Motion Summary

DISPLACEMENT		
LINEAR	**CIRCULAR**	**ROTATIONAL**

• Straight-line distance from the start point to the end point
• $d = x_f - x_i$
	• Distance around the circle
	• Often as a fraction of the circumference (C = 2πr)
	• Can be found from the number of degree difference between the two positions
	• If an object travels 12° around a circle with a circumference of 10 m, the arc length travelled is $\frac{12}{360}\text{x}10 = 0.3\text{m}$
		• Angular displacement based on number of radians
		• 1 radian is equal to angle subtended by an arc length (*l*) equal to one radius (*r*)
		• 1 revolution = 360° = 2π radians
		• $\theta = \frac{l}{r}$
		• Relationship between angular displacement and linear displacement
		  ○ $x = r\theta$
		  ○ $\theta = \frac{x}{r}$

VELOCITY		
LINEAR	**CIRCULAR**	**ROTATIONAL**

• Defined as displacement per unit time
• $v = \frac{d}{t}$
• Base unit is m/s
	• Tangential velocity (v_{tan}) is the velocity of an object tangent to the circular path and perpendicular to the radius
	• Equal to the circumference (C = 2πr) divided by the period (T, time to complete one revolution
	• $v = \frac{2\pi r}{T}$
	• Base unit is m/s
	• Also equal to the length of an arc segment (*l*) divided by the time it takes to travel that distance
	  • $v = \frac{\Delta l}{\Delta t}$
	• **Period (T)** - time to complete one revolution
	  ○ $T = \frac{time}{revolution}$
	• **Frequency (f)** – number of revolutions per unit time
	  ○ $f = \frac{revolutions}{time}$
	• $f = \frac{1}{T}$ $T = \frac{1}{f}$
		• Angular velocity (ω) defined as the change in angular displacement (in radians) per unit time (rad/s)
		• Sometimes referred to as angular frequency (revolutions in radians per unit time)
		• $\omega = \frac{\Delta\theta}{\Delta t}$
		• Angular velocity from period:
		  ○ Period is time per revolution
		  ○ $\frac{\cancel{\text{revolution}}}{\text{period}}\text{x}\frac{2\pi\text{radians}}{\cancel{\text{revolution}}} = \omega$
		• Angular velocity from frequency:
		  ○ Frequency is revolutions per unit time
		  ○ $\frac{\text{revolution}}{\text{time}}\text{x}\frac{2\pi\text{radians}}{\text{revolution}} = \omega$
		  ○ $\omega = 2\pi f$
		• Relationship between velocity and angular velocity
		  ○ $v = r\omega$
		  ○ $\omega = \frac{v}{r}$

ACCELERATION		
LINEAR	**CIRCULAR**	**ROTATIONAL**
• Defined as the change in velocity per unit time ○ $a = \frac{\Delta v}{\Delta t}$	• Tangential acceleration (a_{tan}) is just the change in the ***magnitude*** of the velocity of the object in its circular path (not used very often) ○ $a_{tan} = \frac{\Delta v_{tan}}{\Delta t}$ • Centripetal or radial acceleration (a_c or a_R) is due to the change in ***direction*** of the object's velocity ○ Always directed toward the center of the circular path ○ $a_c = \frac{v^2}{r}$ • You could have tangential and centripetal acceleration going on at the same time, but that's too complicated so we don't do it.	• Average angular acceleration is defined as the change in angular velocity per unit time ○ $\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$ • If the acceleration is uniform, ○ $\alpha = \frac{\Delta\omega}{\Delta t}$ • Relationship between tangential acceleration (a_{tan}) and angular acceleration ○ $\mathrm{a_{tan} = r\alpha}$ • Relationship between centripetal or radial acceleration (a_c or a_R) and angular acceleration ○ $\mathrm{a_R = r\omega^2}$

KINEMATIC EQUATIONS *** Both sets of equations are only valid for constant (uniform) acceleration.**	
LINEAR	**ROTATIONAL**
• $v = v_0 + at$ • $x = v_0 t + 1/2\, at^2$ • $v^2 = v_0^2 + 2ax$ • $\bar{v} = \frac{v + v_0}{2}$	• $\omega = \omega_0 + \alpha t$ • $\theta = \omega_0 t + 1/2\, \alpha t^2$ • $\omega^2 = \omega_0^2 + 2\alpha\theta$ • $\bar{\omega} = \frac{\omega + \omega_0}{2}$

NEWTON'S SECOND LAW		
LINEAR	**CIRCULAR**	**ROTATIONAL**
• $\Sigma F = ma$	• $\Sigma F = ma_c = m\frac{v^2}{r}$	• Torque ○ Defined as force times moment arm ○ Also called the moment of the force ○ Only applies to the component of the force perpendicular to the moment arm ○ $\tau = rF \perp = rF \sin\theta$ • Second Law to Rotation ○ $\Sigma F = ma = mr\alpha$ ○ $\Sigma Fr = mr^2\alpha$ ○ $\Sigma\tau = (\Sigma mr^2)\alpha$ • Moment of Inertia (I) ○ Torque doesn't work for solid objects because they have mass at a continuous range of radii ○ Therefore we use moment of intertia to account for the mass and varying moment arms of the mass ○ $I = \Sigma mr^2$ ▪ The above equation only works for one or more point masses in a system ▪ For solid objects, it must be found using calculus, or (in our case) given • Rotational Equivalent of Newton's Second Law ○ $\Sigma\tau = I\alpha$

<table>
<tr><th colspan="3">KINETIC ENERGY
* Laws of conservation of energy apply to all three situations.</th></tr>
<tr><th>LINEAR</th><th>CIRCULAR</th><th>ROTATIONAL</th></tr>
<tr><td>• $KE = \frac{1}{2}mv^2$</td><td>• $KE = \frac{1}{2}mv^2$</td><td>• If an object is rotating in place (spinning) then it only has rotational kinetic energy
○ $KE = \frac{1}{2}I\omega^2$
• If an object has both rotational and translational motion (rolling), then it will have both rotational and translational kinetic energy
○ $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$</td></tr>
</table>

<table>
<tr><th colspan="2">MOMENTUM
* Laws of conservation of momentum apply to all three situations.</th></tr>
<tr><th>LINEAR</th><th>ROTATIONAL</th></tr>
<tr><td>• Momentum (p) is defined as mass times velocity
○ $p = mv$
• Momentum is conserved if the net force acting on the object is zero.
• If outside forces do act on the object, momentum changes.
• We can re-write the momentum equation in terms of force
○ $F = \frac{\Delta p}{\Delta t}$
○ $\Delta p = F\Delta t$
• Conservation of Momentum
○ $\mathrm{m_1v_1 + m_2v_2 = m_1v_1{}' + m_2v_2{}'}$</td><td>• In a manner similar to kinetic energy, angular momentum (L) is defined as the moment of inertia (I) instead of mass times the angular velocity (ω)
○ $L = I\omega$
• The total angular momentum of a rotating object remains constant if the net torque acting on it.
• If the net torque is not zero, there will be a change in rotational momentum.
• We can re-write the equation in terms of force, but just like in Newton's Second Law, we use torque (τ) instead of force
○ $\tau = \frac{\Delta L}{\Delta t}$
○ $\Delta L = \tau\Delta t$
• Conservation of Momentum
○ $I\omega = I'\omega'$</td></tr>
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