Determining solar characteristics using planetary data

Introduction

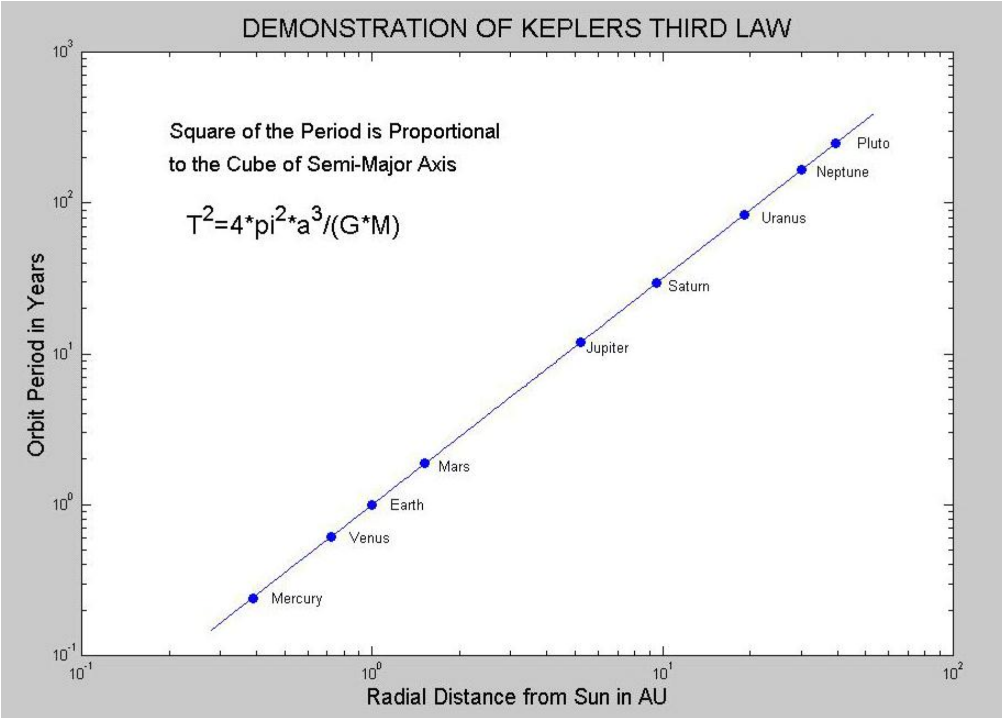
The Sun is a G-type main sequence star at the center of the Solar System around which the planets, including our Earth, orbit. In this investigation I will be examining the Sun and its physical characteristics. I will use a database provided by the European Space Agency, found at http://pdb.estec.esa.int/, to acquire various pieces of information about the planets in the Solar System and then use that to determine some of the Sun's characteristics through the laws of astrophysics.

Feature 1: Mass

The most basic characteristic about the Sun that I can find is its mass. The mass of the Sun must first be found before it is possible to determine characteristics based on gravitational force. I plan to determine the mass of the Sun using data from the planets. This can be done through the application of Johannes Kepler's laws of orbital motion. Kepler's third law states that:

"The square of a planet's orbital period is directly proportional to the cube of its semi-major axis."

This law can be demonstrated using the planets in our Solar System and their orbits around the Sun. I found a graph on the internet which shows the accuracy of Kepler's third law in our Solar System.



http://burro.astr.cwru.edu/Academics/Astr221/Gravity/kepler3.htm

The orbital period of a body is the time it takes to complete one full orbit. The semi-major axis is half of the longer diameter of an orbit's elliptical path.

Any proportionality statement can be expressed in an equation through the addition of a constant. In this case, the constant must relate to the gravitational force which guides the orbits and their circular paths. The full equation for Kepler's third law is given as:

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

Where P is the orbital period, G is the universal gravitational constant ($6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$), M_1 and M_2 are the masses of the two celestial bodies and a is the semi-major axis of the orbit.

This equation can then be rearranged to find the solar mass, M_1:

$$M_1 = \frac{4\pi^2 a^3}{P^2 G} - M_2$$

Now, using data from the planetary database, I can calculate the mass of the Sun. To improve the accuracy of the result, I did this calculation for each of the eight planets.

Sample calculation using data from Mercury:

$$M_1 = \frac{4\pi^2 (57909175670)^3}{(7.60055 \times 10^6)^2 (6.67 \times 10^{-11})} - 3.3 \times 10^{23}$$

$$M_1 = 1.9897 \times 10^{30}\, kg$$

These are the results I obtained for the solar mass using data from each of the planets:

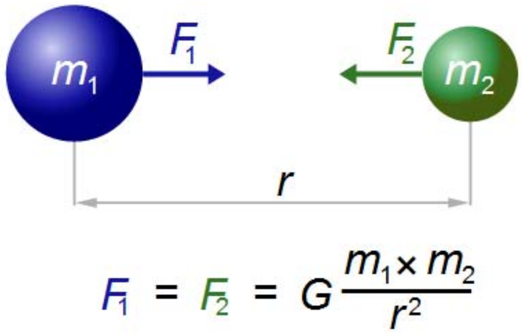
Planet	**Solar Mass (kg)**
Mercury	1.9897×10^{30}
Venus	1.9897×10^{30}
Earth	1.9897×10^{30}
Mars	1.9897×10^{30}
Jupiter	1.9878×10^{30}
Saturn	1.9891×10^{30}
Uranus	1.9896×10^{30}
Neptune	1.9896×10^{30}

All the results are very similar when rounded. The main difference occurred when going from the smaller and lighter inner planets to the larger and heavier outer ones. The greater mass of these planets changed the value calculated slightly. I decided to take an average mass from these results and got my final value for the solar mass as 1.9894×10^{30}kg.

Feature 2: Surface Gravity

After successfully determining the mass of the Sun through Keplerian calculations, the Sun's gravitational force can be calculated. The surface gravity of a body is the acceleration due to the force of its gravity when at its surface. The mathematical formula for acceleration due to gravity is derived from Newton's law of universal gravitation.

Newton's law is shown in the picture below.



Then, taking Newton's second law, $F = mg$ and setting the two forces equal to each other we get:

$$mg = \frac{GMm}{r^2}$$

The masses then cancel out allowing one to solve for the acceleration due to gravity using the following formula:

$$g = \frac{GM}{r^2}$$

Where g is the surface gravity, G is the gravitational constant, M is the mass of the body and r the distance from the body's center of gravity. To calculate the surface gravity, the value for r must be set equal to the body's radius. In the last section I determined the mass of the Sun. Its radius can be found through measuring its angular size in the sky and then, knowing the distance from the Earth to the Sun, the radius can be calculated trigonometrically. I was unable to do this myself, so I obtained the radius of the Sun from my database, as 695500km. Using this data, the surface gravity of the Sun can be calculated.

$$g = \frac{(6.67\times10^{-11})(1.9894\times10^{30})}{(695500000)^2}$$

$$g = 274.3ms^{-2}$$

As a comparison, the surface gravity on Earth is 9.81ms^{-2}, almost 28 times less. Its very large gravitational force is how the Sun manages to keep all the planets in orbit.

Feature 3: Escape Velocity

The escape velocity of a celestial body is the velocity required by an unpowered object to escape that body's gravitational field. At escape velocity, the gravitational potential energy of the object and its kinetic energy will equal zero when added. The formula for escape velocity can be derived through this.

Kinetic energy has the formula $E_k = \frac{1}{2}mv^2$ and gravitational potential energy is given by

$$E_g = \frac{-GM_1M_2}{r}.$$

When these two are added, they must equal zero, so:

$$\frac{1}{2}mv^2 + \frac{-GM_1M_2}{r} = 0$$

This now allows us to rearrange this formula to solve for v, the escape velocity.

$$v = \sqrt{\frac{2GM}{r}}$$

Now the values that we obtained can be inserted into the equation.

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(1.9894 \times 10^{30})}{695500000}}$$
$$v = 617718 ms^{-1}$$

This value I obtained is the velocity an object fired from the surface of the Sun to be fired in order to escape its gravitational field – about 617.7kms^{-1}. However, this value does not have many practical applications as we do not fire objects from the surface of the Sun. A much more useful value to calculate would be the velocity required to escape solar orbit when launched from the Earth. This allows us to launch probes into the interstellar medium. To calculate this value, the Earth's average distance from the Sun, one astronomical unit, must be set as the r value in the equation.

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(1.9894 \times 10^{30})}{149597887100}}$$
$$v = 42118 ms^{-1}$$

In order to escape the Sun's gravity when launched from the Earth, a velocity of 42kms^{-1} is required. This value can be used if probes ever need to be sent on a solar escape trajectory.

Conclusion

Using data provided in an online database, I was able to determine the mass, gravitational force and escape velocity of the Sun. To check the accuracy of the results I calculated, I compared them to the actual accepted values.

The mass of the Sun that I obtained was 1.9894×10^{30}kg. The actual accepted value is 1.9891×10^{30}kg. My result was only 0.015% higher than the accepted value. This shows that through the application of Kepler's third law, celestial masses can be calculated with a very good amount of accuracy.

The next characteristic that I calculated was the surface gravity of the Sun. The value I obtained was 274.3ms^{-1}. The accepted value is 274.0ms^{-1}. My value is 0.15% higher which is once again very close. The reason that my result is higher is because I used the mass of the Sun that I calculated in the first part, which was also slightly higher than the actual mass. However, my calculations were still very accurate.

The final characteristics I calculated were the escape velocities from the surface of the Sun and also the solar escape velocity from Earth. For the escape velocity from the surface of the Sun, I got a value of 617.7kms^{-1}. This is identical to the accepted value, which means that I was able to perform my escape velocity calculation very accurately. The result that I calculated for the solar escape velocity from Earth was 42.1kms^{-1}. This is also identical to the accepted value. My escape velocity calculations were extremely accurate.

Overall, the values I obtained through my calculations in this investigation were very accurate with regards to the actual values and I am pleased with how it turned out. I was able to apply Kepler's and Newton's laws of astrophysics to the planets and the Sun in our Solar System well and through this I was able to confirm the accuracy of these laws.

Evaluation

I chose to do this investigation because of a personal interest in the field of astrophysics. I chose to do our Solar System because it is the system that we are most familiar with and we have an abundance of data to do with the celestial bodies in it. The Sun, being the focal point of the Solar System, was the best choice for me to conduct my investigation on. I knew that I could use the orbital data of the planets to calculate various characteristics of the Sun. For my database, I chose the one provided by the European Space Agency because it has very accurate and precise information with easy accessibility. The database proved very useful throughout my investigation and gave me very accurate results in the end.

Throughout my investigation, I had to apply various physical laws. In order to conduct the first part of my investigation I had to familiarize myself with Kepler's third law, a topic that we only studied briefly in class and I did not know too much of. I was able to use the law properly and accurately to calculate my data. Newton's laws of gravitation were ones that we had studied more extensively so I was also able to apply them in a proper manner and effectively. The accuracy of my results shows a good application of these laws.

I am pleased with how the investigation turned out and the results I managed to obtain. If I could add more to this investigation I would probably look into doing further calculations of different factors, examining characteristics such as the Sun's surface temperature, luminosity and energy output. I would also probably do more calculations for my current factors, such as expanding my solar escape velocities to include those from all the planets. In this investigation I only looked into a small fraction of the solar characteristics available. However, I think that what I did with my investigation went very well.