2\pi t}{0.4}\right) = -0.25\sin(5\pi t)$.

8 We need to graph the equation $a = -\omega^2 x$ where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.57\ \text{s}^{-1}$. The slope would be $\omega^2 = 158\ \text{s}^{-2}$ or just 1.58 since we are plotting cm on the horizontal axis.



9 **a** The defining relation for SHM is that $a = -\omega^2 x$ which implies that a graph of acceleration versus displacement is a straight line through the origin with negative slope just as the given graph.

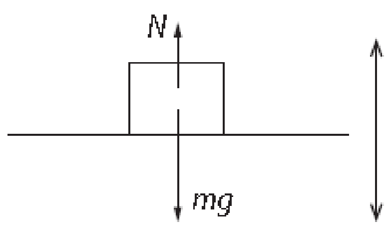
b The slope of the graph gives $-\omega^2$. The measured slope is $\frac{1.5}{0.10} = -15\ \text{s}^{-2}$ and so $\omega = \sqrt{15} = 3.873\ \text{s}^{-1}$. Thus the period is $T = \frac{2\pi}{3.873} = 1.6$ s.

c The maximum velocity is $\omega A = 3.873 \times 0.10 = 0.39\ \text{m s}^{-1}$.

d The maximum net force is $ma = m\omega^2 A = 0.150 \times 15 \times 0.10 = 0.225$ N.

e The total energy is $E_T = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} 0.150 \times 3.873^2 \times 0.10^2 = 0.012$ J.

10 **a** The forces on the mass when the plate is at the top are shown below:



The net force is $mg - N = ma$. Since we have simple harmonic motion $a = \omega^2 x = 4\pi^2 f^2 x$ in magnitude, and the largest acceleration is obtained when $x = A$, the amplitude of the oscillation. The frequency is 5.0 Hz. The critical point is when $N = 0$. I.e. $g = 4\pi^2 f^2 A$ and so $A = \frac{g}{4\pi^2 f^2} = \frac{9.8}{4\pi^2 \times 25} = 0.0099$ m. The amplitude must not exceed this value.

b At the lowest point:

$$N - mg = ma = m4\pi^2 f^2 A$$

$$\Rightarrow N = mg + m4\pi^2 f^2 A$$

$$N = 0.120 \times 9.8 + 0.120 \times 4 \times \pi^2 \times 25 \times 0.0099$$

$$N = 2.35\ \text{N}$$

11 **a** The volume within the sphere of radius x is $\frac{4\pi x^3}{3}$ and that of the entire sphere is $\frac{4\pi R^3}{3}$ therefore the mass enclosed is the fraction $M\frac{x^3}{R^3}$.

b $F = G\frac{\frac{Mx^3}{R^3}m}{x^2} = \frac{GMmx}{R^3}$

c The acceleration of the mass is given by $ma = -\frac{GMmx}{R^3} \Rightarrow a = -\frac{GM}{R^3}x$ which is the condition for SHM with $\omega^2 = \frac{GM}{R^3}$.

d $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$

e $T = 2\pi\sqrt{\frac{(6.4\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}} = 5085\ s = 85\ \text{min}$.

f From gravitation we know that $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$ as in **d**.

12 **a** When extended by an amount x the force pulling back on the body is $2kx$ and so $ma = -2kx \Rightarrow a = -\frac{2k}{m}x$ and so $\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times120}{2.0}} = 10.95\ \text{s}^{-1}$ giving a period of $T = \frac{2\pi}{\omega} = \frac{2\pi}{10.95} = 0.57$ s.

b With the springs connected this way, and the mass pulled to the side by small amount one spring will be compressed and the other extended. Hence the net force on the mass will still be $2kx$ so the period will not change.

13 **a** At the top the woman's total energy is gravitational potential energy equal to mgh where h is the height measured from the lowest position that we seek. At the lowest position all the gravitational potential energy has been converted into elastic energy $\frac{1}{2}kx^2$ and so $mgh = \frac{1}{2}kx^2$. Since $h = 15 + x$ we have that $mgh = \frac{1}{2}k(h-15)^2$.

We must now solve for the height h:

$$60\times10\times h = \frac{1}{2}\times220\times(h-15)^2$$

$$600h = 110(h^2 - 30h + 225)$$

$$110h^2 - 3900h + 24750 = 0$$

$$11h^2 - 390h + 2475 = 0$$

The physically meaningful solution is $h = 27$ m.

b The forces on the woman at the position in (a) are her weight vertically downwards and the tension in the spring upwards. Hence the net force is $F_{net} = T - mg = kx - mg = 220\times(27-15) - 600 = 2040$ N hence $a = \frac{F_{net}}{m} = \frac{2040}{60} = 34\ \text{m s}^{-2}$.

c Let x be the extension of the spring at some arbitrary position of the woman. Then the net force on her is $F_{net} = T - mg = kx - mg$ directed upwards i.e. opposite to the direction of x. So $ma = -(kx - mg)$. The acceleration is not proportional to the displacement so it looks we do not have SHM. But we must measure displacement from an equilibrium position. This is when the extension of the spring is x_0 and $kx_0 = mg$. In other words call the displacement to be $y = x - x_0$. Then $ma = -(k(y + x_0) - mg) = -ky - kx_0 + mg = -ky$ since $kx_0 = mg$. Hence we do have the condition for SHM. And so $a = -\frac{k}{m}y$ so that $\omega^2 = \frac{k}{m} \Rightarrow \omega = 1.91\ \text{s}^{-1}$ and finally $T = \frac{2\pi}{\omega} = 3.28 = 3.3$ s.

d She will come to rest when the tension in the spring equals her weight i.e. when $kx_0 = mg \Rightarrow x_0 = \frac{mg}{k} = \frac{60 \times 10}{220} = 2.7$ m. Hence the distance from the top is $15 + 2.7 = 17.7 = 18$ m.

e It has been converted to other forms of energy mainly thermal energy in the air and at the point of support of the spring.