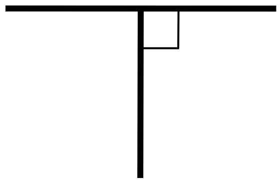
Devil Physics

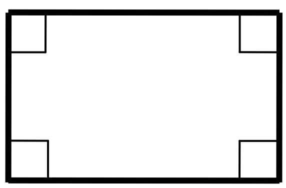
Trigonometry For Vectors

Right Angles

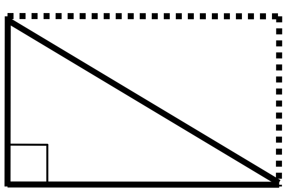


- Right angles are formed when two lines are perpendicular.
- Right angles are equal to 90˚
- Right angles are denoted by a box

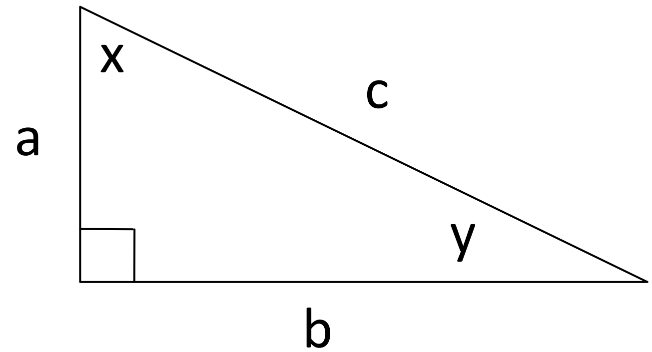
Right Triangles



- Rectangles have 4 right angles
- The total number of degrees in a rectangle is 4x90˚=360˚
- If we cut the rectangle in half, we have a triangle



- The triangle has half the area and half the number of degrees of a rectangle, thus
- The total number of degrees in a triangle is always 180˚
- Here we have a ***right triangle*** because it contains one right angle (can't have more than one)
- Since the right angle is 90˚, the other two angles must add up to 90˚



- **Hypotenuse:** The hypotenuse is the side opposite the right angle. In this triangle, side c is the hypotenuse.
- The hypotenuse is always the largest side of a right triangle because it is opposite the largest angle.
- The other two sides of the triangle are called, well, sides (for those that may confuse this with the baked beans and cole slaw that come with your order of ribs at Sonny's, you can call them legs, unless of course you ordered chicken . . . in that case you can refer to them as "those other things).

Pythagorean Theorem

- In a right triangle, the sum of the squares of the sides equals the square of the hypotenuse (see above triangle).

$$a^2 + b^2 = c^2$$

Solving for individual sides:

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

Example 1. If side a is 3in and side b is 5in, how long is side c?

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{3^2 + 5^2}$

$c = \sqrt{9 + 25}$

$c = \sqrt{34}$

$c = 5.8in$

Check: $a^2 + b^2 = c^2$

$3^2 + 5^2 = 5.8^2$

$9 + 25 = 34$ ✓

Example. 2. If side b is 8cm and side c is 9cm, how long is side a?

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{9^2 - 8^2}$

$a = 4.1cm$

Example. 3. If side a is 5m and side c is 10m, how long is side b?

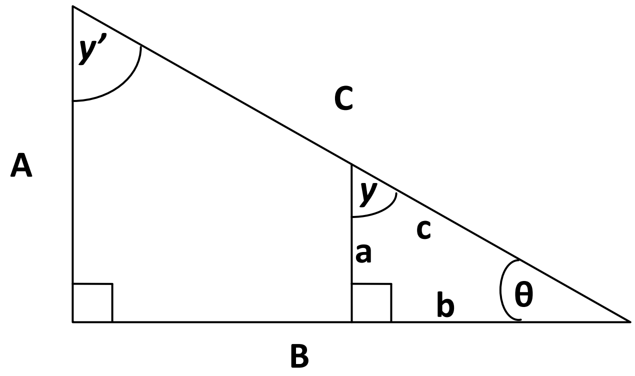
$b = \sqrt{c^2 - b^2}$

$a = \sqrt{10^2 - 5^2}$

$a = 8.7cm$

Proportions In Right Triangles

- In the diagram below, the sides of Δabc have been expanded to form ΔABC. Δabc and ΔABC are similar triangles because they both have one right angle, they both have angle θ as one of their angles, and the third angle is 90˚- θ.



- Since Δabc and ΔABC are similar triangles, they are proportional to each other. As such, the following equalities between proportions exist:
    - $\frac{a}{c} = \frac{A}{C}$
    - $\frac{b}{c} = \frac{B}{C}$
    - $\frac{a}{b} = \frac{A}{B}$
- The value of these ratios are a function of the angle, θ, and **<u>NOT</u>** the lengths of the sides, i.e. no matter what the size of a triangle, if the angles are the same (θ, and 90°-θ), these ratios will be the same in all triangles.
- Likewise, a given ratio of sides corresponds to one and only one angle.
- Therefore, if we know the ratio of two sides, we can determine the value of an angle. Conversely, if we know the angle, we can determine the value of the ratio of two sides. This relationship gives us powerful tools to solve problems involving right triangles which means VECTOR PROBLEMS!!!!

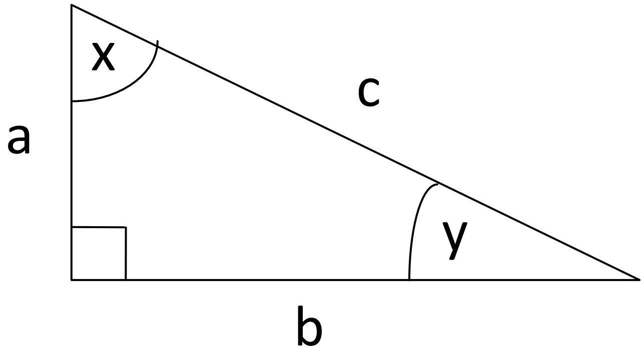
Sine

- We have three functions associated with the proportion of sides in a right triangle: Sine (sin), Cosine (cos) and Tangent (tan).

- **Sine:** The Sine of an angle is equal to the ratio of the side opposite the angle to the hypotenuse. In the triangle below,

$$\sin x = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

$$\sin y = \frac{opposite}{hypotenuse} = \frac{a}{c}$$



Example 4. In the above triangle, a=3cm and b=8cm. Use the Pythagorean Theorem to determine the value of c, then determine the value of sin x and sin y.

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{3^2 + 8^2}$$

$$c = 8.54cm$$

$$\sin x = \frac{b}{c} = \frac{8}{8.54} = 0.9363$$

$$\sin y = \frac{a}{c} = \frac{3}{8.54} = 0.3511$$

**** Your answer may vary slightly based on how you rounded off numbers before performing calculator operations.***

- Note that because it is a ratio, any units associated with the sides cancel out – AS LONG AS THEY ARE THE SAME UNITS!

- Now turn to the inside back cover of your Giancoli textbook for the Trigonometric Table. Go down the Sine column to find the angles associated with the ratios 0.9363 and 0.3511. Because of the rounding, you won't find exact numbers, but find the degree measurement most closely associated with the ratios.

- You should have come up with 69° and 21°. Notice they add to 90° just like they are supposed to!

- Now let's try it with a calculator. Tables and calculators are built to find the ratio of the sides for a given angle, but we are going backwards from the ratio to the angle so we have to use the inverse sine (or arcsine) function. On your calculator:

    - Access the $\sin^{-1}$ function (probably have to press "2nd" first)

    - Once that appears, type in 0.9363, our ratio for sin x.

    - Hit ENTER and you should get 69.44°

    - ***Note: If you get 1.212 instead of 69.44, your calculator is in radians instead of degrees. See the instructions that came with your calculator to switch to degrees.***

    - Do the same for our sin y = 0.3511 and you should get an angle of 20.55°. Notice again that they add to 89.99° which if we use significant figures to account for the lack of precision due to our rounding, we get 90°.

- We can also use our calculators (or the trig table) to find the value of the ratio of the opposite side to the hypotenuse (the sine of the angle) for a given angle. Using your calculator, find the sin of 36° (sin 36°).

- Press SIN
- Type 36
- Press Enter
- You should get 0.5878. This represents the ratio of a side opposite a 36° angle to the hypotenuse in a right triangle.

- Using our basic relationship of $\sin x = \frac{opposite}{hypotenuse}$, we can use Algebra to give us some very useful tools:

$$\sin x = \frac{opp}{hyp}$$

$$(hyp)\sin x = opp$$

$$hyp = \frac{opp}{\sin x}$$

Example 5: One angle of a right triangle is 25°. The hypotenuse is 15m. What is the length of the side opposite the angle?

$$\sin x = \frac{opp}{hyp}$$

$$\sin 25° = \frac{opp}{15\text{m}}$$

$$(15\text{m}) \sin 25° = opp$$

$$(15\text{m})(0.4226) = opp$$

$$6.34m = opp$$

Example 6: One angle of a right triangle is 68°. The side opposite the angle is 42m. What is the length of the hypotenuse?

$$\sin x = \frac{opp}{hyp}$$

$$\sin 68° = \frac{(42m)}{hyp}$$

$$(hyp)(\sin 68°) = \frac{(42m)}{1}$$

$$(hyp) = \frac{(42m)}{(\sin 68°)}$$

$$(hyp) = 45.3m$$

Example 7: In a particular right triangle, the hypotenuse is 22mm and one of the sides is 16mm. What is the measure of the angle opposite the side that is 16mm?

$$\sin x = \frac{opp}{hyp}$$

$$\sin x = \frac{(16mm)}{(22mm)}$$

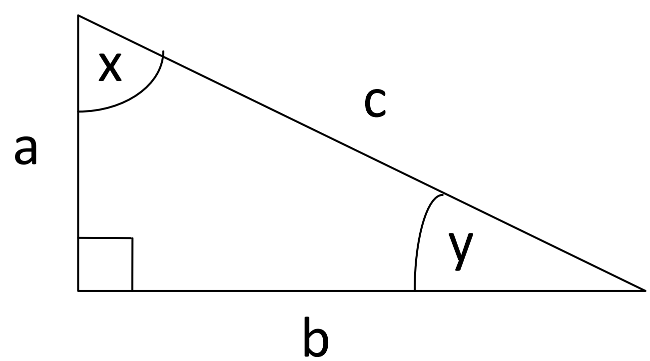
$$x = \sin^{-1} \frac{(16mm)}{(22mm)}$$

$$x = \sin^{-1}(0.7273)$$

$$x = 46.7°$$

Cosine and Tangent

- We have two other relationships to add to our bag of tricks, cosine and tangent. Let's go back to our right triangle to define them.



- **Cosine (cos)** is defined as the ratio of the side adjacent to (next to) the angle to the hypotenuse. Using the right triangle above,

$$\cos x = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

$$\cos y = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

- Cosine can be manipulated algebraically in the same manner as sine.

  $$\cos x = \frac{adj}{hyp}$$

  $$(hyp)\cos x = adj$$

  $$hyp = \frac{adj}{\cos x}$$

- **Tangent (tan)** is defined as the ratio of the side opposite the angle to the side adjacent to (next to) the angle. Using the right triangle above,

$$\tan x = \frac{opposite}{adjacent} = \frac{b}{a}$$

$$\tan y = \frac{opposite}{adjacent} = \frac{a}{b}$$

- Tangent can also be manipulated algebraically in the same manner as sine and cosine.

  $$\tan x = \frac{opp}{adj}$$

  $$(adj)\tan x = opp$$

  $$adj = \frac{opp}{\tan x}$$

- Cosine and tangent can used in the same way as we used sine to find values for sides or angles in a right triangle.
- I could go through the entire procedure for both of these, but I'm like really tired of typing this stuff. Go over the Trigonometry Tool Inventory, ask any questions you have in class, and then give the worksheet a try. With a little practice, it will become second nature.
- Anyone up for a run to Sonny's? I'm hungry.

TURN OVER FOR TRIGONOMETRY TOOL INVENTORY

Trigonometry Tool Inventory

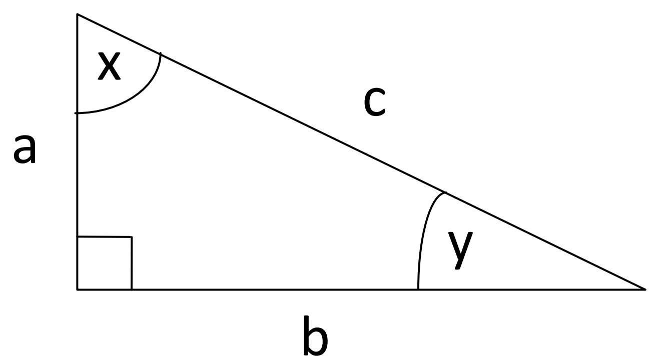
1. Right angles are formed when two lines are perpendicular. Right angles measure 90°.
2. Right triangles have one right angle – can't have more than one. The right angle in a right triangle is identified by a box.
3. The hypotenuse of a right triangle is always the side opposite the right angle. It is also always the longest side of a right triangle.
4. The sum of the angles in any triangle is 180°. Since one of the angles in a right triangle is 90°, the other two must add together to 90°.
5. In a right triangle, ratios of the sides are directly related to the angles of the right triangle. Because of this we have lots of cool stuff to play around with and figure stuff out.
6. The Pythagorean Theorem states that the square of the hypotenuse is equal to the sum or the squares of the other sides. As with any other algebraic equation involving three variables, if we have two of them we can solve for the third. Using our old standby sample triangle,

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$



7. The sine of an angle is equal to the opposite side divided by the hypotenuse

$$\sin x = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

$$\sin y = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

$$\sin x = \frac{opp}{hyp}$$

$$(hyp)\sin x = opp$$

$$hyp = \frac{opp}{\sin x}$$

8. Cosine of an angle is equal to the adjacent side divided by the hypotenuse,

$$\cos x = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

$$\cos y = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos x = \frac{adj}{hyp}$$

$$(hyp)\cos x = adj$$

$$hyp = \frac{adj}{\cos x}$$

9. Tangent of an angle is equal to the opposite side divided by the adjacent side,

$$\tan x = \frac{opposite}{adjacent} = \frac{b}{a}$$

$$\tan y = \frac{opposite}{adjacent} = \frac{a}{b}$$

$$\tan x = \frac{opp}{adj}$$

$$(adj)\tan x = opp$$

$$adj = \frac{opp}{\tan x}$$

10. Mnemonically Speaking:

SOHCAHTOA		
SOH	**CAH**	**TOA**
$\sin x = \frac{opp}{hyp}$	$\cos x = \frac{adj}{hyp}$	$\tan x = \frac{opp}{adj}$