

## How temperature affects the vibration rate of a tuning fork

My teacher told me that we needed to come up with an investigation relating to something that we are interested in and that relates to the real world. Well, as a student of music I often use a tuning fork to help tune my guitar. I simply hit the end and it vibrates at a fixed and known frequency, and so I can tune your guitar string to the same frequency by comparing the tuning fork sound with the guitar sound.

In physics we learned that sound travels at differing speeds in different mediums, and that air could be a different medium if the temperature changed. That is, sound travels faster in hot air compared to cold air. Temperature differences in air can have the effect of bending sound travel, as we learned studying waves. We also learned last semester that when a metal is heated the particle vibrates more and thus the metal object expands slightly.

As temperature increases the metal of the tuning fork will expand, and the result will be slightly longer fork prongs and hence a lower frequency (a longer wavelength) of vibration.

According to known scientific theory, the frequency of a tuning fork is related to a number of qualities. The length "l" is inversely related to frequency (see the equation below). The frequency also relates to the square root of Young's modulus, the moment of inertia, the density and the cross sectional area of the tuning fork. Again, see the equation below.

### Calculation of frequency

The frequency of a tuning fork depends on its dimensions and the material from which it is made.<sup>[6]</sup>

$$f = \frac{1.875^2}{2\pi l^2} \sqrt{\frac{EI}{\rho A}}$$

Where:

- $f$  is the **frequency** the fork vibrates at in **Hertz**.
- 1.875 the smallest positive solution of  $\cos(x)\cosh(x) = -1$ .<sup>[7]</sup>
- $l$  is the length of the prongs in metres.
- $E$  is the **Young's modulus** of the material the fork is made from in **pascals**.
- $I$  is the **second moment of area** of the cross-section in metres to the fourth power.
- $\rho$  is the **density** of the material the fork is made from in **kilogrammes** per cubic metre.
- $A$  is the cross-sectional **area** of the prongs (tines) in square metres.

The ratio  $\frac{I}{A}$  in the equation above can be rewritten as  $r^2/4$  if the prongs are cylindrical of radius  $r$ , and  $a^2/12$  if the prongs have rectangular cross-section of width  $a$  along the direction of motion.

[http://en.wikipedia.org/wiki/Tuning\\_fork](http://en.wikipedia.org/wiki/Tuning_fork)

Although temperature does not appear in this equation, we also know in physics that metals expand with temperature, for the length and area will increase with temperature and so changing the frequency. See [http://en.wikipedia.org/wiki/Thermal\\_expansion](http://en.wikipedia.org/wiki/Thermal_expansion).

For my experiment, then, I investigated three tuning forks, each of a different frequency. I will first cool them in the refrigerator, and then heat them in an oven. Each time I will measure the temperature and the frequency.

	
Thermometer	Frequency Measurement

For **temperature** measurements, I used an infrared thermometer. You point it at the tuning fork and press a button and the temperature read out. This has an accuracy of one degree. It is an *Extech* Pocket IR model IR201 laser pointer thermometer.

For the **frequency** I used my Smartphone with a sound analysis application. The phone catches the sound and reads out the frequency and then you select the FFT histogram on a linear scale. The FFT is called Fast Fourier transformation and it tells me the frequency of the loudest sound. See [http://en.wikipedia.org/wiki/Fourier\\_transform](http://en.wikipedia.org/wiki/Fourier_transform)

This program is an advanced mathematical tool it does all the analysis for me. The app for my Smartphone is from Tektronix and is called “Real Time Audio Analyzer & Oscilloscope 1.1” and costs \$30.

For a method, I left the tuning forks in the refrigerator overnight and tested them the next day. I put the tuning forks on a cookie sheet and heated them in the oven and then tested them. I did not want the temperature to be too hot to handle, a safety precaution. I used a kitchen hot matt for this.

The following are my scientific results.

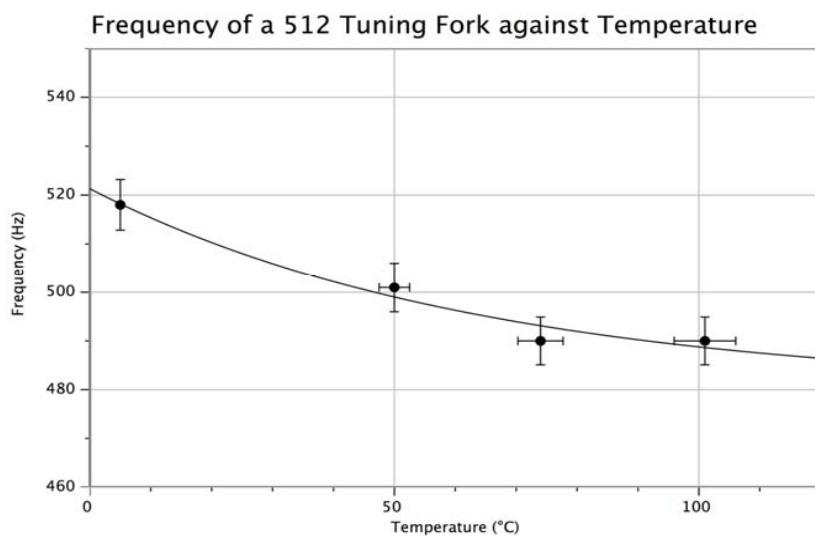
### **Experiment 1: Tuning fork @ 1024 Hz**

4°C	50°C	75°C	100°C
1025	1002	1002	1002

The data here is too poor to make use of. I include here anyway because I did a work to make the measurements. No graph should be draw here as three frequency values are the same.

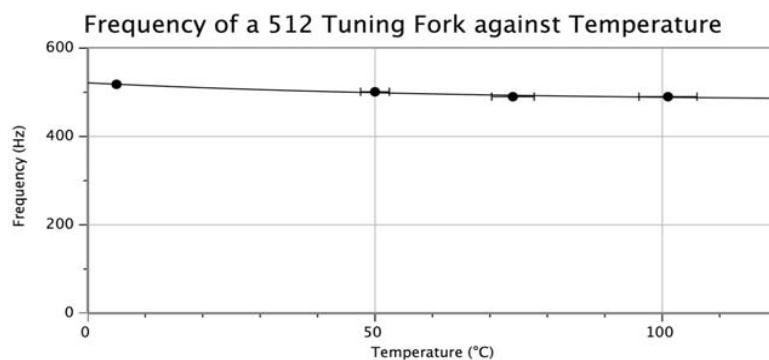
### **Experiment 2: Tuning fork @ 512 Hz**

4°C	50°C	75°C	100°C
518	501	490	490



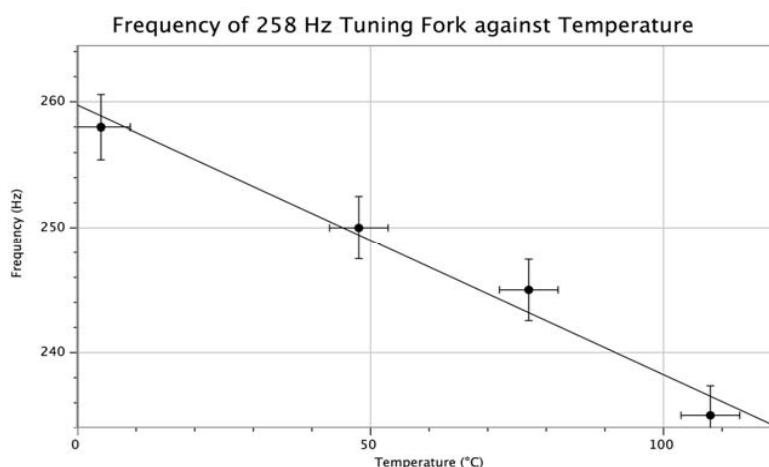
For the uncertainties in frequency and temperature I simple adjusted the percentage number in *LoggerPro* so that the best-fit graph line would include all the data ranges. This was as low as 1% for the frequency and 5% for the temperature; both of these are good for my experiment.

The equation for the relationship is incomprehensible but is clearly an inverse proportional equation, that is, when the temperature increases the frequency decreases. The next graph is even better if the beginning of the frequency axis is shown, as below.



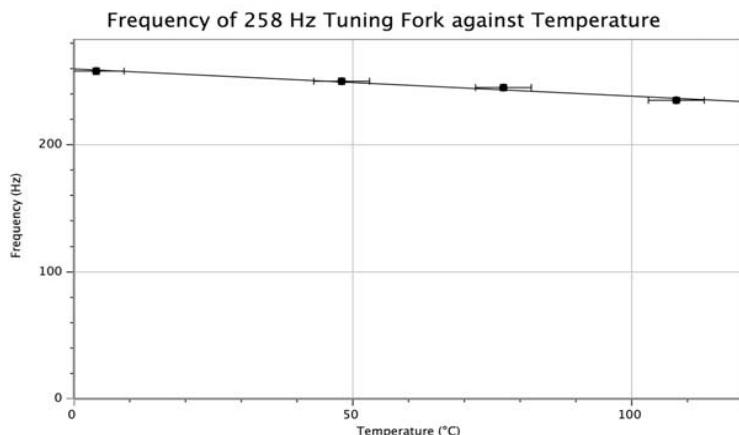
### Experiment 3: Tuning fork @ 258 Hz

4°C	50°C	75°C	100°C
258	250	245	245



Here the uncertainty is 5% for temperature and only 1% for frequency

Next you see the same graph as above but with a full range (from zero) for frequency.



Conclusion. My hypothesis was that the tuning fork would produce a lower frequency at higher temperature. I proved this is true. When the fork was heated it expanded causing the length of the tines to increase. This increase in length caused the frequency to be lower because the length of the tines depicts the frequency (wavelength) of the fork. The longer tuning forks have lower frequency while the smaller ones have a high frequency. The frequency of the tuning forks is inversely proportional to the length of the tines squared. My experiment was a success.

For improvements I would (1) repeat the experiment many times, and (2) do the experiment with more and different tuning forks.