Obtaining Wien's displacement law of electromagnetic radiation

We are told that the temperature at the surface of the sun is 5778 K and the temperature of the universe is 2.735 degrees above absolute zero.

I ask myself: How do scientists determine these temperatures? These claims are hard to believe but my physics IA will help answer this.

§1. Research Project

There is a law in physics stating that the wavelength of light carrying the maximum intensity is inversely proportional to the temperature of the radiating body. In other words, the hotter a body is, the shorter the emitted wavelength at the peak intensity. The relationship has the form $\lambda_{peak} T$ = constant, where λ_{peak} is the wavelength at maximum intensity and T is the absolute temperature of the radiating body. The law is named after German physicist Wilhelm Wien. [# 1]

This purpose of this investigation is two-fold: first, to determine Wien's displacement law constant using a computer simulation, and second, to recognize a confusing representation found in some textbooks about Wien's law.

§2. History

Wilhelm Wien was a physics professor at the University of Würzburg in Germany. In 1911 he was awarded the Nobel Prize. The Academy wrote the following comments of appreciation.

> *"Professor Wien. The Swedish Academy of Sciences has awarded to you this year's Nobel Prize for Physics for your discoveries concerning the laws of thermal radiation. You have devoted your researches to one of the most difficult and spectacular problems of physics, and among the researchers now living it is you who has succeeded in making the greatest and most significant contributions to the solution of the problem. In admiration of the completed task and with the wish that further success may be granted to you in future work, the Academy now calls upon you to receive the prize from the hands of his Majesty the King."*

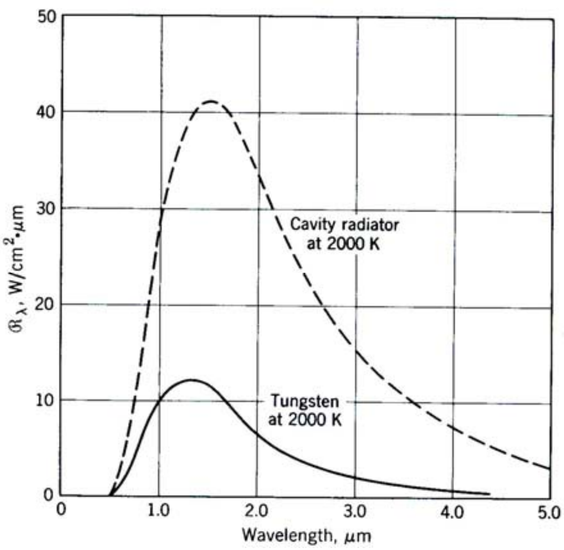
From Nobel Lectures, *Physics 1901-1921*,
Elsevier Publishing Company, Amsterdam, 1967 [# 2 & #3]

§3. Using Wien's Law to Measure Temperature

Wien's law can be used to measure the temperature of the sun, about 5778 K, and the temperatures of stars. The temperature of a burning wood campfire can be determined to be about 1500 K, and the temperature of a fire iron pulled out of a campfire can be determined be about 800 K. The temperature of an asteroid can be determined to be very cold, about 190 K (or –80 °C). Wien's law can be used to determine the temperature of the universe; that is, the background radiation resulting from the Big Bang gives the current temperate of about 2.7 K. [#4] These claims and more are amazing to me and motivate me to produce this investigation.

§4. Light Emitting Bodies

In the box below, there is a graph of the typical intensity against wavelength for a tungsten filament at a temperature of 2000 K. The graph line can be described as a 'radiancy curve.' [#5] Note that the curve goes to zero at both long and short wavelengths. The curve peaks at a certain wavelength for a certain temperature, this peak intensity wavelength is called the maximum wavelength, λ_{peak}.



Graph of Intensity of Light against Wavelength

The graph shows intensity (power per unit surface area) against wavelength. The solids graph line is real data for a tungsten filament glowing at a temperature of 2000 K. The dashed line is the ideal black body curve appropriate for any ideal body radiating at a temperature of 2000 K. You can see that there is a difference in real wavelength as well as intensity. [#8]

For every material there exists a family of spectral radiancy curves, one curve for every temperature. When such families of curves are compared, no obvious regularities stand out.

Different materials are not exactly the same. A quantitative understanding in terms of theory here is not understood, but can be approximated. [#6]

§5. Theory and Reality

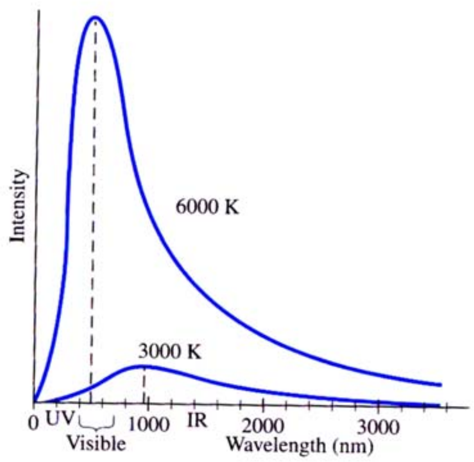
Instead of studying the radiation from one or another material surface we may idealize this situation and consider the radiation emerging from a small hole in the wall of a closed material surface kept at a fixed temperature. This is like an oven, with a small opening on one side. We direct our measurements at the opening and thereby measure the radiant energy emerging from the interior of the enclosure. This idealization is called a 'back body' and blackbody radiation refers to an object that radiates energy that is characteristic of this radiating system only, not dependent upon the type of radiation or the material of the body radiating, or on the size or shape of the body. [#7]

The radiated energy is produced by standing waves or resonant modes of the cavity that is radiating. The equation for this idealized 'black body' is obtained from modern quantum physics, and the details and derivations are beyond the scope of this study. However, there are several good sources of the theory. See the explanation in the ***Hyperphysiscs*** web site as well as the Wolfram mathematical web site and numerous other sources. [# 9]

The computer simulation as well as most applications of Wien's law will be based on the black body radiation characteristic. This is an important assumption, but the difference between the real and ideal is very small and it is appropriate to ignore this difference.

§6. A Basic Explanation of Wien's Law

The graph tells us that as the temperature increases then the wavelength of the peak intensity emission decreases. A shorter wavelength corresponds to a higher frequency, and we know from the Einstein-Plank equation that energy and frequency are related.



(Right) Spectrum of wavelength emitted by a blackbody at two different temperatures. [#10]

Page 3

The peak wavelength λ_{peak} at which the maximum amount of energy is radiated decreases with temperature. Textbooks tell us the λ_{peak} is inversely proportional to absolute temperature. This is **Wien's Displacement Law**, and the equation is written as

$$\lambda_{\text{peak}} T \approx 0.00290 \text{ m} \cdot \text{K}$$

The wavelength is in metres (λ / m) and the temperature on the Kelvin scale (T / K). A more precise value of the proportionally constant is **0.0028977685 m K**. This is given by CODA (International Council for Science, The Committee on Data for Science and Technology, 2006, listed on Wikipedia) and has an uncertainty in the last two significant figures. [#11]

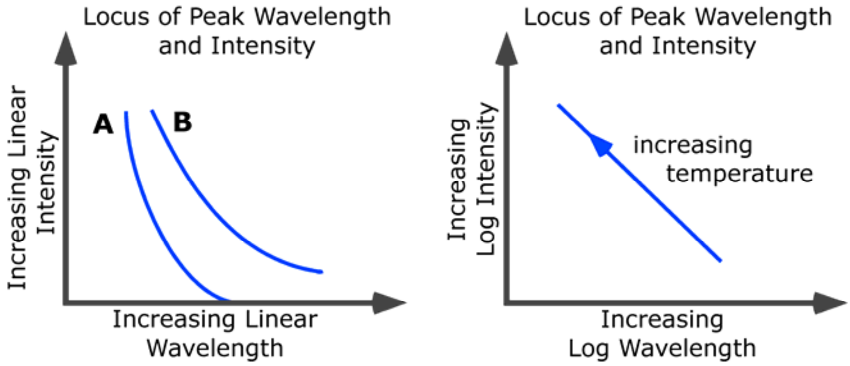
§7. Representing Wien's Proportionality Constant

Wien's law states that the absolute temperature of a radiating body is inversely proportional to the peak intensity wavelength. In this investigation I record wavelength at peak intensity, λ_{peak}, and the corresponding absolute temperatures, T_{K}, from a computer simulation. Then a graph of absolute temperature against the reciprocal of the wavelength is made, and a linear best-fit line yields a gradient that represents Wien's proportional constant. This is compared to the accepted value.

Wien's constant defines **the locus of peak wavelengths**. This locus or line connecting peak wavelength as a function of temperature for different temperatures is represented in various textbooks. To my surprise, there are different interpretations of this locus. Only one can be correct, and why textbooks would publish the wrong graph is beyond me. However, the theory is clear, and the correct graph should have been obvious to the textbook authors. Correcting this misunderstanding is not part of my IA, but rather rather explained here and suggest as an extension in my conclusion.

§8. When Textbooks Get It Wrong

Different textbooks represent **the locus of peak wavelengths** in different ways. See the sketches below.



The various representations of the locus of peak intensities against wavelengths as found in different textbooks. Most books do not draw the locus of peak values, but some do. The graph sketched above on the left shows two different representations, A and B, of the peak wavelength line as a function of wavelength. The graph on the right shows how we would establish the correct representation (see my extension idea in the conclusion, and footnote #14).

Line A above can be found in "Physics For Use In The IB Diploma Program" by Greeg Keer and Paul Ruth, and in "Astrophysics: University of Bath Science 16–19" by Nigel Ingham. And, by extrapolation, line A can be found in "Physics for the IB Diploma" by K.A. Tsokos. [#12]

Line B can be found in some textbooks, such as in "Physics: Algebra/Trig" by Eugene Hecht. [#13]

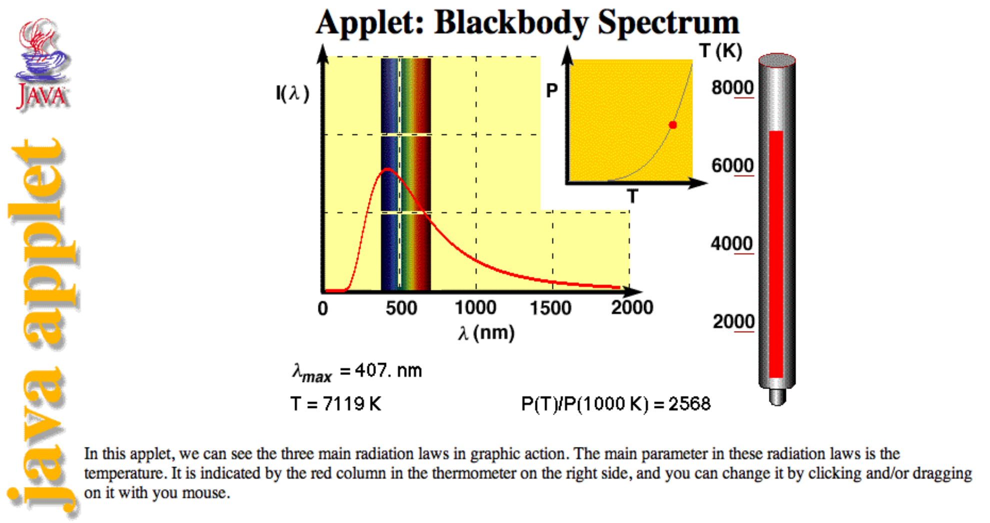
It is not clear what the mathematical shape of either curve, A or B, actually is. Logarithms might resolve this. The more accurate representation of the locus line would be the linear line represented when the data is graphed in a log-log graph (see the sketch on the right, above). The only example of this that I could find was in an online article by Tlaczala. [# 14]

§9. My Search for Data

The first simulation I considered was from the Physics Educations Technology web site (PhET). [# 15] This interactive simulation was found at the University of Colorado at Boulder web site. The measurements from this model required reading an analogue scale and hence offered low quality data. So I moved on. The next simulation I considered was found at the Open Source Physics [# 16] web site. This simulation worked well, and the data was more precise than the last simulation, but was not easy to read the values so I keep searching. Overall, I found a total of 13 simulations.

§10. Data Generation Applet and Data

I finally used the Applet I found among many excellent Java Applets produced by Professor Bauer (in 1999) at the Michigan University web site for online lectures. [#17]



http://lectureonline.cl.msu.edu/~mmp/applist/blackbody/black.htm

I recorded the temperature in Kelvin (K) and the wavelength in nanometers (nm). I used the spreadsheet from Vernier's ***LoggerPro 3.8.4*** software [#18] for graphing and for calculating the reciprocal of the wavelength.

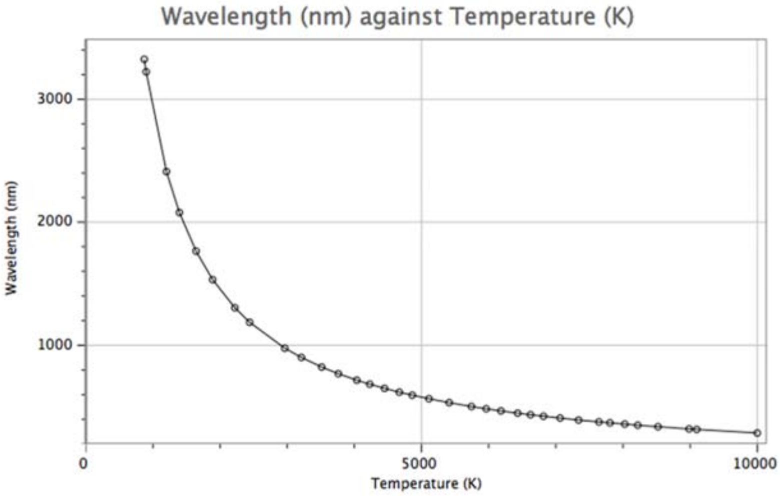
I recorded temperatures from 10 kK to down to 871 K with separate 35 measurements. This should be more than enough data points The least count for temperature is ±1K and for wavelength it is ±1 nm. As this is computer-generated data, I will not process the uncertainties (and just make use of the best straight line graph error compared to the accepted value). Because the data came from a computer simulation there was no need to take repeated measurements.

Data Set

	Temperature T (K)	λ (nm)	λ^{-1} (nm^{-1})
1	10000	289	0.00346021
2	9099	318	0.00314465
3	8985	322	0.00310559
4	8522	340	0.00294118
5	8220	352	0.00284091
6	8027	361	0.00277008
7	7807	371	0.00269542
8	7642	379	0.00263852
9	7339	394	0.00253807
10	7064	410	0.00243902
11	6816	425	0.00235294
12	6623	437	0.00228833
13	6431	450	0.00222222
14	6183	468	0.00213675
15	5963	485	0.00206186
16	5743	504	0.00198413
17	5412	535	0.00186916
18	5110	567	0.00176367
19	4862	596	0.00167785
20	4669	620	0.00161290
21	4449	651	0.00153610
22	4229	685	0.00145985
23	4036	717	0.00139470
24	3761	770	0.00129870
25	3513	824	0.00121359
26	3211	902	0.00110865
27	2963	977	0.00102354
28	2440	1187	0.00084246
29	2220	1305	0.00076628
30	1889	1533	0.00065232
31	1642	1764	0.00056689
32	1394	2078	0.00048123
33	1201	2411	0.00041477
34	899	3223	0.00031027
35	871	3325	0.00030075

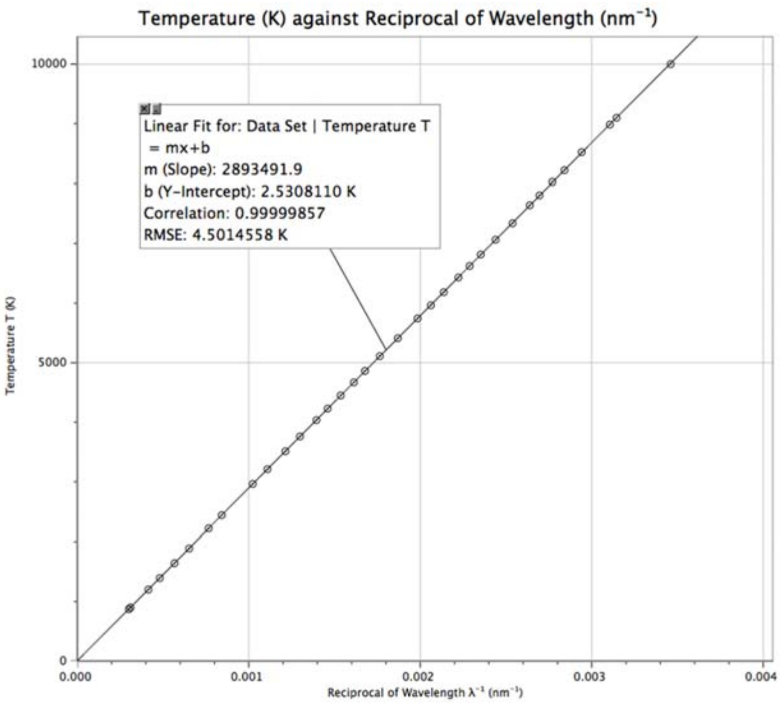
§11. Graph of Wavelength against Temperature

This graph shows that as the temperature increases the peak wavelength decreases.



To find the inverse proportionality constant behind this graph I next graphed the temperature against the reciprocal the peak wavelengths.

§12. Graph of the Temperature against the Reciprocal the Peak Wavelength



This graph is beautifully linear and has only a 2.5 K systematic shift off from being proportional. Perhaps that is the temperature of the universe! The gradient of this graph is used in the conclusion.

§13. Conclusion

The gradient of the above graph is given as 2893491.9 nm K. A nanometer (nm) is 10^{-9} metres. **Hence in SI units the gradient is 0.0028934919 m K.**

A theoretical value of the proportionally constant is **0.0028977685 m K** and the uncertainty for this is only in the last two significant figures. [#19]

My result, when compared to the official value, is only about 0.15% off. Given that I had only three and four significant figures for the wavelength measurements, the 0.15% error is almost amazing. If I had propagated uncertainties based on no other information than significant figures and least count, my gradient would be good to only about 0.4%. I suspect that the error here, if it can be considered an error or uncertainty, is due to the limit of significant figures and the rounding effect used in the computer programming. The standard deviation generated by the graphing program is insignificant here. The *y*-intercept is only 2.5K, probably due to rounding errors. As a percentage it is insignificant.

Because this entire investigation is based on a computer simulation my determination of Wien's constant is an academic exercise, and has no physical meaning other than uncovering information in the computer model that generated the data. Still, I learned an important lesson. By understanding Wien's laws and how the locus of peak wavelength should have been drawn, I was able to correct the ambiguities expressed in some textbooks. This alone made the experiment worthwhile.

I can see no significant improvements for this investigation. No refinement of data, more precision or more values, would change the effect of determining a relationship embedded in the equations that generate the data. The uncertainty of the data was the least count on the readouts, and I motioned in the text why I ignored these.

An interesting extension of my study would be to resolve my original observation about conflicting textbook representation of intensity and wavelength. I would remake all my measurements as before but also record the value of the peak intensity, and then a log-intensity against log-wavelength graph would establish the correct intensity against wavelength graph. With more time I would do this, or in retrospect I could have done this too.

§14. FOOTNOTES

#1: “Physics: Principles with Applications” by Dogulas Giancoli (Prentice Hall, 6th edition, 2005), Pages 756–758.

#2: http://www.nobelprize.org/nobel_prizes/physics/laureates/1911/press.html

3: http://www.nobelprize.org/nobel_prizes/physics/laureates/1911/wien-bio.html

4: http://en.wikipedia.org/wiki/File:Wiens_law.svg

#5: Physics” by Halliday and Resnick, Physics, Parts I and II (Wiley & Sons, 1978), Pages 1091–1092.

#6: “Quantum Physics: Berkeley Physics Course—Volume 4” by Eyvind Wichmann, (McGraw-Hill Publishing Company, 1971), Pages 23–24.

#7: “Quantum Physics” Berkeley Physics Course Volume 4” by E. H. Wichmann (McGraw-Hill Publishing, 1971), page 24.

#8: “Physics” by Halliday and Resnick, Physics, Parts I and II (Wiley & Sons, 1978), Page 1092.

#9: Hyperphyiscs is found at http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html.
Also see the article “Obtaining Wien’s Displacement Law form Planck’s Law of Radiation” by Biman Das, in the journal *The Physics Teacher,* Volume 40, March 2002. Other online derivations can be found at the following web sites:
http://scienceworld.wolfram.com/physics/WiensDisplacementLaw.html
and http://www.wolframalpha.com

#10: Graph copied from “Physics: Principles with Applications” by Douglas Giancoli (5th edition, Prentice Hall Publishers), page 827, figure 27-4.

#11: http://en.wikipedia.org/wiki/Wien's_displacement_law

#12: “Physics For Use In The IB Diploma Program” by Greeg Keer and Paul Ruth (2rd edition, IBID Press), page 605, and in “Astrophysics: University of Bath Science 16–19” by Nigel Ingham (Nelson Publications), page 76. And, by extrapolation, line A can be found in “Physics for the IB Diploma” by K.A. Tsokos (5th edition, Cambridge University Press), page 497.

#13: “Physics: Algebra/Trig” by Eugene Hecht (2nd edition, Brooks/Cole Publishing Company), page 1022.

#14: “Computer simulation and modeling in virtual physics” by W. Tlaczala published in *Current Developments in Technology-Assisted Education* (2006, page 1199).

#15: http://phet.colorado.edu/en/simulation/blackbody-spectrum

#16: http://www.opensourcephysics.org/
http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=1037.0
http://www.compadre.org/OSP/document/ServeFile.cfm?ID=10528&DocID=2030

#17: http://lectureonline.cl.msu.edu/~mmp/applist/blackbody/black.htm

#18: http://www.vernier.com

#19: See the International Council for Science, The Committee on Data for Science and Technology, 2006, quoted on Wikipedia. http://en.wikipedia.org/wiki/Wien's_displacement_law

§ 15. The following web sites were looked at during my research for this physics exploration.

http://jersey.uoregon.edu/cdrom.html

http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=1037.0
http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=427.msg3833#new
http://sydney.edu.au/science/uniserve_science/school/curric/stage6/phys/physapplets.html
http://www.mhhe.com/physsci/astronomy/applets/Blackbody/frame.html
http://webphysics.davidson.edu/alumni/milee/java/bb_mjl.htm
http://www.yteach.co.uk/page.php/resources/view_all?id=p5_radiation_object_black_body_Stefan_Boltzmann_law_Wien_grey_star_t_page_12&from=search
http://www.uni.edu/morgans/ajjar/Astrophysics/wiens.html
http://itg1.meteor.wisc.edu/wxwise/AckermanKnox/chap2/planck_curve.html
http://www.mhhe.com/physsci/astronomy/applets/Blackbody/frame.html
http://phet.colorado.edu/sims/blackbody-spectrum/blackbody-spectrum_en.html
http://csep10.phys.utk.edu/astr162/lect/light/planck.html
http://lectureonline.cl.msu.edu/~mmp/applist/blackbody/black.htm
http://thermofluids.sdsu.edu/testhome/javaapplets/planckRadiation/blackbody.html
http://cas.sdss.org/dr5/en/proj/basic/color/physlet/blackbody.asp#eq
http://cas.sdss.org/dr5/en/proj/basic/color/physlet/blackbody.asp#eq
http://highered.mcgraw-hill.com/sites/0072482621/student_view0/interactives.html#