Accelerometers: Theory and Operation

Accelerometers measure accelerations by measuring forces. The vertical accelerometer in this kit consists of a cylindrical weight hung from a spring. Its operation can be understood in terms of Hooke's law: $F_s = -kx$

where F_s is the force applied by the spring to the weight, x is the extension of the spring, and k is a constant that depends on the spring. The negative sign indicates that the force is in the direction opposite to the extension. As the force applied by the spring to the weight increases, the stretch increases in direct proportion. Thus the position of the end of the spring indicates the amount of force being applied to the weight by the spring.

Calibration of the device can be in newtons for the spring force, or, in the ratio $\frac{F_s}{m} = a$

where a is an acceleration, since the mass of the cylinder remains constant for all uses. With the unstretched spring position taken as the zero point, the weight of a single cylinder defines the position corresponding to a restoring force which has magnitude equal to the weight of the cylinder, or $\frac{F_s}{m} = 9.8\frac{m}{s^2}$.

Note that if the device is calibrated in units of "g" instead of m/sec^2, it should be pointed out that the unit "g" used here is related to the local acceleration due to gravity only in that it has the same magnitude. Since the symbol "g" means local gravitational field strength, a reading of 2.0 g on an accelerometer does not mean that the gravitational field has increased. It means that the rider feels a force which is twice the magnitude of the rider's weight.

When the device is held vertically, the net force on the cylinder is given by: $F_{net} = F_s - mg$

where mg is the weight of the cylinder.

A diagram of the spring and weight is shown in Figure 1. When the accelerometer is held at rest (1a), the spring force is equal to the weight but in the opposite direction, so the net acceleration is zero.

$$F_{net} = 0 = F_s = mg$$
$$F_s = mg$$

and the scale reads "1g".

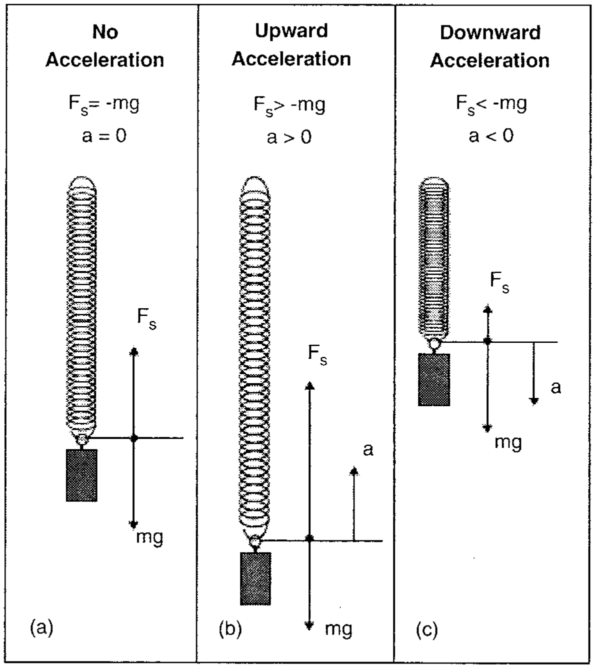

Figure 1: Diagram of the Vertical Accelerometer

All the spring is doing is supporting the weight of the cylinder. This is also true if the device is moving up or down with constant velocity.

If the weight is accelerating upward, the spring must exert not only the weight but enough additional upward force to provide the acceleration (1b). With F_s greater than mg, the net acceleration is greater than zero and upward. In this case, the spring will have stretched more than when at rest and the weight will be below the "1g" position.

If the weight is accelerating downward (1c), the spring must be applying less force than the weight It will have stretched less than when at rest and the cylinder will be above the "1g" position. In this case, the weight helps to accelerate the mass downward.

The device registers the acceleration as seen in the frame of reference of the rider. Consider the weight of the accelerometer to be a "plumb bob". Its direction response is the same as that of a plumb bob. In this case, the amount of stretch of the spring gives the weight of the cylinder in the combined gravitational and acceleration fields of the ride.

You cannot tell the gravitational field in one direction from an acceleration in the opposite direction. You cannot feel the difference between a force due to gravity and a force due to the ride pushing on you. The scale readings give what you feel is the local gravitational field. Since it registers the acceleration in the reference frame of the rider, the accelerometer readings agree with what the rider "feels."

A negative or downward acceleration occurs after the tops of roller coaster hills, when an elevator begins its downward trip, or when one begins to slide downhill. Riders have a sinking feeling because less force is being applied upward than they are accustomed to. On some rides the downward force is partly a push from the safety bar. This downward push feels as if the rider has suddenly become lighter and is rising out of the seat. Sure enough, the accelerometer reads less than one "g."

Upward or positive accelerations are felt in elevators as they begin to rise, and at the bottom of vertical loops on roller coasters and swings. As the elevator begins to rise, the floor must push up with a force greater than the rider's weight. The rider interprets this as an increase in downward force and feels heavier. The accelerometer spring, stretching to provide the additional force for the weight, registers more than one "g". Both the direction and magnitude of the readings agree with the rider's feeling of an altered gravitational field.

Upside down, at the top of a vertical circle such as a roller coaster loop or rotating ride, the rider may feel little if any force from the seat. The rider feels almost "weightless". At the same point the accelerometer shows little if any pull being applied by the spring. They are in agreement. At the bottom of the same loop the strong upward push from the seat feels like a force pushing the rider down into the ground. This upward force is applied to the cylinder by the spring which stretches strongly giving a large reading. In both cases, the rider sees the spring being pulled "down" toward the rider's seat, which conforms with what the rider feels.

The Horizontal Accelerometer

With horizontal accelerometers, as opposed to vertical accelerometers, there is not the same confusion between the subjective experience and the accelerometer reading. At rest, the BB's in the horizontal accelerometer settle to the bottom of the curved plastic tube. There is no horizontal force applied and no horizontal acceleration.

When the BB's are above the bottom, as in Figure 2, the inside of the curved plastic tube applies a force to them. The applied force has a vertical component equal to the weight of the BB's and a horizontal component equal to the mass of the BB's times their horizontal acceleration. The applied force acts along the line making the angle q with the vertical, center line of the accelerometer.

Since the components are perpendicular to one another and the horizontal force, ma, is opposite the angle θ:

$$\tan\theta = \frac{ma}{mg}$$

and

$$ma = mg \tan \theta.$$

We can divide both sides by the mass of the BB's to obtain:

$$a = g \tan \theta;$$

where a, the horizontal acceleration, is always directed forward toward the front of the device.

To measure the horizontal acceleration in the direction you are moving, just hold the accelerometer level with the straw pointed in the direction you are moving.

Multiply *g* by the tangent of the angle to the center of the BB.

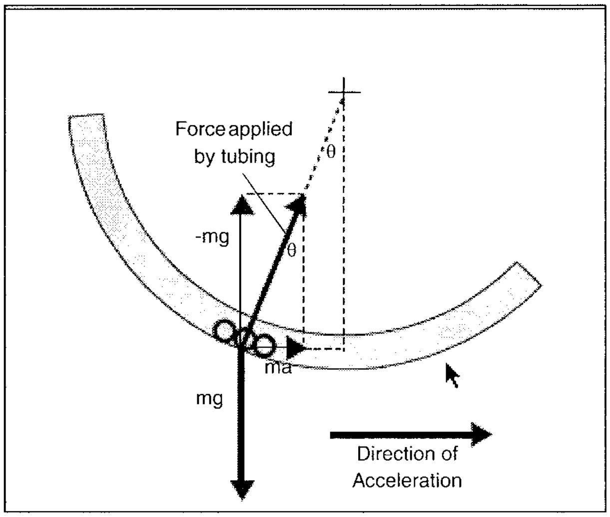

Figure 2: Diagram of the Horizontal Accelerometer

To use the horizontal accelerometer to measure horizontal centripetal accelerations, hold it perpendicular to the direction in which you are headed and as level as possible. For example, on the rotor ride at an amusement park, where you are in a rotating cylinder feeling mashed to the wall, hold the accelerometer with the short side pressed to the wall. It will be level with the floor and, since you are traveling sideways, perpendicular to the direction of travel.

Before the motion begins, the BBs sit in the bottom of the tube. When the ride begins to rotate, a centripetal force is needed to make them go in a circle. The BBs will ride up the side nearest the wall, as if forced outward. In fact, the tube will be exerting a horizontal force on them directed in toward the center of the ride. They will ride up until the angle is large enough to give the necessary horizontal acceleration. In circular motion

$$a = \frac{v^2}{r}$$

where *v* is the linear speed along the circumference and *r* is the radius of the circle. As the ride picks up speed, the BBs will travel farther up the curve.

Using the Horizontal Accelerometer as a Sextant

The horizontal accelerometer can be used to measure the heights of objects that are too high to measure directly, such as measuring the height from the ground to the top of King Kong's head, in Figure 3. You can measure these distances with reasonable accuracy using just the accelerometer, a piece of string that is marked out in meters, and a little trigonometry. The procedure is as follows:

1. Measure the distance S with a piece of string marked out in meters.

S is the horizontal distance between your point of observation and a point directly below the object of interest.

2. Sight through the straw to the top of King Kong's head, and measure the angle θ on the accelerometer.

θ is the angle that the center BB aligns with on the horizontal accelerometer. It is also the angle between your horizontal line of sight and your line of sight to the top of King Kong's head.

3. Measure h_0, the vertical distance between the base of your height measurement and your observation point.

As long as the ground is level between you and the building on which King Kong is standing, h_0 is just the distance from the ground to your eyes.

4. Then:

$$H = h_0 + h_1 = h_0 + S \tan \theta.$$

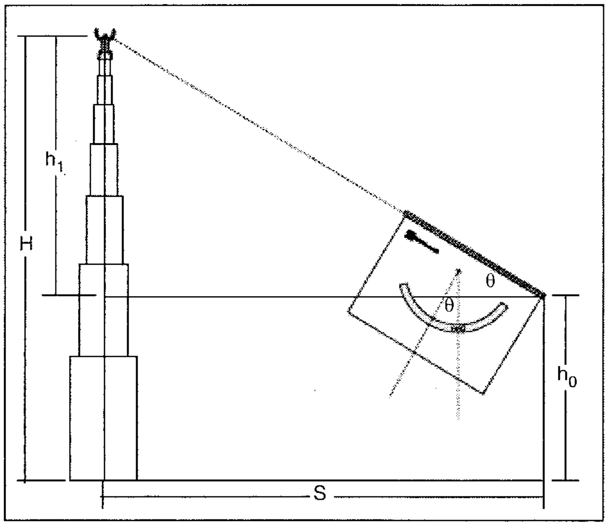

Figure 3: Measuring Heights with the Horizontal Accelerometer