## AP Physics

Name: $\qquad$

Period: $\qquad$ Date: $\qquad$

| AP EXAM |  | CHAPTER TEST |  |
| :---: | :---: | :---: | :---: |
| 50 Multiple Choice <br> - 45 Single Response <br> - 5 Multi-Response | $90 \mathrm{~min}, 1$ point each | 25 Multiple Choice <br> - 22 Single Response <br> - 3 Multi-Response | 45 min |
| Free Response <br> - 3 Short Free Response <br> - 2 Long Free Response | 90 min <br> - 13 min ea, 7 pts ea <br> - 25 min ea, 12 pts ea | Free Response <br> - 2 Short Free Response <br> - 1 Long Free Response | 45 min <br> - 12 min ea, 7 pts ea <br> - 20 min ea, 12 pts ea |

## CHAPTER 4 TEST REVIEW -- Answer Key

## MULTIPLE CHOICE

1. You are standing in a moving bus, facing forward, and you suddenly fall forward. You can imply from this that the bus's
a. velocity decreased
b. velocity increased
c. speed remained the same, but it's turning to the right
d. speed remained the same, but it's turning to the left
e. speed remained the same, but you have vertigo
2. A net force F acts on a mass m and produces an acceleration a . What acceleration results if a net force 2 F acts on mass 4 m ?
a. $\quad \mathrm{a} / 2$
b. 8 a
c. 4 a
d. 2 a
e. a
3. If you blow up a balloon, and then release it, the balloon will fly away. This is an illustration of
a. Newton's first law
b. Newton's second law
c. Newton's third law
d. Galileo's law of inertia
e. Ideal Gas law
4. Who has a greater weight-to-mass ratio, a person weighing 400 N or a person weighing 600 N ?
a. the person weighing 400 N
b. the person weighing 600 N
c. neither, their ratios are the same
d. the question can't be answered with the information given
e. the person eating the Fig Newtons
5. A person standing on a horizontal floor feels two forces: the downward pull of gravity and the upward supporting force from the floor. These two forces,
a. have equal magnitudes and form an action/reaction pair
b. have equal magnitudes and but do not form an action/reaction pair - forces only act on one object
c. have unequal magnitudes and form an action/reaction pair
d. have equal magnitudes and do not form an action/reaction pair
e. none of the above
6. If all of the forces acting on an object balance so that the net force is zero, then
a. the object must be at rest
b. the object's speed will decrease
c. the object will follow a parabolic trajectory
d. the object's direction of motion can change, but not its speed
e. None of the above

Change in direction means an acceleration which means an unbalanced force. If forces are balanced, it could be at rest but it could be moving at constant velocity.
7. A block of mass $\boldsymbol{m}$ is at rest on a frictionless, horizontal table placed in a laboratory on the surface of the earth. An identical block is at rest on a frictionless, horizontal table placed on the surface of the moon. Let $\boldsymbol{F}$ be the net force necessary to give the earth-bound block an acceleration of $\boldsymbol{a}$ across the table. Given that $\boldsymbol{g}_{\text {moon }}$ is one-sixth of $\boldsymbol{g}_{\text {earth }}$ and that air resistance is neglected, the force necessary to give the moon-bound block the same acceleration $\boldsymbol{a}$ across the table is
a. $\boldsymbol{F} / 12$
b. $\boldsymbol{F} / 6$
c. $\boldsymbol{F} / 3$
d. $\boldsymbol{F}$
e. $6 \boldsymbol{F}$

## If frictionless, gravity does not affect horizontal motion

8. A crate of mass 100 kg is at rest on a horizontal floor. The coefficient of static friction between the crate and the floor is 0.4 , and the coefficient of kinetic friction is 0.3 . A force $\boldsymbol{F}$ of magnitude 344 N is then applied to the crate, parallel to the floor. Which of the following is true?
a. the crate will accelerate across the floor at $0.5 \mathrm{~m} / \mathrm{s}^{2}$
b. the static friction force will also have a magnitude of 344 N
c. the crate will slide across the floor at a constant speed of $0.5 \mathrm{~m} / \mathrm{s}$
d. the crate will not move
e. none of the above

$$
F_{f}=F_{N} \mu_{s}=m g \mu_{s}=392 \mathrm{~N}
$$

Since the force of friction is greater than the applied force, the crate won't move
9. Two crates are stacked on top of each other on a horizontal floor; Crate \#1 is on the bottom and Crate \#2 is on the top. Both crates have the same mass. Compared to the strength of the force $\boldsymbol{F}_{\boldsymbol{I}}$ necessary to push Crate \#1 by itself at a constant speed, the strength of the force $\boldsymbol{F}_{2}$ necessary to push the two crates stacked together at constant speed is greater than $\boldsymbol{F}_{\boldsymbol{I}}$ because
a. the normal force on Crate \#1 is greater
b. the coefficient of kinetic friction between Crate \#1 and the floor is greater
c. the force of kinetic friction, but not the normal force, on Crate \#1 is greater
d. the coefficient of static friction between Crate \#1 and the floor is greater
e. the weight of Crate $\# 1$ is greater
10. The amount of force needed to keep a 0.2 kg hockey puck moving at a constant speed of $7 \mathrm{~m} / \mathrm{s}$ on frictionless ice is
a. zero
b. 0.2 N
c. 0.7 N
d. 7 N
e. 70 N

## Newton's First Law

## 11. Friction

a. can only occur between two surfaces which are moving relative to one another
b. is equal to the normal force divided by the coefficient of friction
c. opposes the relative motion between two surfaces in contact
d. only depends on one of the surfaces in contact
e. is always equal to the applied force
12. A person who weighs 800 N steps onto a scale that is on the floor of an elevator car. If the elevator accelerates upward at a rate of $5 \mathrm{~m} / \mathrm{s}^{2}$, what will the scale read? (use $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )
a. 400 N
b. 800 N
c. 1000 N
d. 1200 N
e. 1600 N

$$
\begin{gathered}
\sum F=m a, F_{R}-F_{g}=m a \\
F_{R}=m a+m g \\
m=\frac{800 \mathrm{~N}}{10 \mathrm{~m} / \mathrm{s}^{2}}=80 \mathrm{~kg} \\
F_{R}=(80 * 5)+(80 * 10)=1200 \mathrm{~N}
\end{gathered}
$$

13. A frictionless inclined plane has a slant length of 20 m and a maximum vertical height of 5 m . If an object of mass 2 kg is placed on the plane, which of the following best approximates the net force it feels? (use $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )
a. 5 N
b. 10 N
c. 15 N

d. 20 N
e. 30 N

$$
\begin{gathered}
\sin \theta=\frac{5}{20} \\
F_{g_{x}}=m g \sin \theta=(2)(10)(5 / 20)=5 \mathrm{~N}
\end{gathered}
$$

14. A 20 N block is being pushed across a horizontal table by an 18 N force. If the coefficient of kinetic friction between the block and the table is 0.4 , find the acceleration of the block. (use $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )
a. $0.5 \mathrm{~m} / \mathrm{s}^{2}$
b. $1 \mathrm{~m} / \mathrm{s}^{2}$
c. $5 \mathrm{~m} / \mathrm{s}^{2}$
d. $7.5 \mathrm{~m} / \mathrm{s}^{2}$
e. $9 \mathrm{~m} / \mathrm{s}^{2}$

$$
\begin{gathered}
\sum F=m a, F_{p}-F_{f}=m a \\
\frac{F_{p}-m g \mu}{m}=a \\
m g=20 \mathrm{~N}, m=20 \mathrm{~N} / g=2 \mathrm{~kg} \\
\frac{(18)-(20)(0.4)}{2}=a=5
\end{gathered}
$$

15. The coefficient of static friction between a box and a ramp is 0.5 . The ramp's incline angle is $30^{\circ}$. If the box is placed at rest on the ramp, the box will
a. accelerate down the ramp
b. accelerate briefly down the ramp but then slow down and stop
c. move with constant velocity down the ramp
d. not move
e. cannot be determined from the information given


$$
\begin{gathered}
\sum F=m a \\
F_{g-x}-F_{f}=m a
\end{gathered}
$$

$$
\begin{gathered}
F_{g-x}=m g \sin \theta=0.5 \mathrm{mg} \\
F_{f}=F_{N} \mu \\
F_{N}=F_{g-y}=m g \cos \theta \\
F_{f}=m g \cos \theta \mu=0.43 \mathrm{mg} \\
0.5 \mathrm{mg}-0.43 \mathrm{mg}=\mathrm{ma} \\
0.07 g=a=0.7 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$

16. Assuming the pulley above is frictionless and massless, determine the acceleration of the blocks once they are released from rest
a. $\frac{m}{M+m} g$
b. $\frac{M}{M+m} g$
c. $\frac{M}{m} g$
d. $\frac{M+m}{M-m} g$
e. $\frac{M-m}{M+m} g$
if we say acceleration is the downward direction is positive,

$$
\begin{gathered}
\sum F=m a \\
M_{g}-m_{g}=(M+m) a \\
(M-m) g=(M+m) a \\
\frac{(M-m)}{(M+m)} g=a
\end{gathered}
$$

17. A force of 26 N is needed to overcome a frictional force of 5 N to accelerate a 3 kg mass across a floor. What is the acceleration of the mass?
a. $4 \mathrm{~m} / \mathrm{s}^{2}$
b. $5 \mathrm{~m} / \mathrm{s}^{2}$
c. $7 \mathrm{~m} / \mathrm{s}^{2}$
d. $20 \mathrm{~m} / \mathrm{s}^{2}$
e. $60 \mathrm{~m} / \mathrm{s}^{2}$

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{p}}-\mathrm{F}_{\mathrm{f}}=\mathrm{ma} \\
& 26-5=(3) \mathrm{a}
\end{aligned}
$$

18. A force of 100 N directed at an angle of $45^{\circ}$ from the horizontal pulls a 70 kg sled across a frozen frictionless pond. The acceleration of the sled is most nearly
a. $1 \mathrm{~m} / \mathrm{s}^{2}$
b. $0.7 \mathrm{~m} / \mathrm{s}^{2}$
c. $7 \mathrm{~m} / \mathrm{s}^{2}$
d. $35 \mathrm{~m} / \mathrm{s}^{2}$
e. $50 \mathrm{~m} / \mathrm{s}^{2}$

$$
\begin{aligned}
\mathrm{F}_{\mathrm{p}-\mathrm{x}} & =\mathrm{F}_{\mathrm{p}} \cos \theta \\
\mathrm{~F}=\mathrm{ma}, \mathrm{a} & =\frac{\mathrm{F}}{\mathrm{~m}}=\frac{F_{p} \cos \theta}{m}
\end{aligned}
$$

19. Two blocks of mass $\boldsymbol{m}$ and $\mathbf{5 m}$ are connected by a light string which passes over a pulley of negligible mass and friction. What is the acceleration of the masses in terms of the acceleration due to gravity, g ?
a. 4 g
b. 5 g
c. 6 g
d. $4 / 5 \mathrm{~g}$
e. $2 / 3 \mathrm{~g}$

$$
\begin{gathered}
\text { Block } \mathrm{m}: F_{T}-m g=m a \\
F_{T}=m a+m g \\
\text { Block } 5 \mathrm{~m}: F_{T}-5 m g=-5 m a \\
F_{T}=5 m g-5 m a \\
m a+m g=5 m g-5 m a \\
6 a=4 g \\
a=\frac{4}{6} g=\frac{2}{3} g
\end{gathered}
$$

20. A $1-\mathrm{kg}$ block rests on a frictionless table and is connected by a light string to another block of mass 2 kg . The string is passed over a pulley of negligible mass and friction, with the 2 kg mass hanging vertically. What is the acceleration of the masses?
a. 5 g
b. 6.7 g
c. 10 g
d. 20 g
e. 30 g

$$
\begin{gathered}
\text { Block } \mathrm{m}_{1}: F_{T}=m_{1} a \\
\text { Block } \mathrm{m}_{2}: F_{T}-m_{2} g=-m_{2} a \\
F_{T}=m_{2} g-m_{2} a \\
m_{1} a=m_{2} g-m_{2} a \\
m_{1} a+m_{2} a=m_{2} g \\
\left(m_{1}+m_{2}\right) a=m_{2} g \\
a=\frac{\left(m_{2}\right)}{\left(m_{1}+m_{2}\right)} g=\frac{2}{3} g=6.7
\end{gathered}
$$

21. A $2-\mathrm{kg}$ wooden block rests on an inclined plane as shown below. The frictional force between the block and the plane is most nearly

a. 2 N
b. 10 N
c. 12 N
d. 17 N
e. 20 N

$$
\begin{gathered}
\sum F=m a=0 \\
F_{g-x}=F_{f} \\
F_{g-x}=m g \sin \theta=10 N=F_{f}
\end{gathered}
$$

22. A hockey puck with a mass of 0.3 kg is sliding along ice that can be considered frictionless. The puck's velocity is $20 \mathrm{~m} / \mathrm{s}$. The puck now crosses over onto a floor that has a coefficient of kinetic friction equal to 0.35 . How far will the puck travel across the floor before it stops?
a. 3 m
b. 87 m
c. 48 m
d. 92 m
e. 58 m

$$
\begin{gathered}
\sum F=m a \\
F_{f}=m a \\
F_{f}=F_{N} \mu_{k}=m g \mu_{k} \\
m g \mu_{k}=m a \\
a=g \mu_{k}=(9.81)(0.35)=3.43 \\
\mathcal{Z}^{z}=v_{0}^{2}-2 a x \\
2 a x=v_{0}^{2} \\
x=\frac{v_{0}^{2}}{2 a}=\frac{20^{2}}{2(3.43)}=58.3 m
\end{gathered}
$$

23. A $20-\mathrm{N}$ force is pushing two blocks horizontally along a frictionless floor as shown below


What is the force that the $8-\mathrm{kg}$ mass exerts on the $2-\mathrm{kg}$ mass?
a. 4 N
b. 8 N
c. 16 N
d. 20 N
e. 24 N

$$
\begin{gathered}
2 \mathrm{~kg} \text { block: } F-F_{R}=(2 \mathrm{~kg}) a \\
F-(2 \mathrm{~kg}) a=F_{R} \\
8 \mathrm{~kg} \text { block: } F_{R}=(8 \mathrm{~kg}) a \\
F_{R}=F_{R} \\
F-(2 \mathrm{~kg}) a=(8 \mathrm{~kg}) a \\
F=(8 \mathrm{~kg}) a+(2 \mathrm{~kg}) a \\
F=(10 \mathrm{~kg}) a \\
\frac{F}{10 \mathrm{~kg}}=a=2 \mathrm{~m} / \mathrm{s}^{2} \\
F_{R}=(8 \mathrm{~kg}) a=16 \mathrm{~N} \text { OR } F-(2 \mathrm{~kg}) a=F_{R}=16 \mathrm{~N}
\end{gathered}
$$

24. According to the diagram below, what is the tension in the connecting string if the table is frictionless?

a. $\quad 6.4 \mathrm{~N}$
b. 13 N
c. 20 N
d. 25 N
e. 32 N

$$
\begin{gathered}
2 \mathrm{~kg} \text { block: } F_{T}-(2 \mathrm{~kg}) g=(2 \mathrm{~kg}) a \\
F_{T}=(2 \mathrm{~kg}) a+(2 \mathrm{~kg}) g \\
4 \mathrm{~kg} \text { block: } F-F_{T}=(4 \mathrm{~kg}) a \\
F-(4 \mathrm{~kg}) a=F_{T} \\
F_{T}=F_{T} \\
F-(4 \mathrm{~kg}) a=(2 \mathrm{~kg}) a+(2 \mathrm{~kg}) g
\end{gathered}
$$

$$
\begin{gathered}
F=(4 \mathrm{~kg}) a+(2 \mathrm{~kg}) a+(2 \mathrm{~kg}) g \\
F=(6 \mathrm{~kg}) a+(2 \mathrm{~kg}) \mathrm{g} \\
\frac{F-(2 \mathrm{~kg}) \mathrm{g}}{6 \mathrm{~kg}}=a=0.0633 \mathrm{~m} / \mathrm{s}^{2} \\
F_{T}=(2 \mathrm{~kg}) a+(2 \mathrm{~kg}) g=19.7 \mathrm{~N} \text { OR } \\
F-(4 \mathrm{~kg}) a=F_{T}=19.7 \mathrm{~N}
\end{gathered}
$$

25. A mass $M$ is released from rest on an incline that makes a $42^{\circ}$ angle with the horizontal. In 3 s , the mass is observed to have gone a distance of 3 m . What is the coefficient of kinetic friction between the mass and the surface of the incline?
a. 0.8
b. 0.7
c. 0.6
d. 0.5
e. 0.3

$$
\begin{gathered}
\sum F=m a \\
F_{g-x}-F_{f}=m a \\
F_{g-x}=m g \sin \theta \\
F_{f}=F_{N} \mu=F_{g-y} \mu=m g \cos \theta \mu \\
x=\mathrm{x}_{\theta}+v_{\theta} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
\frac{2 x}{\mathrm{t}^{2}}=\mathrm{a}=0.67 \mathrm{~m} / \mathrm{s}^{2} \\
F_{g-x}-F_{f}=m a \\
F_{g-x}-m a=F_{f} \\
m g \sin \theta-m a=m g \cos \theta \mu \\
\frac{g \sin \theta-a}{g \cos \theta}=\mu=0.808
\end{gathered}
$$

## FREE RESPONSE

26. This question concerns the motion of a crate being pulled across a rough, horizontal floor by a rope. In the diagram below, the mass of the crate is $\boldsymbol{m}$, the coefficient of kinetic friction between the crate and the floor is $\boldsymbol{\mu}$, and the tension in the rope is $\boldsymbol{F}_{\boldsymbol{1}}$.

a. Draw and label (using the given variables) a free-body diagram showing all the forces acting on the crate.

b. Compute the normal force acting on the crate in terms of $\boldsymbol{m}, \boldsymbol{F}_{\boldsymbol{T}}, \boldsymbol{\theta}$, and $\boldsymbol{g}$.

$$
\begin{array}{ll}
\Sigma F=m a & \sin \theta=\frac{F_{T-y}}{F_{T}} \\
\underline{F_{N}+F_{T-y}-F_{g}=0} & F_{T} \sin \theta=F_{T-y} \\
\hline \boldsymbol{F}_{N}=F_{g}-F_{T-y} & F_{g}=m g \\
& F_{N}=m g-F_{T} \sin \theta \\
\hline
\end{array}
$$

c. Compute the acceleration force of the crate in terms of $\boldsymbol{m}, \boldsymbol{F}_{\boldsymbol{T}}, \boldsymbol{\theta}, \boldsymbol{\mu}$, and $\boldsymbol{g}$.

$$
\begin{array}{ll}
\sum F=m a & \cos \theta=\frac{F_{T-x}}{F_{T}} \\
\frac{F_{T-x}-F_{f}=m a}{} & F_{T} \cos \theta=F_{T-x} \\
\frac{F_{T-x}-F_{f}}{m}=a & F_{f}=F_{N} \mu=\left(m g-F_{T} \sin \theta\right) \mu \\
\hline & \frac{F_{T} \cos \theta-\left(m g-F_{T} \sin \theta\right) \mu}{m}=a
\end{array}
$$

27. In the diagram below, a massless string connects two blocks (masses $\boldsymbol{m}_{\boldsymbol{1}}$ and $\boldsymbol{m}_{\mathbf{2}}$ respectively) on a flat, frictionless tabletop. A force F pulls on Block \#2, as shown.

Block \#1 Block \#2

a. Draw and label (using the given variables) a free-body diagram showing all the forces acting on Block \#1.

b. Draw and label (using the given variables) a free-body diagram showing all the forces acting on Block \#2.

c. What is the acceleration of Block \#1 in terms of $\boldsymbol{F}, \boldsymbol{m}_{\boldsymbol{1}}$ and $\boldsymbol{m}_{2}$ ?

From the Block 1 FBD: $\quad F_{T}=\boldsymbol{F}_{T}$
$F_{T}=m_{1} a$
$\boldsymbol{F}-\boldsymbol{m}_{2} a=m_{1} a$
From the Block 2 FBD:
$\boldsymbol{F}=\boldsymbol{m}_{1} \boldsymbol{a}+\boldsymbol{m}_{2} \boldsymbol{a}$
$\underline{F-F_{T}}=\boldsymbol{m}_{2} \boldsymbol{a}$
$\boldsymbol{F}=\left(\boldsymbol{m}_{1}+\boldsymbol{m}_{2}\right) \boldsymbol{a}$
$\underline{\boldsymbol{F}-m_{2}} \boldsymbol{a}=\boldsymbol{F}_{T} \quad \frac{\boldsymbol{F}}{\left(m_{1}+m_{2}\right)}=\boldsymbol{a}$
d. What is the tension in the string connecting the two blocks in terms of $\boldsymbol{F}, \boldsymbol{m}_{1}$ and $\boldsymbol{m}_{2}$ ?

From the Block 1 FBD:
From the Block 2 FBD:
$\boldsymbol{F}_{T}=m_{1} a \quad \boldsymbol{F}-\boldsymbol{F}_{T}=\boldsymbol{m}_{2} a$
$\boldsymbol{F}_{T}=m_{1}\left(\frac{F}{\left(m_{1}+m_{2}\right)}\right) \quad \boldsymbol{F}-\boldsymbol{m}_{7} \boldsymbol{a}=\boldsymbol{F}_{T}$

$\frac{\left(m_{1}+m_{2}\right) F}{\left(m_{1}+m_{2}\right)}-\left(\frac{m_{2} F}{\left(m_{1}+m_{2}\right)}\right)=F_{T}$
$\frac{\left(m_{1}+m_{2}-m_{2}\right) F}{\left(m_{1}+m_{2}\right)}=F_{T}$
$\longrightarrow \frac{\left(m_{1}\right) \boldsymbol{F}}{\left(m_{1}+m_{2}\right)}=\boldsymbol{F}_{T}$
e. If the string connecting the two blocks were not massless, but instead had a mass of m, figure out
i. the acceleration of Block \#1

From the Block 1 FBD: $\quad \boldsymbol{F}_{T}=\boldsymbol{F}_{T}$
$\underline{F_{T}}=\boldsymbol{m}_{1} a+m a=\left(m_{1}+\boldsymbol{m}\right) a \quad \boldsymbol{F}-\boldsymbol{m}_{2} a=\left(\boldsymbol{m}_{1}+\boldsymbol{m}\right) a$
From the Block 2 FBD: $\quad F=\left(m_{1}+m\right) a+m_{2} a$
$\underline{F-F_{T}}=\boldsymbol{m}_{2} \boldsymbol{a} \quad \boldsymbol{F}=\left(\boldsymbol{m}_{1}+\boldsymbol{m}+\boldsymbol{m}_{2}\right) \boldsymbol{a}$

ii. The difference between the strength of the force that the connecting string exerts on Block \#2 and the strength of the force that the connecting string exerts on Block \#1.
From the Block 1 FBD: $\quad F_{T 2}-F_{T 1}=F-m_{2} a-m_{1} a$
$\boldsymbol{F}_{T 1}=\boldsymbol{m}_{1} a \quad \boldsymbol{F}_{T 2}-\boldsymbol{F}_{T 1}=\boldsymbol{F}-\left(\boldsymbol{m}_{2}+\boldsymbol{m}_{1}\right) \boldsymbol{a}$
From the Block 2 FBD: $\quad \boldsymbol{F}_{T 2}-\boldsymbol{F}_{T 1}=\boldsymbol{F}-\left(\boldsymbol{m}_{2}+\boldsymbol{m}_{1}\right)\left(\frac{\boldsymbol{F}}{\left(m_{1}+m_{1+} m_{2}\right)}\right)$
$\underline{\boldsymbol{F}-\boldsymbol{F}_{T 2}=\boldsymbol{m}_{2} \boldsymbol{a}} \quad \boldsymbol{F}_{T 2}-\boldsymbol{F}_{T 1}=\frac{\left(m_{1}+m+m_{2}\right)}{\left(m_{1}+m+m_{2}\right)} \boldsymbol{F}-\frac{\left(m_{2}+m_{1}\right)}{\left(m_{1}+m+m_{2}\right)} \boldsymbol{F}$
$\underline{\boldsymbol{F}-\boldsymbol{m}_{2}} \boldsymbol{a}=\boldsymbol{F}_{T 2} \quad \quad \boldsymbol{F}_{T 2}-\boldsymbol{F}_{T 1}=\frac{\left(\boldsymbol{m}_{1}\right)}{\left(m_{1}+m^{2}+m_{2}\right)} \boldsymbol{F}$
28. In the figure shown below, assume that the pulley is frictionless and massless.

a. Derive an equation for the acceleration of mass $\boldsymbol{m}_{\boldsymbol{I}}$.
define positive as going up the ramp and the hanging weight going downward
From the $m_{1}$ FBD: $\quad F_{T}=F_{T}$
$\underline{F_{T}-m_{1} g \sin \theta=m_{1} a \quad m_{1} a+m_{1} g \sin \theta=m_{2} g-m_{2} a}$
$\underline{F_{T}}=m_{1} a+m_{1} g \sin \theta \quad m_{1} a+m_{2} a=m_{2} g-m_{1} g \sin \theta$
From the $m_{2}$ FBD: $\quad\left(m_{1}+m_{2}\right) a=m_{2} g-m_{1} g \sin \theta$
$\underline{m_{2} g-F_{T}=m_{2} a} \quad a=\frac{m_{2} g-m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)}$
$\underline{\boldsymbol{m}_{2}} \boldsymbol{g}-\boldsymbol{m}_{2} a=F_{T}$
b. If the surface of the inclined plane is frictionless, determine what value(s) of $\theta$ will cause the box of mass to
i. accelerate up the ramp
define positive as going up the ramp and the hanging weight going downward
$\underline{\text { Given }=\frac{m_{2} g-m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)} \text {, the box will accelerate up the ramp if } a>0}$
Therefore $m_{2} g-m_{1} g \sin \theta>0 \quad m_{2} g>m_{1} g \sin \theta$
$\left[\frac{m_{2 \xi}}{m_{1 \xi}}>\sin \theta\right.$
$\left[\sin \theta<\frac{m_{2}}{m_{1}}\right.$

$$
\theta<\sin ^{-1} \frac{m_{2}}{m_{1}}
$$

ii. slide up the ramp at constant speed
define positive as going up the ramp and the hanging weight going downward
$\underline{\text { Given }=\frac{m_{2} g-m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)} \text {, the box will slide up the ramp at constant speed if } a=0}$
Therefore $m_{2} g-m_{1} g \sin \theta=0 \quad m_{2} g=m_{1} g \sin \theta$

| $\frac{m_{2} g}{m_{1} \theta}=\sin \theta$ |
| :--- |
| $\sin \theta=\frac{m_{2}}{m_{1}}$ |
| $\theta=\sin ^{-1} \frac{m_{2}}{m_{1}}$ |

$$
\begin{aligned}
& \frac{m_{2} g}{m_{1} \theta}=\sin \theta \\
& \sin \theta=\frac{m_{2}}{m_{1}} \\
& \theta=\sin ^{-1} \frac{m_{2}}{m_{1}}
\end{aligned}
$$

c. If the coefficient of kinetic friction between the surface of the inclined plane and the box of mass $\boldsymbol{m}_{\boldsymbol{I}}$ is $\boldsymbol{\mu}_{k}$, derive an equation for $\boldsymbol{a}$. Then, derive an equation in terms of $\boldsymbol{m}_{\boldsymbol{I}}, \boldsymbol{m}_{\boldsymbol{2}}, \boldsymbol{\theta}$, and $\boldsymbol{\mu}_{\boldsymbol{k}}$ which provides the condition for which the box of mass $\boldsymbol{m}_{\boldsymbol{I}}$ will slide up the ramp at constant speed.

define positive as going up the ramp and the hanging weight going downward
$\underline{F}_{g-x}=m g(\sin \theta), F_{g-y}=m g(\cos \theta), F_{f}=F_{N} \mu=F_{g-y} \mu=m_{1} g(\cos \theta) \mu$
From the $m_{1}$ FBD: $\quad F_{T}=F_{T}$
$F_{T}-m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu=m_{1} a \quad m_{1} a+m_{1} g \sin \theta+m_{1} g(\cos \theta) \mu=m_{2} g-m_{2} a$
$F_{T}=m_{1} a+m_{1} g \sin \theta+m_{1} g(\cos \theta) \mu \quad m_{1} a+m_{2} a=m_{2} g-m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu_{K}$
From the $m_{2}$ FBD: $\quad\left(m_{1}+m_{2}\right) a=m_{2} g-m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu_{K}$
$\underline{m_{2} g-F_{T}=m_{2} a \quad a=\frac{m_{2} g-m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu_{K}}{\left(m_{1}+m_{2}\right)}}$
$\underline{m}_{2} g-m_{2} a=F_{T} \quad$ For the box to travel up the ramp at constant speed

| $m_{2} g-m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu_{K}=0$ |
| :---: |
| $m_{2} g=m_{1} g \sin \theta-m_{1} g(\cos \theta) \mu_{K}$ |
| $m_{2} g=\left(m_{1} g\right)\left(\sin \theta-\cos \theta \mu_{K}\right)$ |
| $\frac{m_{2}}{m_{1}}=\sin \theta-\cos \theta \mu_{K}$ |

29. A block of mass $m$ rests on an air table (no friction), and is pulled with a force probe, producing the Force vs Acceleration graph shown below.

a. Determine the mass of the block.
$\underline{F}=m a$
$\frac{F}{a}=m$
slope $=\frac{F}{a}=\frac{(15-0)}{(5-0)}=3 \mathrm{~kg}$

The block is now placed on a rough horizontal surface having a coefficient of static friction $\mu_{s}=0.2$, and a coefficient of kinetic friction $\boldsymbol{\mu}_{\boldsymbol{k}}=0.1$.
b. What is the minimum value of the force $\boldsymbol{F}$ which will cause the block to just begin to move?

$$
F=F_{f}=m g \mu_{c}=(3)(10)(.2)=6 \mathrm{~N}
$$

c. After the block begins to move, the same force determined in part b. continues to act on the block. What is the acceleration of the block?
$\underline{F-F_{f_{k}}}=m a$
$F_{f}=m g \mu_{K}=(3)(10)(1)=3 N$
$\frac{F-F_{f_{k}}}{\mp m}=a=\frac{6-3}{3}=1 \mathrm{~m} / \mathrm{s}^{2}$
d. The force $\boldsymbol{F}$ is now tripled to $\mathbf{3 \boldsymbol { F }}$, which then pulls the block up an incline of angle $\boldsymbol{\theta}=20^{\circ}$ and having a coefficient of kinetic friction $\boldsymbol{\mu}_{\boldsymbol{k}}=0.1$.
i. Draw the free-body diagram for the block as it is being pulled up the incline.

ii. Determine the magnitude of the frictional force $\mathrm{F}_{\mathrm{f}}$ acting on the block as it slides up the incline. $\boldsymbol{F}_{f}=F_{N} \boldsymbol{\mu}=F_{g-y} \boldsymbol{\mu}=m g \cos \theta \mu=(3)(10)(\cos 20)(.1)=2.8 N$
iii. Determine the acceleration of the block as it is pulled up the incline.
$\underline{\Sigma} \boldsymbol{F}=\boldsymbol{m} \boldsymbol{a}$

$$
3 F-F_{f}-F_{g-x}=m a
$$

| $\underline{3 F-F_{f}}-F_{g-x}=m a$ | $\frac{3 F-F_{f}-F_{g-x}}{m}=a$ |
| :--- | :--- |
| $\boldsymbol{F}_{f}=F_{N} \mu=F_{g-y} \mu=m g \cos \theta \mu$ | $\frac{3(6)-(3)(10)(\cos 20)(0.1)-(3)(10)(\sin 20)}{3}=a=1.64 \mathrm{~N}$ |

$\underline{F}_{g-x}=m g \sin \theta$


