

Errors and uncertainties in physics internal assessment

The treatment of errors and uncertainties is directly relevant in the internal assessment of:

- data collection and processing, aspects 1, 2 and 3 (recording raw data, processing raw data, and presenting processed data)
- conclusion and evaluation, aspects 1 and 2 (concluding, and evaluating procedure(s))—a reasonable interpretation, with justification, may include the appreciation of errors and uncertainties, and evaluation of procedures may, if relevant, include the appreciation of errors and uncertainties.

The **core** physics syllabus covers errors and uncertainties in the following section of the *Physics guide* (2007):

- Measurement and uncertainties (topic 1.2).

Both standard and higher level students are to be assessed by the same syllabus content and the same assessment criteria.

Expectations at standard level and higher level

All physics students are expected to deal with uncertainties throughout their investigations. Students can make statements about the minimum uncertainty in raw data based on the least significant figure in a measurement. They can calculate the uncertainty using the range of data in a repeated measurement, and they can make statements about the manufacturer's claim of accuracy. Students can estimate uncertainties in compound measurements, and can make educated guesses about uncertainties in the method of measurement. If uncertainties are small enough to be ignored, the student should note this fact.

Students may express uncertainties as absolute, fractional, or percentages. They should be able to propagate uncertainties through a calculation—addition and subtraction, multiplication and division, as well as squaring and trigonometric functions.

All students are expected to construct, where relevant, uncertainty bars on graphs. In many cases, only one of the two axes will require such uncertainty bars. In other cases, uncertainties for both quantities may be too small to construct uncertainty bars. A brief comment by the student on why the uncertainty bars are not included is then expected. If there is a large amount of data, the student need only draw uncertainty bars for the smallest value datum point, the largest value datum point, and several data points between these extremes. Uncertainty bars can be expressed as absolute values or percentages.

Arbitrary or made-up uncertainty bars will not earn the student credit. Students should be able to use the uncertainty bars to discuss, qualitatively, whether or not the plot is linear, and whether or not the two plotted quantities are in direct proportion. In respect of the latter, they should also be able to recognize if a systematic error is present.

Using the uncertainty bars in a graph, students should be able to find the minimum and maximum slopes, and then use these to express the overall uncertainty range in an experiment.

Qualitative and quantitative comments about errors and uncertainties may be relevant in the data collection and processing criterion, aspect 1. Qualitative comments might include parallax problems in reading a scale, reaction time in starting and stopping a timer, random fluctuation in the read-out, or difficulties in knowing just when a moving ball passes a given point. Students should do their best to quantify these observations. For example, one student measured a voltage from an unstable power supply, and wrote the following qualitative and quantitative comments:

The voltage varied slightly over time; it went up and down by several hundredths of a volt. Therefore, the values recorded have an uncertainty greater than the least significant digit of each measurement. The uncertainty was estimated to be more like ± 0.04 V.

Interpreting the relevant assessment criteria

Data collection and processing: Aspect 1 (recording raw data)

Table 1: DCP aspect 1 = “complete”	Table 2: DCP aspect 1 = “partial”																																												
<p>For a complete, students need to present raw data in a clear and comprehensible way, including the name of the quantities, the symbols and units, and an estimated raw uncertainty for each raw data quantity (table 1). Uncertainties are always relevant in raw data, even if they are small enough to ignore.</p>	<p>For a partial, students need to present raw data in an appropriate manner, but there may be some mistakes or omissions. In the example in table 2, awarded a partial, the student again records raw data appropriately in a table, but the symbols are not given, there are no estimated uncertainties and the raw data is recorded with an inconsistent number of significant figures.</p>																																												
<p>Table 1</p> <table border="1" data-bbox="181 685 580 1568"> <thead> <tr> <th>Voltage V/V $\Delta V = 0.0$ V</th> <th>Current I/mA $\Delta I = \pm 0.3$ mA</th> </tr> </thead> <tbody> <tr><td>1.00</td><td>0.9</td></tr> <tr><td>2.00</td><td>2.1</td></tr> <tr><td>3.00</td><td>2.8</td></tr> <tr><td>4.00</td><td>4.1</td></tr> <tr><td>5.00</td><td>5.0</td></tr> <tr><td>6.00</td><td>5.9</td></tr> <tr><td>7.00</td><td>7.1</td></tr> <tr><td>8.00</td><td>8.0</td></tr> <tr><td>9.00</td><td>8.9</td></tr> <tr><td>10.0</td><td>9.9</td></tr> </tbody> </table>	Voltage V/V $\Delta V = 0.0$ V	Current I/mA $\Delta I = \pm 0.3$ mA	1.00	0.9	2.00	2.1	3.00	2.8	4.00	4.1	5.00	5.0	6.00	5.9	7.00	7.1	8.00	8.0	9.00	8.9	10.0	9.9	<p>Table 2</p> <table border="1" data-bbox="817 685 1241 1478"> <thead> <tr> <th>Voltage / V</th> <th>Current / mA</th> </tr> </thead> <tbody> <tr><td>1</td><td>0.9</td></tr> <tr><td>2</td><td>2.1</td></tr> <tr><td>3</td><td>2.8</td></tr> <tr><td>4</td><td>4.1</td></tr> <tr><td>5</td><td>5</td></tr> <tr><td>6</td><td>5.9</td></tr> <tr><td>7</td><td>7.1</td></tr> <tr><td>8</td><td>8</td></tr> <tr><td>9</td><td>8.9</td></tr> <tr><td>10</td><td>9.9</td></tr> </tbody> </table>	Voltage / V	Current / mA	1	0.9	2	2.1	3	2.8	4	4.1	5	5	6	5.9	7	7.1	8	8	9	8.9	10	9.9
Voltage V/V $\Delta V = 0.0$ V	Current I/mA $\Delta I = \pm 0.3$ mA																																												
1.00	0.9																																												
2.00	2.1																																												
3.00	2.8																																												
4.00	4.1																																												
5.00	5.0																																												
6.00	5.9																																												
7.00	7.1																																												
8.00	8.0																																												
9.00	8.9																																												
10.0	9.9																																												
Voltage / V	Current / mA																																												
1	0.9																																												
2	2.1																																												
3	2.8																																												
4	4.1																																												
5	5																																												
6	5.9																																												
7	7.1																																												
8	8																																												
9	8.9																																												
10	9.9																																												

Table 3: DCP aspect 1 = “not at all”		
<p>A student may earn a not at all if they forget to record any raw data or if the presentation and details are incomprehensible or if essential information is missing such as units (table 3).</p>		
<p>Table 3</p> <table border="1" data-bbox="201 1832 1362 1980"> <thead> <tr> <th>Raw Data: Voltage and Current</th> </tr> </thead> <tbody> <tr> <td>1 @ 0.9, 2 @ 2.1, 3 @ 2.8, 4 @ 4.1, 5 @ 5, 6 @ 5.9, 7 @ 7.1, 8 @ 8, 9 @ 8.9, 10 @ 9.9</td> </tr> </tbody> </table>	Raw Data: Voltage and Current	1 @ 0.9, 2 @ 2.1, 3 @ 2.8, 4 @ 4.1, 5 @ 5, 6 @ 5.9, 7 @ 7.1, 8 @ 8, 9 @ 8.9, 10 @ 9.9
Raw Data: Voltage and Current		
1 @ 0.9, 2 @ 2.1, 3 @ 2.8, 4 @ 4.1, 5 @ 5, 6 @ 5.9, 7 @ 7.1, 8 @ 8, 9 @ 8.9, 10 @ 9.9		

Data collection and processing: Aspect 2 (processing raw data)

Data processing is usually understood as combining and manipulating raw data to determine the value of a physical quantity. Often raw data is multiplied or divided, added or subtracted from other values or constants. When this is done, errors and uncertainties should be propagated. However, there are cases where the raw data is appropriate for graphing and for establishing a conclusion. For example, in an Ohm's law experiment, voltages and currents may be recorded and graphed. In such cases processing will be understood as transferring the data to an appropriate graph, constructing a best-fit line and determining the slope. Students will not be penalized under aspect 2 if their investigation is of this type. The processing of uncertainty consists in correctly constructing the relevant uncertainty bars on the graph and correctly calculating the slope of the graph.

When students process data by product or quotient, sum or difference, or some other mathematical function, such as averaging, then how well the student processes the raw data determines assessment under aspect 2.

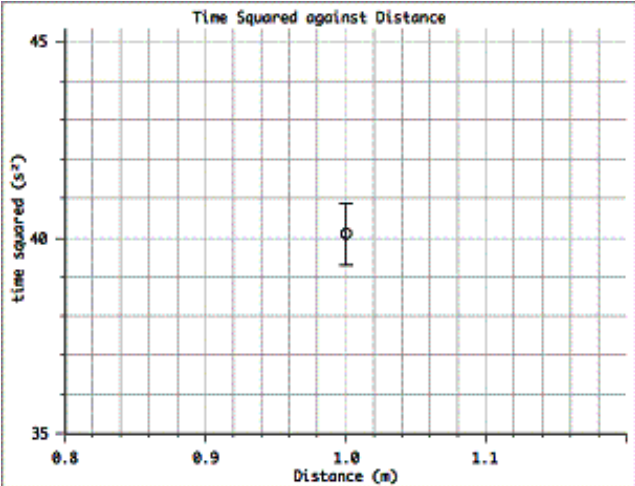
Table 4: DCP aspect 2 = "complete"	Example 1: DCP aspect 2 = "complete"						
<p>In the example in table 4, the student finds the average of three trial measurements of the time it takes for a ball to roll down a 1.00 m inclined plane. She clearly and correctly calculates the average time and the uncertainty thus earning a complete.</p>	<p>In this next example, the student calculates the square of the average time for three trial runs as shown above and also determines the uncertainty. Again, the student earns a complete.</p>						
<p>Table 4</p> <table border="1" data-bbox="178 913 730 1164"> <thead> <tr> <th>Distance s / m $\Delta s = \pm 0.00$ m</th> <th>Time t / s $\Delta t = \pm 0.01$ s</th> <th>Average time t_{ave} / s $\Delta t_{ave} = \pm 0.06$ s</th> </tr> </thead> <tbody> <tr> <td>1.00</td> <td>6.28 6.39 6.31</td> <td>6.33</td> </tr> </tbody> </table> $t_{ave} = \frac{t_1 + t_2 + t_3}{3} = \frac{(6.28 + 6.39 + 6.31) \text{ s}}{3} \approx 6.33 \text{ s}$ $\Delta t_{ave} = \frac{\text{range}}{2} = \frac{t_{max} - t_{min}}{2} = \frac{(6.39 - 6.28) \text{ s}}{2} \approx 0.06 \text{ s}$ $t_{ave} \pm \Delta t_{ave} = (6.33 \pm 0.06) \text{ s}$	Distance s / m $\Delta s = \pm 0.00$ m	Time t / s $\Delta t = \pm 0.01$ s	Average time t_{ave} / s $\Delta t_{ave} = \pm 0.06$ s	1.00	6.28 6.39 6.31	6.33	<p>Example 1</p> <p>The average time and uncertainty is: $t_{ave} \pm \Delta t_{ave} = (6.33 \pm 0.06) \text{ s}$ The uncertainty in average time as a percentage: $\Delta t_{ave} \% = \frac{0.06}{6.33} \times 100 \approx 1\%$</p> <p>The average time squared is: $t_{ave}^2 = (6.33 \text{ s})^2 \approx 40.1 \text{ s}^2$</p> <p>The uncertainty in time squared is: $\Delta t_{ave}^2 \% = 2 \times 1\% \approx 2\%$</p> <p>The average time squared and its uncertainty is thus: $t_{ave}^2 \pm \Delta t_{ave}^2 = 40.1 \text{ s}^2 \pm 2\% = (40.1 \pm 0.8) \text{ s}^2$</p> <p>The datum and its uncertainty are now correctly processed as an uncertainty bar on a graph (see aspect 3) of time squared against distance (figure 1).</p> 
Distance s / m $\Delta s = \pm 0.00$ m	Time t / s $\Delta t = \pm 0.01$ s	Average time t_{ave} / s $\Delta t_{ave} = \pm 0.06$ s					
1.00	6.28 6.39 6.31	6.33					

Table 5: DCP aspect 2 = “partial”

In the next example the student again finds the average of three trial measurements of the time it takes for a ball to roll down a 1.00 m inclined plane but expresses the average with too many significant figures (table 5) and does not appreciate the propagation of uncertainty. This earns a **partial**.

Table 6: DCP aspect 2 = “not at all”

Finally, earning a **not at all**, a student could either fail to show any processing of data or processes it incorrectly, as shown in table 6.

Table 5

Distance s / m	Time t / s $\Delta t = \pm 0.01$ s	Average time t_{ave} / s $\Delta t_{ave} = \pm 0.01$ s
1.00	6.28 6.39 6.31	6.3266

The average time and its uncertainty is:

$$t_{ave} \pm \Delta t_{ave} = (6.3266 \pm 0.01) \text{ s}$$

Next the student calculates the square of the average time.

The average time squared is:

$$t_{ave}^2 = (6.3266 \text{ s})^2 = 40.02586 \text{ s}^2 \approx 40.03 \text{ s}^2$$

Then the student simply carries forward the raw data uncertainty, which is incorrect:

$$t_{ave}^2 \pm \Delta t_{ave}^2 \% = (40.03 \pm 0.01) \text{ s}^2$$

The error bar is insignificant on the graph (figure 2), but this is a mistake due to incorrect data processing.

Figure 2

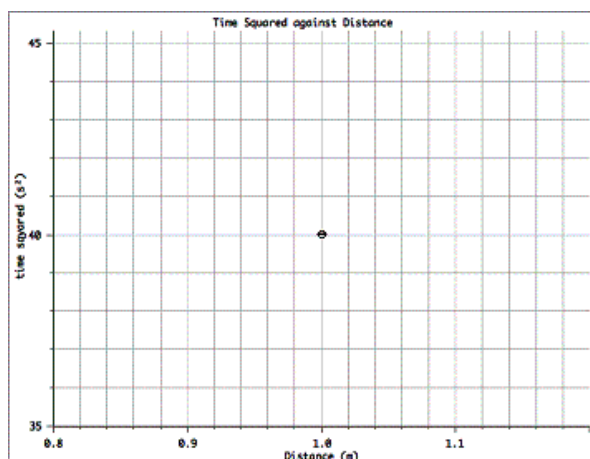


Table 6

Distance s / m	Time t / s	Average time t_{ave} / s
1.00	6.28 6.39 6.31	6.32666

Next the student calculates (but incorrectly records) the square of the average time.

The average time squared is:

$$t_{ave}^2 = (6.32666 \text{ s})^2 = 38.9439 \text{ s}$$

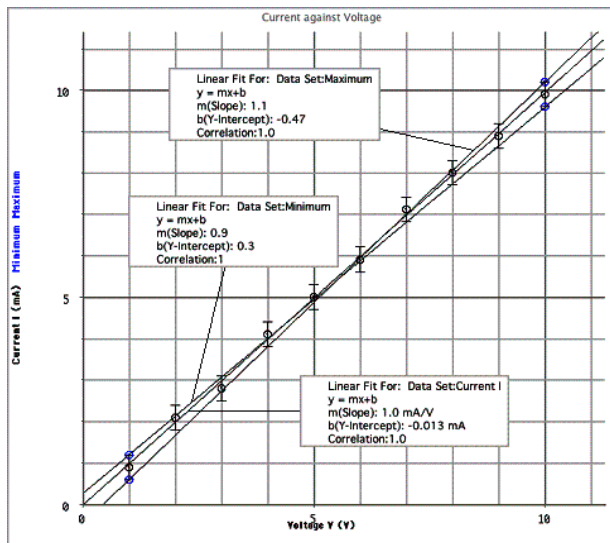
There is a major error in the square of the average time. No uncertainties are appreciated here and the data is transferred to the graph as a data point.

Data collection and processing: Aspect 3 (presenting processed data)

Figure 3: DCP aspect 3 = “complete”

A student constructs a graph of current against voltage in an Ohm’s law experiment (figure 3). He uses the slope of the graph and the uncertainties in the current to establish the resistance and its uncertainty. The information has been correctly processed and presented. This example earns a **complete**.

Figure 3



The computer generates the best-fit line with a gradient $m = 1.0 \text{ mA V}^{-1}$.

The resistance is then calculated with this value:

$$R = \frac{V}{I} = \frac{\Delta V}{\Delta I} = \frac{1}{\frac{\Delta I}{\Delta V}} = \frac{1}{m} = \frac{1}{1.0 \times 10^{-3} \text{ A V}^{-1}} = 1.0 \times 10^3 \Omega = 1.0 \text{ k}\Omega$$

The minimum and maximum experimental values of resistance are calculated based on the uncertainty bars for current using the first and last data points:

$$R_{\max} = \frac{1}{m_{\min}} = \frac{1}{0.9 \times 10^{-3} \text{ A V}^{-1}} = 1.1 \text{ k}\Omega$$

$$R_{\min} = \frac{1}{m_{\max}} = \frac{1}{1.1 \times 10^{-3} \text{ A V}^{-1}} = 0.9 \text{ k}\Omega$$

The resistance uncertainty is then:

$$\Delta R = \pm \frac{R_{\max} - R_{\min}}{2} = \pm \frac{(1.1 - 0.9) \text{ k}\Omega}{2} = \pm 0.1 \text{ k}\Omega$$

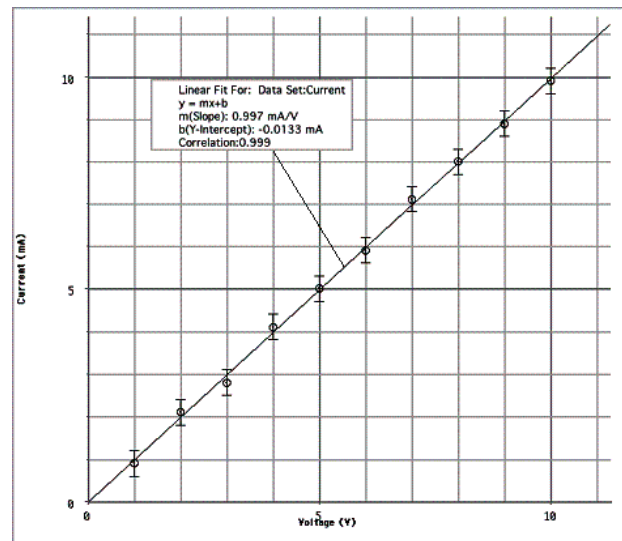
The overall resistance and its uncertainty are thus:

$$R \pm \Delta R = (1.0 \pm 0.1) \text{ k}\Omega.$$

Figure 4: DCP aspect 3 = “partial”

The same data is used but this time the student earns only a **partial**. He has correctly constructed the graph and included error bars (figure 4). He has failed to determine maximum and minimum slopes using the uncertainties in the current. As a result he has not been able to determine the range in the resistance values and hence the uncertainty in the calculated value of resistance.

Figure 4



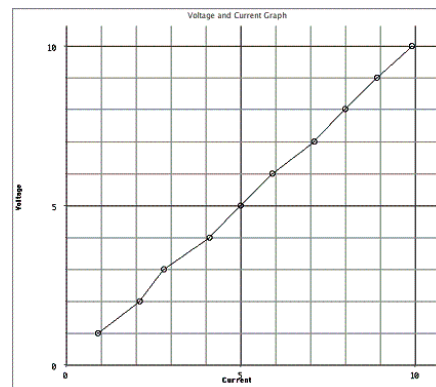
The resistance is

$$R = \frac{V}{I} = \frac{\Delta V}{\Delta I} = \frac{1}{\text{slope}} = \frac{1}{0.99 \text{ mA V}^{-1}} \approx 1.0 \text{ k}\Omega$$

Figure 5: DCP aspect 3 = “not at all”

Next, the student draws an inappropriate graph with major errors (figure 5), thus earning a **not at all**.

Figure 5



$$\text{Resistance} = R = \frac{V}{I} = \frac{10}{10} = 1 \Omega$$

Conclusion and evaluation: Aspects 1 and 2 (concluding and evaluating procedure(s))

Errors and uncertainties are often relevant in aspects 1 and 2 of the conclusion and evaluation criterion because students are expected to reach a reasonable and justified interpretation of the data, and to appreciate the quality of the procedure (producing a measure of precision and accuracy).

CE aspect 1 = “complete”	CE aspect 1 = “partial”
<p>After constructing the graph of current against voltage (see figure 3 in data collection and processing, aspect 3, above), the student makes a reasonable and justified interpretation of the data when they state the following.</p> <p style="padding-left: 40px;">The best straight line clearly lies well within the range of the uncertainty bars and passes nearly through the origin, thus establishing a linear and proportional relationship. Using the gradient of the graph the resistance is $R = 1.0 \text{ k}\Omega$. Using the minimum and maximum gradients I am confident that the resistance is $R \pm \Delta R = (1.0 \pm 0.1) \text{ k}\Omega$. For the given range of voltage and current, the relationship is constant and obeys Ohm’s law.</p> <p>This conclusion would earn a complete for aspect 1 of conclusion and evaluation. The student may refer to a theory or hypothesis in the conclusion, but such reference is not required to earn a complete.</p>	<p>Consider the current against voltage graph as shown in figure 4 in data collection and processing, aspect 3, which earned a “partial”. Although the graph shows uncertainty bars, they are not used to establish a conclusion. Only the gradient of the graph is used, and no comment is given about the quality of the data. The student wrote the following:</p> <p style="padding-left: 40px;">The graph is a straight line through the origin so for this given resistor we find the resistance as 1000 ohms. This is a good result.</p> <p>If there is no justification of the limits of the data, then the conclusion earns a partial. A reasonable interpretation is really an incomplete interpretation.</p>
	CE aspect 1 = “not at all”
	<p>The student whose graph is shown above in figure 5 under data collection and processing, aspect 3, which earned a “not at all”, has no appreciation of errors or uncertainties and does not construct a best straight line. Moreover, the student incorrectly calculates the resistance as 1 ohm.</p> <p style="padding-left: 40px;">The graph is good; it gives me a resistance of exactly 1 ohm. The experiment was a success.</p>

When attempting to measure an already known and accepted value of a physical quantity, such as the charge of an electron or the wavelength of a laser light, students need to appreciate whether or not the accepted value lies within the experimental value range.

Perhaps a student conducts the Young’s double-slit experiment and determines that the laser light wavelength is 610 nm. With experimental uncertainty, the student decides that $\lambda_{\text{exp}} \pm \Delta\lambda_{\text{exp}} = (6.1 \pm 0.2) \times 10^2 \text{ nm}$. The manufacturer’s literature that came with the laser gives a wavelength of $\lambda = 632.8 \text{ nm}$. The student might write the following:

The accepted value is $6.328 \times 10^2 \text{ nm}$ while my experimental value is $(6.1 \pm 0.2) \times 10^2 \text{ nm}$. The accepted value lies just outside the experimental range, which is from $5.9 \times 10^2 \text{ nm}$ to $6.3 \times 10^2 \text{ nm}$. My estimation of errors and uncertainties needs to be re-examined. Nonetheless, my results are close to the accepted value, about 4% too low.

In addition to the above comment, students may also comment on errors in the assumptions of any theory being tested, and errors in the method and equipment being used. For example:

- Perhaps a graph of voltage against current does not form a linear and proportional line. It may be that the load resistance is changing as the current changes, so an ohmic relationship does not hold.
- Measuring the magnetic field alongside a current-carrying wire may confirm the inverse relationship, but for the smallest distances and the largest distances the data does not line up. The induction coil has a finite size, and the centre of it is assumed to be zero. This may not be the case. At large distances, the radius is similar in magnitude to the length of the wire, and the inverse law for the magnetic field assumed an infinite wire length.
- When using the sonic detector, the software was not calibrated with the speed of sound first, and so the measured distances were inaccurate. This error was due to an unexamined assumption, but it was appreciated when the experimental results were evaluated.
- The experiment was done to determine the efficiency of an electric motor. As the investigation was carried out, the battery may have lost power. This would have affected the results.

Overall, students can critically appreciate limitations in their experimental results due to assumptions in the theory, in the experimental techniques, and in the equipment used. Qualitative comments, based on a careful reading of graphed results, will guide the student’s criticism.