Investigating the lift force of a toy helicopter

I am an active member of my school's aeronautics club. We occasionally fly gas powered model aircraft and we spend endless hours at the realistic controls of a computer based flight simulator. I have always been fascinated by flight. Recently my physics teacher bought a neat little toy helicopter[1], one that operates by batteries and infrared remote control. This toy was the stimulus for my required physics investigation. A little helicopter theory research and the help of my teacher soon revealed an equation[2] that gave me the purpose of my experiment, namely, to confirm the relationship between theory and experiment.

PE The student has some interest here. This could turn out to be a good IA.

EX The research project is defined and appropriately extracted from the relevant scientific equation. Under Evaluation, this is the research question to be answered.

As early as 400 BC the Chinese made toy helicopters. In 1483 Leonardo de Vinci designed a working helicopter; we do not know if one was ever made. Then in 1754 the Russian Mikhail Lomonosov developed a helicopter with a motor. The first sustainable flight helicopter was make in 1860 in France, and the design has improved every since. From 1922 to today helicopters have played an important role in all aspects of aviation.[3]

So here is the helicopter equation[4]: $f^2 = \dfrac{F}{8\pi^2\rho\lambda^2R^2} = \dfrac{mg}{8\pi^2\rho\lambda^2R^2}$

The rotational frequency of the rotor blade is denoted *f*. Frequency is measured in hertz (Hz). The lifting force *F* (measured in newton's) is the force required to make a helicopter hover in air with no up or down motion. I think that the constant $8\pi^2$ is derived by the theory of the equation. The density of air ρ, units of kilograms per cubic meter, $\mathrm{kg\,m^3}$. Lambda, λ, is a parameter called the *rotor inflow ratio*. Lambda relates to the flow of air about the rotor. The radius of the rotor blade is *R*. Using Newton's second law of motion, I changed *F* into mg (mass and gravity) in the above equation.

C Here and in a few other places we find minor mistakes, but this does not affect the overall investigation.

By varying the rotor blade frequency (by adjusting the motor power supply) I then measured the lifting effect (on a digital balance). A graph of frequency squared again lifting mass would confirm or deny the equation to my toy helicopter. The other quantities, such as the value of gravity, the density of air, the rotor radius and the physical characteristic of the rotor inflow ratio are all constants, so we can ignore them as far as testing the equation.

Unlike airplanes that have propellers, helicopters have blades. Helicopters are also known as rotary wing aircraft. When helicopter blades are turning they hit the air and air is deflected downward, producing lift. This gives some lift and also reduces air pressure. It is complicated but explained online.[5] With rotating blades the helicopter lifts off the ground or, if forces (weight and lift) are balanced, the helicopter can hover at a fixed distance above the ground.

EX More technical details here would help the quality of the report. Lift is due to both the pushing down of air by the blade and by a partial vacuum produced above the blade. If the student was curious they would have looked into this some more.

When a helicopter is close to the ground the helicopter experiences even more lift, and this is called *ground effect*[6]. This happens because the air is hitting the ground and bounces back. But the helicopter doesn't require this in order to fly. The air is constantly pushing back providing the force needed to create lift. Changing the angle of attack of the blades varies the amount of lift produced, in much the same way you change the angle of attack of your hand as you hold it out the car window speeding down the freeway. Adjusting the entire plane of the rotor controls the direction of the lift. Tilting it forward causes the helicopter to move forward[7].

Here is the Toy Helicopter[8]

I used the *Extech Stroboscope Tachometer* stroboscope[9], model 46180. It has variable frequency and a digital readout. I considered the uncertainty here to be ±1 Hz (one digit of the least count). I adjusted the strobe frequency to obtain a stationary and a single image of the rotor blade.

I used an *OHAUS* digital balance[10] model *Adventurer*™. It has a resolution down to one milligram, so the uncertainty was ±1 mg = ±0.001 g.

The helicopter mass alone was 6.721 g and the helicopter when at rest with the mounting block had a total mass of 172.303 g.

The quantity used in my investigation is the unit of mass, which is not the unit of force, but the lifting force is directly proportional to the calculated lift mass, hence my investigation concerns the 'lift mass' of the helicopter as a function of the rotor frequency. Although the digital balance reads mass to three decimal places, I did not accept measurements with this precision. The reason was that when the helicopter engine was running there was sufficient vibration and noticeable variable airflow and so that the digital balance reading kept changing. As a result, I could only read with confidence (that is, a steady value) to one decimal place. Hence my lift mass uncertainty is limits to ±0.1 g.

EV The student is perceptive with comments here and repeated elsewhere about the quality of the data.

A The student is perceptive here and makes a sound judgment about an appropriate uncertainty.

Analysis.
I took gravity as 9.8 N kg^{-1}.

I took air density as 1.2 kg m^{-1}.

C There is a minor error in the units of density but it does not matter in this case.

The rotor radius was measured to be 6.5 cm.

A Uncertainties here and with gravity and density are not required. Significant figures are not considered, but again they are not relevant.

Here is what I did.

PE It is not clear what the student designed and what the teacher or the journal article did. It turns out that the student just copied the method, so there is little independent thinking or initiative here.

(1) First, I secured the helicopter to a block of wood. My teacher suggested this method. The wood mass was sufficiently large to keep the helicopter on the digital balance even when the helicopter is spinning its rotor at maximum speed. Then I read the balance value without the motor running.

PE It is clear that the teacher has informed much of this investigation, thus restricting PE to levels of insight or creativity (neither of which are obvious through the report).

(2) I next started the helicopter engine by using the remote control. I wore safety goggles and did not stand too close to the experiment. I also made sure other students were not near by for safety sake. When the lowest power was applied, it was impossible to get a low rotation frequency. The motor did not move until it started moving at the lowest frequency, f_{Low}, about 74 Hz. The highest frequency, at the most power, is denoted as f_{High}, about 97 Hz. So that was my range of values. I wanted more.

EX The student took appropriate safety precautions. There are no ethical or environmental issues related here.

(3) The mass lifted at any given frequency was determined by a simple calculation. The lifting mass was: $m_{\text{Lift}} = m_{\text{Stationary}} - m_{\text{Frequency}}$. The lift produced by the helicopter reduces the mass displayed by the electronic balance. Increasing the amount of power produced increasing lift. The mass lifted by the helicopter is the difference between the mass at rest of the helicopter (plus wooden platform) with no power applied and the mass reading when the rotor is spinning. I used a spreadsheet to do the calculations.

A This method is simple, direct and most appropriate.

(4) A graph of frequency squared against lift mass was then used to determine a linear and proportional relationship and confirm the scientific theory.

(5) The physical limits of the helicopter restrict the range of measurable frequencies and lift masses. I obtained only 9 sets of data, but this seems reasonable enough.

A The helicopter characteristics clearly restrict the range of data, and nine values within this range is sufficient. However, repeated measurement would be expected in this situation.

There were a few uncertainties in my experiment.

- The least count on the mass measure was ±0.001 g. However, my uncertainty here was much larger due to the irregular flow of air and the vibrations of the helicopter motor. The digital reading always jumped about so I was only able to obtain a reliable value with an uncertainty of ±0.1 g, so I used this precision for my experiment.

- The strobe light frequency was absolute with the least count of ±0.1 Hz.

Here is my data.

Data Set 1, Basic Data

The mass of the helicopter and the wood mount was a constant mass of 172.3 g. The rotor was started at the lowest possible frequency, 74.1 Hz and increased to a maximum of 97.1 Hz. The corresponding value of the scale measurements were recorded, and the lift mass was calculated simply as the total rest mass minus the reading mass:

	Data Set			
	m(S) (g)	m(M) (g)	Mass Lift (g)	Frequency (Hz)
1	172.3	166.0	6.3	74.1
2	172.3	165.5	6.8	76.3
3	172.3	165.0	7.3	80.1
4	172.3	164.5	7.8	82.4
5	172.3	164.0	8.3	84.2
6	172.3	163.5	8.8	87.7
7	172.3	163.0	9.3	89.3
8	172.3	162.5	9.8	91.3
9	172.3	161.5	10.8	97.1

A Raw and processed data are presented in a clear and reasonable manner. Some uncertainties are mentioned in the text.

$$m_{\text{Lift}} = m_{\text{Stationary}} - m_{\text{Frequency}}$$

Data Set 2, Graphing Data

Here is the data for my graph: lift mass, frequency, and then frequency squared.

	Lift Mass (g)	Frequency (Hz)	f^2 (Hz^2)
1	6.3	74.1	5490.8
2	6.8	76.3	5821.7
3	7.3	80.1	6416.0
4	7.8	82.4	6789.8
5	8.3	84.2	7089.6
6	8.8	87.7	7691.3
7	9.3	89.3	7974.5
8	9.8	91.3	8335.7
9	10.8	97.1	9428.4

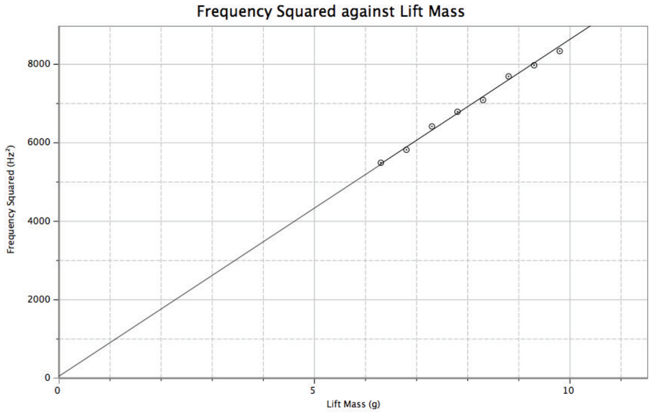
EX The data collection, processing and graph are all relevant to the research question. The standard of error analysis is low but the overall technique is most appropriate.

A There are no uncertainties mentioned here, and they are important.

Next I consider a graph[11] of frequency squared again mass lifted, as the original theory suggested. All the other equation values are constants.

A The graph is relevant and appropriately presented. However, uncertainty bars are missing. Apparently the computer generated the gradient and y-axis intercept but these are not shown.

Frequency Squared against Lift Mass



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Looking at my graph it is clear that the square of frequency is related to the lift mass. The graph line is reasonably understood as being linear because it touches all the data points in a straight line, and it touches the original so I can say the relationship is also proportional.

The computer said that the gradient was 856.8 ± 22.86 Hz^2 / g. This means my conclusion is justified to $\frac{\Delta m}{m} \times 100\% = \frac{22.86}{856.8} \times 100\% = 2.67\%$. My teacher said this error of about 3% was not bad at all, given the uncertainties, so I am happy with my results. The theory is correct.

Improvements and extension ideas include the following:

- The frequency range of possible lift values was limited by the remote control. Perhaps a more powerful power supply would allow me to extend the range but I would not want to burn out the motor.

- The constant vibration of the helicopter on the balance and the variation of air currents restricted the precision of mass life measurements. Perhaps a smooth running motor could be mounted on the block and then higher quality data with a wider range of values could be obtained.

- If I had more time I would repeat the experiment several times and take an average, but I ran out of time as my teacher said I played around too much with the helicopter and there was no more class time to work on the IA.

- A further investigation or extension would include the study of wind power generators.

Overall I enjoyed working with the toy helicopter, especially just flying it about the physics lab. Perhaps some day I will be a pilot.

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EV Although the student is correct here, more thought and detailed appreciation is expected for a conclusion.

A The data is good but more detail is expected in an IA. For instance, the offset of the zero-zero origin and the uncertainty there would have helped establish a proportional relationship.

A The student appreciates the quality of the revealed relationship with the calculation of uncertainty here, presumably based on the standard deviation of all the data points and the best straight line.

EV This error, or rather uncertainty, is indeed good. Given the data, method and materials, perhaps it is even excellent.

EV We know nothing of the theory, only that the given equation is appropriate for the limited data range. More physics context would have helped here. It is hard to find any justification here, other than good quality data on the graph. More depth is required for a justification.

EV The student's brief comments here are appropriate, although the wind power extension means nothing as it stands.

FOOTNOTE REFERENCES

(1) The toy helicopter is an AH-64 *Apache Havoc*™ made by Air Hogs RC.

(2) There are several sources of this information. See Wayne Johnson, "Helicopter Theory" (Dover Books, 1994), the web site: http://www.ultraligero.net/Cursos/helicoptero/modern_helicopter_aero dynamics.pdf as well as the article "Investigating Flight with a Toy Helicopter" by Michael Liebl, *The Physics Teacher*, Volume 48, October 2010, pages 458-460.

(3) For much more history about this see the web article: http://terpconnect.umd.edu/~leishman/Aero/history.html

(4) Page 25 in "Helicopter Theory" (Dover Books, 2012) by Wayne Johnson

(5) http://mitchellscience.com/physics

(6) http://answers.yahoo.com/question/index?qid=20080927232141AAPOnzf

(7) See the online Rotorcraft Flying Hanbook at: http://www.faa.gov/library/manuals/aircraft/media/faa-h-8083-21.pdf

(8) The toy helicopter is an AH-64 Apache Havoc™ made by Air Hogs RC. It runs on a rechargeable battery and is controlled by an infrared remote control unit. See http://www.air-hogs-helicopter.net/air-hogs-helicopter-air-hogs-rc-ah-64-apa che-havoc-heli-indoor-infrared-micro-helicopter/

(9) http://www.extech.com/instruments/resources/manuals/461830_831_um.pdf

(10) http://www.ohausadventurerpro.discountscales.com

(11) My graph was made using *LoggerPro* V 3.8.4 software from Vernier, see: http://www.vernier.com/